SAFE 2016
Design of Slabs, Beams and Foundations
Reinforced and Post-Tensioned Concrete

Reinforced Concrete Design
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References
SAFE automates several slab and mat design tasks. Specifically, it integrates slab design moments across design strips and designs the required reinforcement; it checks slab punching shear around column supports and concentrated loads; and it designs beam flexural, shear, and torsion reinforcement. The design procedures are outlined in the chapter entitled "SAFE Design Features" in the Key Features and Terminology manual. The actual design algorithms vary based on the specific design code chosen by the user. This manual describes the algorithms used for the various codes.

It should be noted that the design of reinforced concrete slabs is a complex subject and the design codes cover many aspects of this process. SAFE is a tool to help the user in this process. Only the aspects of design documented in this manual are automated by SAFE design capabilities. The user must check the results produced and address other aspects not covered by SAFE.
This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code ACI 318-08 [ACI 2008] is selected. Various notations used in this chapter are listed in Table 2-1. For referencing to the pertinent sections or equations of the ACI code in this chapter, a prefix “ACI” followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

## 2.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{cp}$</td>
<td>Area enclosed by the outside perimeter of the section, sq-in</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Gross area of concrete, sq-in</td>
</tr>
</tbody>
</table>
### Table 2-1 List of Symbols Used in the ACI 318-08 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>Area of longitudinal reinforcement for torsion, sq-in</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Area enclosed by the shear flow path, sq-in</td>
</tr>
<tr>
<td>$A_{oh}$</td>
<td>Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, sq-in</td>
</tr>
<tr>
<td>$A'_{s}$</td>
<td>Area of compression reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_t/s$</td>
<td>Area of closed shear reinforcement per unit length of member for torsion, sq-in/in</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Area of shear reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_v/s$</td>
<td>Area of shear reinforcement per unit length, sq-in/in</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, in</td>
</tr>
<tr>
<td>$a_{max}$</td>
<td>Maximum allowed depth of compression block, in</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of section, in</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Effective width of flange (flanged section), in</td>
</tr>
<tr>
<td>$b_o$</td>
<td>Perimeter of the punching shear critical section, in</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), in</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Width of the punching shear critical section in the direction of bending, in</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Width of the punching shear critical section perpendicular to the direction of bending, in</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, in</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, in</td>
</tr>
<tr>
<td>$d'$</td>
<td>Distance from compression face to compression reinforcement, in</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, psi</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, psi</td>
</tr>
<tr>
<td>$f'_{c}$</td>
<td>Specified compressive strength of concrete, psi</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, psi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Specified yield strength of flexural reinforcement, psi</td>
</tr>
<tr>
<td>$f_{st}$</td>
<td>Specified yield strength of shear reinforcement, psi</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section, in</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Height of the flange, in</td>
</tr>
<tr>
<td>$M_u$</td>
<td>Factored moment at a section, lb-in</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Factored axial load at a section occurring simultaneously with $V_u$ or $T_u$, lb</td>
</tr>
<tr>
<td>$P_u$</td>
<td>Factored axial load at a section, lb</td>
</tr>
<tr>
<td>$p_{cp}$</td>
<td>Outside perimeter of concrete cross-section, in</td>
</tr>
<tr>
<td>$p_h$</td>
<td>Perimeter of centerline of outermost closed transverse torsional reinforcement, in</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of shear reinforcement along the beam, in</td>
</tr>
<tr>
<td>$T_{cr}$</td>
<td>Critical torsion capacity, lb-in</td>
</tr>
<tr>
<td>$T_u$</td>
<td>Factored torsional moment at a section, lb-in</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear force resisted by concrete, lb</td>
</tr>
<tr>
<td>$V_{max}$</td>
<td>Maximum permitted total factored shear force at a section, lb</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Shear force resisted by transverse reinforcement, lb</td>
</tr>
<tr>
<td>$V_u$</td>
<td>Factored shear force at a section, lb</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>Punching shear scale factor based on column location</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching shear critical section</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Factor for obtaining depth of the concrete compression block</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in the concrete</td>
</tr>
<tr>
<td>$\varepsilon_{c\ max}$</td>
<td>Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)</td>
</tr>
</tbody>
</table>
Table 2-1 List of Symbols Used in the ACI 318-08 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in the reinforcement</td>
</tr>
<tr>
<td>$\varepsilon_{s,\text{min}}$</td>
<td>Minimum tensile strain allowed in the reinforcement at nominal strength for tension controlled behavior (0.005 in/in)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Strength reduction factor</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Fraction of unbalanced moment transferred by flexure</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Fraction of unbalanced moment transferred by eccentricity of shear</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shear strength reduction factor for light-weight concrete</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of compression diagonals, degrees</td>
</tr>
</tbody>
</table>

2.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-08, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (ACI 9.2.1):

- 1.4D (ACI 9-1)
- 1.2D + 1.6L (ACI 9-2)
- 1.2D + 1.6 (0.75 PL) (ACI 13.7.6.3, 9-2)
- 0.9D ± 1.6W (ACI 9-6)
- 1.2D + 1.0L ± 1.6W (ACI 9-4)
- 0.9D ± 1.0E (ACI 9-7)
- 1.2D + 1.0L ± 1.0E (ACI 9-5)
- 1.2D + 1.6L + 0.5S (ACI 9-2)
- 1.2D + 1.0L + 1.6S (ACI 9-3)
- 1.2D + 1.6S ± 0.8W (ACI 9-3)
- 1.2D + 1.0L + 0.5S ± 1.6W (ACI 9-4)
- 1.2D + 1.0L + 0.2S ± 1.0E (ACI 9-5)
These are the default design load combinations in SAFE whenever the ACI 318-08 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

### 2.3 Limits on Material Strength

The concrete compressive strength, \( f_{c}' \), should not be less than 2,500 psi (ACI 5.1.1). If the input \( f_{c}' \) is less than 2,500 psi, SAFE continues to design the members based on the input \( f_{c}' \) and does not warn the user about the violation of the code. The user is responsible for ensuring that the minimum strength is satisfied.

### 2.4 Strength Reduction Factors

The strength reduction factors, \( \phi \), are applied to the specified strength to obtain the design strength provided by a member. The \( \phi \) factors for flexure, shear, and torsion are as follows:

\[
\phi = 0.90 \text{ for flexure (tension controlled)} \\
\phi = 0.75 \text{ for shear and torsion}
\]

These values can be overwritten; however, caution is advised.

### 2.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:
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- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

2.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

2.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

2.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, $f_y$, is limited to 80 ksi (ACI 9.4), even if the material property is defined using a higher value.
The design procedure is based on the simplified rectangular stress block, as shown in Figure 2-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 10.3.4) when the concrete in compression reaches its assumed strain limit of 0.003. When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed \( (0.1f'_{c}A_g) \) (ACI 10.3.5), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than \( (0.1f'_{c}A_g) \) and axial tensions are always included in flexural and shear design.

2.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, \( M_u \) (i.e., designing top or bottom reinforcement), the depth of the compression block is given by \( a \) (see Figure 2-1), where,

\[
a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85f'_{c} \phi b}}
\]

(ACI 10.2)

and the value of \( \phi \) is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, \( c_{\text{max}} \), is calculated based on the limitation that the tension reinforcement strain shall not be less than \( \varepsilon_{\text{min}} \), which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

\[
c_{\text{max}} = \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}} + \varepsilon_{\text{min}}} d
\]

(ACI 10.2.2)
Figure 2-1 Rectangular Beam Design

where,

\[ \varepsilon_{c,\text{max}} = 0.003 \]  
\[ \varepsilon_{s,\text{min}} = 0.005 \]  

The maximum allowable depth of the rectangular compression block, \( a_{\text{max}} \), is given by:

\[ a_{\text{max}} = \beta_1 c_{\text{max}} \]  

where \( \beta_1 \) is calculated as:

\[ \beta_1 = 0.85 - 0.05 \left( \frac{f'_c - 4000}{1000} \right), \quad 0.65 \leq \beta_1 \leq 0.85 \]  

- If \( a \leq a_{\text{max}} \) (ACI 10.3.4), the area of tension reinforcement is then given by:
This reinforcement is to be placed at the bottom if $M_u$ is positive, or at the top if $M_u$ is negative.

- If $a > a_{\text{max}}$, compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

  - The compressive force developed in the concrete alone is given by:
    \[
    C = 0.85 f'_c b a_{\text{max}} \tag{ACI 10.2.7.1}
    \]
    and the moment resisted by concrete compression and tension reinforcement is:
    \[
    M_{uc} = \phi C \left( d - \frac{a_{\text{max}}}{2} \right) \tag{ACI 9.3.2.1}
    \]
    - Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:
      \[
      M_{us} = M_u - M_{uc}
      \]
    - The required compression reinforcement is given by:
      \[
      A'_s = \frac{M_{us}}{\phi (f'_s - 0.85 f'_c)(d - d')} , \text{ where}
      \]
      \[
      f'_s = E_s \epsilon_{c\text{max}} \left[ \frac{c_{\text{max}} - d'}{c_{\text{max}}} \right] \leq f_y \tag{ACI 10.2.2, 10.2.3, 10.2.4}
      \]
    - The required tension reinforcement for balancing the compression in the concrete is:
      \[
      A_{sl} = \frac{M_{uc}}{\phi f_y \left( d - \frac{a_{\text{max}}}{2} \right)}
      \]
and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A_{s2} = \frac{M_{us}}{\phi f_y (d - d')} \]

Therefore, the total tension reinforcement is \( A_s = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A' \). \( A_s \) is to be placed at the bottom and \( A' \) is to be placed at the top if \( M_u \) is positive, and vice versa if \( M_u \) is negative.

### 2.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 2-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 2-1 is assumed on the compression side.

![Figure 2-2 T-Beam Design](image)
2.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M_u$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

2.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85f'_c \phi b_f}}$$  \hspace{1cm} (ACI 10.2)

where, the value of $\phi$ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, $c_{\text{max}}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{\text{min}}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\text{max}} = \frac{\varepsilon_{\text{c max}}}{\varepsilon_{\text{c max}} + \varepsilon_{\text{s min}}}d$$  \hspace{1cm} (ACI 10.2.2)

where,

$$\varepsilon_{\text{max}} = 0.003$$  \hspace{1cm} (ACI 10.2.3)

$$\varepsilon_{\text{min}} = 0.005$$  \hspace{1cm} (ACI 10.3.4)

The maximum allowable depth of the rectangular compression block, $a_{\text{max}}$, is given by:

$$a_{\text{max}} = \beta_1 c_{\text{max}}$$  \hspace{1cm} (ACI 10.2.7.1)

where $\beta_1$ is calculated as:

$$\beta_1 = 0.85 - 0.05 \left( \frac{f'_c - 4000}{1000} \right), \hspace{1cm} 0.65 \leq \beta_1 \leq 0.85$$  \hspace{1cm} (ACI 10.2.7.3)
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- If \( a \leq h_f \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as \( b_f \). Compression reinforcement is required if \( a > a_{max} \).

- If \( a > h_f \), the calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \), as shown in Figure 2-2. \( C_f \) is given by:

\[
C_f = 0.85 f'_c \left( b_f - b_w \right) \min(h_f, a_{max})
\]  

(ACI 10.2.7.1)

Therefore, \( A_{s1} = \frac{C_f}{f_y} \) and the portion of \( M_u \) that is resisted by the flange is given by:

\[
M_{uf} = \phi C_f \left( d - \frac{\min(h_f, a_{max})}{2} \right)
\]  

(ACI 9.3.2.1)

Again, the value for \( \phi \) is 0.90 by default. Therefore, the balance of the moment, \( M_u \), to be carried by the web is:

\[
M_{uw} = M_u - M_{uf}
\]

The web is a rectangular section with dimensions \( b_w \) and \( d \), for which the design depth of the compression block is recalculated as:

\[
a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi b_w}}
\]  

(ACI 10.2)

- If \( a_1 \leq a_{max} \) (ACI 10.3.4), the area of tension reinforcement is then given by:

\[
A_{s2} = \frac{M_{uw}}{\phi f_y \left( d - \frac{a_1}{2} \right)}, \text{ and}
\]

\[
A_s = A_{s1} + A_{s2}
\]

This reinforcement is to be placed at the bottom of the flanged beam.
- If $a_t > a_{max}$, compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

  - The compressive force in the web concrete alone is given by:
    \[
    C_w = 0.85 f'_c b_w a_{max}
    \]
    \[
    (ACI 10.2.7.1)
    \]
    Therefore the moment resisted by the concrete web and tension reinforcement is:
    \[
    M_{uc} = C_w \left( d - \frac{a_{max}}{2} \right) \phi
    \]
    and the moment resisted by compression and tension reinforcement is:
    \[
    M_{us} = M_{uw} - M_{uc}
    \]

  Therefore, the compression reinforcement is computed as:
  \[
  A_t' = \frac{M_{us}}{(f'_s - 0.85 f'_c)(d - d') \phi}, \text{ where}
  \]
  \[
  f'_s = E_s \varepsilon_{c_{max}} \left[ \frac{c_{max} - d'}{c_{max}} \right] \leq f_y \]
  \[
  (ACI 10.2.2, 10.2.3, 10.2.4)
  \]

The tension reinforcement for balancing compression in the web concrete is:
  \[
  A_{s2} = \frac{M_{uc}}{f_y \left[ d - \frac{a_{max}}{2} \right] \phi}
  \]

and the tension reinforcement for balancing the compression reinforcement is:
  \[
  A_{s3} = \frac{M_{us}}{f_y (d - d') \phi}
  \]
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The total tension reinforcement is \( A_s = A_{s1} + A_{s2} + A_{s3} \), and the total compression reinforcement is \( A'_s \). \( A_s \) is to be placed at the bottom and \( A'_s \) is to be placed at the top.

### 2.5.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the two following limits:

\[
A_{s,\text{min}} = \max \left( \frac{3 \sqrt{f'_e}}{f_y} b w d, \frac{200}{f_y} b w d \right) \quad \text{(ACI 10.5.1)}
\]

\[
A_s \geq \frac{4}{3} A_{s(\text{required})} \quad \text{(ACI 10.5.3)}
\]

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

\[
A_s \leq \begin{cases} 
0.4bd & \text{Rectangular beam} \\
0.4b_w d & \text{Flanged beam} 
\end{cases}
\]

\[
A'_s \leq \begin{cases} 
0.4bd & \text{Rectangular beam} \\
0.4b_w d & \text{Flanged beam} 
\end{cases}
\]

### 2.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, \( V_u \).
- Determine the shear force, \( V_c \), that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.
2.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

2.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, \( V_c \), is calculated as:

\[
V_c = 2\lambda \sqrt{f'_c} b_n d
\]

(ACI 11.2.1.2, 11.2.1.2, 11.2.2.3)

A limit is imposed on the value of \( \sqrt{f'_c} \) as \( f'_c \leq 100 \) (ACI 11.1.2)

The value of \( \lambda \) should be specified in the material property definition.

2.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[
V_{\text{max}} = V_c + \left( 8 \sqrt{f'_c} \right) b_n d
\]

(ACI 11.4.7.9)

Given \( V_u \), \( V_c \), and \( V_{\text{max}} \), the required shear reinforcement is calculated as follows where, \( \phi \), the strength reduction factor, is 0.75 (ACI 9.3.2.3). The flexural reinforcement strength, \( f_{yt} \), is limited to 60 ksi (ACI 11.5.2) even if the material property is defined with a higher value.

- If \( V_u \leq 0.5 \phi V_c \),
  \[
  \frac{A_v}{s} = 0
  \]
  (ACI 11.5.6.1)

- If \( 0.5 \phi V_c < V_u \leq \phi V_{\text{max}} \),
  \[
  \frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{yt} d}
  \]
  (ACI 11.4.7.1, 11.4.7.2)

- If \( V_u > \phi V_{\text{max}} \), a failure condition is declared. (ACI 11.4.7.9)
If $V_u$ exceeds the maximum permitted value of $\phi V_{\text{max}}$, the concrete section should be increased in size (ACI 11.4.7.9).

Note that if torsion design is considered and torsion reinforcement is required, the equation given in ACI 11.5.6.3 does not need to be satisfied independently. See the subsequent section Design of Beam Torsion Reinforcement for details.

If the beam depth $h$ is

$$h \leq 10'' \text{ for rectangular,}$$

$$h \leq \min \left\{ 24'', \max \left( 2.5h_f, \frac{b}{2} \right) \right\} \text{ for T-beam,}$$

the minimum shear reinforcement given by ACI 11.4.6.3 is not enforced (ACI 11.4.6.1).

$$\frac{A_v}{s} \geq \max \left( \frac{0.75\lambda \sqrt{f_{\text{c}}} b_w}{f_{\text{y}}} \frac{50b_w}{f_{\text{y}}} \right) \quad \text{(ACI 11.4.6.3)}$$

The maximum of all of the calculated $A_v/s$ values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

2.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, $T_u$.
- Determine special section properties.
• Determine critical torsion capacity.
• Determine the torsion reinforcement required.

2.5.3.1 Determine Factored Torsion
In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 11.6.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design $T_u$ is permitted to be reduced in accordance with the code (ACI 11.6.2.2). However, the program does not automatically redistribute the internal forces and reduce $T_u$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

2.5.3.2 Determine Special Section Properties
For torsion design, special section properties, such as $A_{cp}$, $A_{oh}$, $A_o$, $p_{cp}$, and $p_h$, are calculated. These properties are described in the following (ACI 2.1).

\[
\begin{align*}
A_{cp} & = \text{Area enclosed by outside perimeter of concrete cross-section} \\
A_{oh} & = \text{Area enclosed by centerline of the outermost closed transverse torsional reinforcement} \\
A_o & = \text{Gross area enclosed by shear flow path} \\
p_{cp} & = \text{Outside perimeter of concrete cross-section} \\
p_h & = \text{Perimeter of centerline of outermost closed transverse torsional reinforcement}
\end{align*}
\]

In calculating the section properties involving reinforcement, such as $A_{oh}$, $A_o$, and $p_h$, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to a 1.5 inch clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion rein-
force calculation. However, the flange is considered during $T_{cr}$ calculation. With this assumption, the special properties for a rectangular beam section are given as:

\[
A_{cp} = bh 
\]

\[
A_{oh} = (b-2c)(h-2c) 
\]  

\[
A_o = 0.85 A_{oh} 
\]  

\[
p_{cp} = 2b + 2h 
\]  

\[
p_h = 2(b-2c) + 2(h-2c) 
\]

where, the section dimensions $b$, $h$, and $c$ are shown in Figure 2-3. Similarly, the special section properties for a flanged beam section are given as:

\[
A_{cp} = b_w h + (b_f - b_w) h_f 
\]

\[
A_{oh} = (b_w - 2c)(h - 2c) 
\]

\[
A_o = 0.85 A_{oh} 
\]

\[
p_{cp} = 2b_f + 2h 
\]

\[
p_h = 2(h - 2c) + 2(b_w - 2c) 
\]

where the section dimensions $b_f$, $b_w$, $h$, $h_f$, and $c$ for a flanged beam are shown in Figure 2-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h-h_f)$ (ACI 13.2.4).

### 2.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, $T_{cr}$, for which the torsion in the section can be ignored is calculated as:

\[
T_{cr} = \phi \lambda \sqrt{f'c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{N_u}{4A_g \lambda \sqrt{f'c}}} 
\]  

\[
(ACI 11.5.1(c)) 
\]

where $A_{cp}$ and $p_{cp}$ are the area and perimeter of the concrete cross-section as described in the previous section, $N_u$ is the factored axial force (compression
positive), $\phi$ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 9.3.2.3), and $f'_c$ is the specified concrete compressive strength.

### 2.5.3.4 Determine Torsion Reinforcement

If the factored torsion $T_u$ is less than the threshold limit, $T_{cr}$, torsion can be safely ignored (ACI 11.6.1). In that case, the program reports that no torsion reinforcement is required. However, if $T_u$ exceeds the threshold limit, $T_{cr}$, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI R11.6.3.6). Note that the longitudinal reinforcement strength, $f_y$, is limited to 80 ksi (ACI 9.4) and the transverse reinforcement strength, $f_{yt}$, is limited to 60 ksi, even if the material property is defined with a higher value.

If $T_u > T_{cr}$, the required closed stirrup area per unit spacing, $A_t/s$, is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan \theta}{\phi 2 A_o f_{ys}} \quad \text{(ACI 11.5.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{T_u p_h}{\phi 2 A_o f_y \tan \theta} \quad \text{(ACI 11.5.3.7, 11.5.3.6)}$$

where, the minimum value of $A_t/s$ is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{ys}} b_w \quad \text{(ACI 11.5.5.3)}$$

and the minimum value of $A_l$ is taken as:

$$A_l = 5 \lambda \sqrt{\frac{f'_c A_p}{f_y}} - \left( \frac{A_t}{s} \right) p_h \left( \frac{f_{yt}}{f_y} \right) \quad \text{(ACI 11.5.5.3)}$$

In the preceding expressions, $\theta$ is taken as 45 degrees. The code allows any value between 30 and 60 degrees (ACI 11.5.3.6).
An upper limit of the combination of $V_u$ and $T_u$ that can be carried by the section is also checked using the equation:

$$\left( \frac{V_u}{b_u \phi} \right)^2 + \left( \frac{T_u p_b}{1.7 A_{oh}} \right)^2 \leq \phi \left( \frac{V_c}{b_u \phi} + 8 \sqrt{f'_c} \right)$$

(ACI 11.5.3.1)

For rectangular sections, $b_u$ is replaced with $b$. If the combination of $V_u$ and $T_u$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left( \frac{A_v}{s} + 2 \frac{A_s}{s} \right) \geq \max \left\{ 0.75 \lambda \frac{\sqrt{f'_c}}{f_{yt}} b_{u}, \frac{50 b_{u}}{f_y} \right\}$$

(ACI 11.5.5.2)
If this equation is not satisfied with the originally calculated \(A_v/s\) and \(A_t/s\), \(A_v/s\) is increased to satisfy this condition. In that case, \(A_v/s\) does not need to satisfy the ACI Section 11.4.6.3 independently.

The maximum of all of the calculated \(A_t/s\) and \(A_v/s\) values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 2.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (ACI 318-08) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

#### 2.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

2.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

2.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

2.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2):

\[
A_{s,\text{min}} = 0.0020 \times bh \quad \text{for } f_y = 40 \text{ ksi or 50 ksi} \quad \text{(ACI 7.12.2.1(a))}
\]

\[
A_{s,\text{min}} = 0.0018 \times bh \quad \text{for } f_y = 60 \text{ ksi} \quad \text{(ACI 7.12.2.1(b))}
\]

\[
A_{s,\text{min}} = \frac{0.0018 \times 60000}{f_y} \times bh \quad \text{for } f_y > 60 \text{ ksi} \quad \text{(ACI 7.12.2.1(c))}
\]
In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

2.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following sections.

2.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (ACI 11.11.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.11.1.3). Figure 2-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 2-4 Punching Shear Perimeters](image)
2.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$.

\[
\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \quad \text{(ACI 13.5.3.2)}
\]

\[
\gamma_v = 1 - \gamma_f \quad \text{(ACI 13.5.3.1)}
\]

For flat plates, $\gamma_v$ is determined from the following equations taken from ACI 421.2R-07 Seismic Design of Punching Shear Reinforcement in Flat Plates [ACI 2007].

For interior columns,

\[
\gamma_{vx} = 1 - \frac{1}{1 + (2/3)\sqrt{l_y/l_x}} \quad \text{(ACI 421.2 C-11)}
\]

\[
\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x/l_y}} \quad \text{(ACI 421.2 C-12)}
\]

For edge columns,

$\gamma_{vx} = \text{same as for interior columns}$ \quad \text{(ACI 421.2 C-13)}

\[
\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x/l_y} - 0.2} \quad \text{(ACI 421.2 C-14)}
\]

$\gamma_{vy} = 0 \text{ when } l_x/l_y \leq 0.2$

For corner columns,

$\gamma_{vx} = 0.4$ \quad \text{(ACI 421.2 C-15)}

$\gamma_{vy} = \text{same as for edge columns}$ \quad \text{(ACI 421.2 C-16)}

**NOTE:** Program uses ACI 421.2-12 and ACI 421.2-15 equations in lieu of ACI 421.2 C-14 and ACI 421.2 C-16 which are currently NOT enforced.
where \( b_1 \) is the width of the critical section measured in the direction of the span and \( b_2 \) is the width of the critical section measured in the direction perpendicular to the span. The values \( l_x \) and \( l_y \) are the projections of the shear-critical section onto its principal axes, \( x \) and \( y \), respectively.

### 2.6.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

\[
\nu_c = \min \left\{ \phi \left( 2 + \frac{4}{\beta_c} \right) \lambda \sqrt{f'_c} \right., \left. \phi \left( 2 + \frac{\alpha_s d}{b_0} \right) \lambda \sqrt{f'_c} \right., \left. \phi 4 \lambda \sqrt{f'_c} \right\} \tag{ACI 11.11.2.1}
\]

where, \( \beta_c \) is the ratio of the maximum to the minimum dimensions of the critical section, \( b_0 \) is the perimeter of the critical section, and \( \alpha_s \) is a scale factor based on the location of the critical section.

\[
\alpha_s = \begin{cases} 
40 & \text{for interior columns,} \\
30 & \text{for edge columns,} \\
20 & \text{for corner columns.} 
\end{cases} \tag{ACI 11.11.2.1}
\]

A limit is imposed on the value of \( \sqrt{f'_c} \) as:

\[
\sqrt{f'_c} \leq 100 \tag{ACI 11.1.2}
\]

### 2.6.2.4 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.
\[ V_U = \frac{V_{ul}}{bd} + \gamma v^2 \left[ M_{u2} - V_{ul}(y_3 - y_1) \right] \left[ I_{22} (y_4 - y_3) - I_{23} (x_4 - x_3) \right] \]
\[ \frac{I_{22} I_{33} - I_{23}^2}{I_{22} I_{33} - I_{23}^2} \]

\[ I_{22} = \sum_{sides = 1}^{n} T_{22}, \text{ where "sides" refers to the sides of the critical section for punching shear} \]

\[ I_{33} = \sum_{sides = 1}^{n} T_{33}, \text{ where "sides" refers to the sides of the critical section for punching shear} \]

\[ I_{23} = \sum_{sides = 1}^{n} T_{23}, \text{ where "sides" refers to the sides of the critical section for punching shear} \]

The equations for \( T_{22}, T_{33}, \text{ and } T_{23} \) are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figure 2-5.

\[ T_{22} = Ld(y_2 - y_3)^2, \text{ for the side of the critical section parallel to the 2-axis} \]

\[ T_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2, \text{ for the side of the critical section parallel to the 3-axis} \]

\[ T_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2, \text{ for the side of the critical section parallel to the 2-axis} \]

\[ T_{33} = Ld(x_2 - x_3)^2, \text{ for the side of the critical section parallel} \]
for side of critical section parallel to 3-axis

$$T_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis}$$

Eq. 7

**NOTE:** $T_{23}$ is explicitly set to zero for corner condition.

where,

$b_0 = \text{Perimeter of the critical section for punching shear}$

$d = \text{Effective depth at the critical section for punching shear based on the average of } d \text{ for 2 direction and } d \text{ for 3 direction}$
2.6.2.5 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported.
as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

### 2.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 11.11.3). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

#### 2.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

\[ v_c \leq \phi 2\lambda \sqrt{f'_c} \]  
for shear links \hspace{1cm} (ACI 11.11.3.1)

\[ v_c \leq \phi 3\lambda \sqrt{f'_c} \]  
for shear studs \hspace{1cm} (ACI 11.11.5.1)

#### 2.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ V_{max} = 6 \sqrt{f'_c} b_d d \]  
for shear links \hspace{1cm} (ACI 11.11.3.2)

\[ V_{max} = 8 \sqrt{f'_c} b_d d \]  
for shear studs \hspace{1cm} (ACI 11.11.5.1)

Given \( V_u \), \( V_c \), and \( V_{max} \), the required shear reinforcement is calculated as follows, where, \( \phi \), the strength reduction factor, is 0.75 (ACI 9.3.2.3).
\[
\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d}
\]  
(ACI 11.4.7.1, 11.4.7.2)

\[
\frac{A_v}{s} \geq \frac{2f'c' b_o}{f_y}
\]  
for shear studs  
(ACI 11.11.5.1)

- If \( V_u > \phi V_{\max} \), a failure condition is declared.  
(ACI 11.11.3.2)

- If \( V_u \) exceeds the maximum permitted value of \( \phi V_{\max} \), the concrete section should be increased in size.

### 2.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 2-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

![Typical Studrail (only first and last studs shown) - Interior Column, Edge Column, Corner Column](image)

*Figure 2-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone*

The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \)(ACI R11.3.3, 11.11.5.2). The spacing between adjacent...
shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

### 2.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $2d$, and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

$$s_o \leq 0.5d \quad \text{(ACI 11.11.5.2)}$$

$$s \begin{cases} 0.75d & \text{for } \nu_u \leq 6\phi\lambda \sqrt{f'_{c}} \\ 0.50d & \text{for } \nu_u > 6\phi\lambda \sqrt{f'_{c}} \end{cases} \quad \text{(ACI 11.11.5.2)}$$

$$g \leq 2d \quad \text{(ACI 11.11.5.3)}$$

The limits of $s_o$ and the spacing, $s$, between for the links are specified as:

$$s_o \leq 0.5d \quad \text{(ACI 11.11.3)}$$

$$s \leq 0.50d \quad \text{(ACI 11.11.3)}$$
Chapter 3
Design for AS 3600-01

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Australian code AS 3600-2001 [AS 2001] is selected. Various notations used in this chapter are listed in Table 3-1. For referencing to the pertinent sections of the AS code in this chapter, a prefix “AS” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

3.1 Notations

<table>
<thead>
<tr>
<th>Table 3-1 List of Symbols Used in the AS 3600-2001 Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_g$</td>
</tr>
<tr>
<td>$A_l$</td>
</tr>
<tr>
<td>$A_t$</td>
</tr>
</tbody>
</table>
### Table 3-1 List of Symbols Used in the AS 3600-2001 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{sc}$</td>
<td>Area of compression reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{st}$</td>
<td>Area of tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{s}(\text{required})$</td>
<td>Area of required tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}$</td>
<td>Area of shear reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv,\text{min}}$</td>
<td>Minimum area of shear reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}/s$</td>
<td>Area of shear reinforcement per unit length, mm$^2$/mm</td>
</tr>
<tr>
<td>$A_{sv}/s$</td>
<td>Area of shear reinforcement per unit length consisting of closed ties, mm$^2$/mm</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Area of a polygon with vertices at the center of longitudinal bars at the corners of a section, mm$^2$</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$a_b$</td>
<td>Depth of compression block at balanced condition, mm</td>
</tr>
<tr>
<td>$a_{\text{max}}$</td>
<td>Maximum allowed depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of member, mm</td>
</tr>
<tr>
<td>$b_{ef}$</td>
<td>Effective width of flange (flanged section), mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Concrete cover to compression reinforcement, mm</td>
</tr>
<tr>
<td>$d_o$</td>
<td>Distance from the extreme compression fiber to the centroid of the outermost tension reinforcement, mm</td>
</tr>
<tr>
<td>$d_{om}$</td>
<td>Mean value of $d_o$, averaged around the critical shear perimeter, mm</td>
</tr>
<tr>
<td>$D$</td>
<td>Overall depth of a section, mm</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Thickness of slab (flanged section), mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, MPa</td>
</tr>
<tr>
<td>$f'_{c}$</td>
<td>Specified compressive strength of concrete, MPa</td>
</tr>
<tr>
<td>$f'_{cf}$</td>
<td>Characteristic flexural tensile strength of concrete, MPa</td>
</tr>
</tbody>
</table>
### Table 3-1 List of Symbols Used in the AS 3600-2001 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{cv}$</td>
<td>Concrete shear strength, MPa</td>
</tr>
<tr>
<td>$f_{sy}$</td>
<td>Specified yield strength of flexural reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{sy,f}$</td>
<td>Specified yield strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$f'_{s}$</td>
<td>Stress in the compression reinforcement, MPa</td>
</tr>
<tr>
<td>$J_t$</td>
<td>Torsional modulus, mm³</td>
</tr>
<tr>
<td>$k_u$</td>
<td>Ratio of the depth to the neutral axis from the compression face, to the effective depth, $d$</td>
</tr>
<tr>
<td>$M_{ud}$</td>
<td>Reduced ultimate strength in bending without axial force, N-mm</td>
</tr>
<tr>
<td>$M^*$</td>
<td>Factored moment at section, N-mm</td>
</tr>
<tr>
<td>$N^*$</td>
<td>Factored axial load at section, N</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of shear reinforcement along the beam, mm</td>
</tr>
<tr>
<td>$T_{uc}$</td>
<td>Torsional strength of section without torsional reinforcement, N-mm</td>
</tr>
<tr>
<td>$T_{u\text{,max}}$</td>
<td>Maximum permitted total factored torsion at a section, N-mm</td>
</tr>
<tr>
<td>$T_{us}$</td>
<td>Torsion strength of section with torsion reinforcement, N-mm</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Factored torsional moment at a section, N-mm</td>
</tr>
<tr>
<td>$u_t$</td>
<td>Perimeter of the polygon defined by $A_t$, mm</td>
</tr>
<tr>
<td>$V^*$</td>
<td>Factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_{u\text{,max}}$</td>
<td>Maximum permitted total factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_{u\text{,min}}$</td>
<td>Shear strength provided by minimum shear reinforcement, N</td>
</tr>
<tr>
<td>$V_{uc}$</td>
<td>Shear force resisted by concrete, N</td>
</tr>
<tr>
<td>$V_{us}$</td>
<td>Shear force resisted by reinforcement, N</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Factor for obtaining depth of compression block in concrete</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching critical section</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in concrete</td>
</tr>
<tr>
<td>$\varepsilon_{c\text{,max}}$</td>
<td>Maximum usable compression strain allowed in extreme concrete fiber, (0.003 mm/mm)</td>
</tr>
</tbody>
</table>
Table 3-1 List of Symbols Used in the AS 3600-2001 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in reinforcement</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Strength reduction factor</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>Angle of compression strut for torsion, degrees</td>
</tr>
<tr>
<td>$\theta_v$</td>
<td>Angle of compression strut for shear, degrees</td>
</tr>
</tbody>
</table>

3.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For AS 3600-01, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be defined (AS 3.3.1):

1.35D (AS/NZS 1170.0-02, 4.2.2(a))
1.2D + 1.5L (AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 1.5(0.75 PL) (AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 0.4L + 1.0S (AS/NZS 1170.0-02, 4.2.2(g))
0.9D ± 1.0W (AS/NZS 1170.0-02, 4.2.2(e))
1.2D ± 1.0W (AS/NZS 1170.0-02, 4.2.2(d))
1.2D + 0.4L ± 1.0W (AS/NZS 1170.0-02, 4.2.2(d))
1.0D ± 1.0E (AS/NZS 1170.0-02, 4.2.2(f))
1.0D + 0.4L ± 1.0E (AS/NZS 1170.0-02, 4.2.2(f))

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in SAFE whenever the AS 3600-2001 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.
3.3 Limits on Material Strength

The upper and lower limits of $f'_c$ are 65 MPa and 20 MPa, respectively, for all framing types (AS 6.1.1.1(b)).

\[
 f'_c \leq 65 \text{ MPa} \quad \text{(AS 6.1.1.1)}
\]

\[
 f'_c \geq 20 \text{ MPa} \quad \text{(AS 6.1.1.1)}
\]

The upper limit of $f_{sy}$ is 500 MPa for all frames (AS 6.2.1, Table 6.2.1).

The code allows use of $f'_c$ and $f_{sy}$ beyond the given limits, provided special care is taken regarding the detailing and ductility (AS 6.1.1, 6.2.1, 19.2.1.1).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

3.4 Strength Reduction Factors

The strength reduction factor, $\phi$, is defined as given in AS 2.3(c), Table 2.3:

\[
 \phi = 0.80 \text{ for flexure (tension controlled)} \quad \text{(AS 2.3(c))}
\]

\[
 \phi = 0.70 \text{ for shear and torsion} \quad \text{(AS 2.3(c))}
\]

These values can be overwritten; however, caution is advised.

3.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at each station along the length of the beam.
Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

### 3.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

#### 3.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

#### 3.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added
when the applied design moment exceeds the maximum moment capacity of a
singly reinforced section. The user has the option of avoiding compression rein-
forcement by increasing the effective depth, the width, or the strength of the
concrete. Note that the flexural reinforcement strength, $f_y$, is limited to 500MPa
(AS 6.2.1), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as
shown in Figure 3-1 (AS 8.1.2.2).

The following assumptions are used for the stress block used to compute the
flexural bending capacity of rectangular sections (AS 8.1.2.2).

- The maximum strain in the extreme compression fiber is taken as 0.003.
- A uniform compressive stress of $0.85f'_c$ acts on an area bounded by:
  - The edges of the cross-sections.
  - A line parallel to the neutral axis at the strength limit under the loading
    concerned, and located at a distance $\gamma k_u d$ from the extreme compression
    fiber.

The maximum allowable depth of the rectangular compression block, $a_{\text{max}}$, is
given by

$$a_{\text{max}} = \gamma k_u d \quad \text{where,}$$

$$\gamma = \left[0.85 - 0.007(f'_c - 28)\right]$$

$$0.65 \leq \gamma \leq 0.85 \quad \text{(AS 8.1.2.2)}$$

$$k_u = 0.4$$

The design procedure used by SAFE for both rectangular and flanged sections
(L- and T-beams) is summarized in the following subsections. It is assumed
that the design ultimate axial force does not exceed $(A_{uc}f'_c > 0.15N^*)$ (AS
10.7.1a); hence, all beams are designed for major direction flexure, shear, and
torsion only.
3.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, $M'$ (i.e., designing top or bottom reinforcement), the depth of the compression block is given by $a$ (see Figure 3-1), where,

$$a = d - \sqrt{d^2 - \frac{2|M'|}{0.85 f'_{c} \phi b}}$$

(AS 8.1.2.2)

where, the value of $\phi$ is taken as that for a tension controlled section ($k_u \leq 0.4$), which by default is 0.80 (AS 2.3) in the preceding and following equations.

- If $a \leq a_{\text{max}}$, the area of tension reinforcement is then given by:

$$A_{st} = \frac{M'}{\phi f_{xy} \left( d - \frac{a}{2} \right)}$$
This reinforcement is to be placed at the bottom if $M^*$ is positive, or at the top if $M^*$ is negative.

- If $a > a_{\text{max}}$, i.e., $k_u > 0.4$, compression reinforcement is required (AS 8.1.3) and is calculated as follows:

The compressive force developed in the concrete alone is given by:

$$ C = 0.85 f'_c b a_{\text{max}} $$  \hspace{1cm} (AS 8.1.2.2)

and the moment resisted by concrete compression and tension reinforcement is:

$$ M_{uc} = C \left( d - \frac{a_{\text{max}}}{2} \right) \phi $$

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$ M_{us} = M^* - M_{uc} $$

The required compression reinforcement is given by:

$$ A_{sc} = \frac{M_{us}}{(f' - 0.85 f'_c)(d - d')\phi}, \text{ where} $$

$$ f'_s = 0.003 E_s \left[ \frac{c - d'}{c} \right] \leq f_{sy} \hspace{1cm} (AS 8.1.2.1, 6.2.2) $$

The required tension reinforcement for balancing the compression in the concrete is:

$$ A_{st} = \frac{M_{uc}}{f_{sy} \left[ d - \frac{a_{\text{max}}}{2} \right] \phi} $$

and the tension reinforcement for balancing the compression reinforcement is given by:
Therefore, the total tension reinforcement is $A_{st} = A_{st1} + A_{st2}$, and the total compression reinforcement is $A_{sc}$. $A_{st}$ is to be placed at the bottom and $A_{sc}$ is to be placed at the top if $M^*$ is positive, and vice versa if $M^*$ is negative.

### 3.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 3-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 3-1 is assumed on the compression side (AS 8.1.3).

![Figure 3-2 T-Beam Design](image-url)
3.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M^*$ (i.e., designing top reinforcement), the calculation of the reinforcement is exactly the same as above, i.e., no flanged beam data is used.

3.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M^* > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85f'_c \phi b_f}}$$

where, the value of $\phi$ is taken as that for $k_u \leq 0.4$, which is 0.80 by default (AS 2.3) in the preceding and the following equations.

The maximum allowable depth of the rectangular compression block, $a_{\text{max}}$, is given by:

$$a_{\text{max}} = \gamma k_u d \text{ where, } k_u = 0.4$$  \hspace{1cm} (AS 8.1.3)

- If $a \leq D_s$, the subsequent calculations for $A_{st}$ are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as $b_f$. Compression reinforcement is required when $a > a_{\text{max}}$.

- If $a > D_s$, the calculation for $A_{st}$ has two parts. The first part is for balancing the compressive force from the flange, $C_f$, and the second part is for balancing the compressive force from the web, $C_w$, as shown in Figure 3-2. $C_f$ is given by:

$$C_f = 0.85 f'_c \left(b_{ef} - b_w\right) \times \min(D_s, a_{\text{max}})$$  \hspace{1cm} (AS 8.1.2.2)

Therefore, $A_{st} = \frac{C_f}{f_{sy}}$ and the portion of $M^*$ that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min(D_s, a_{\text{max}})}{2}\right)$$

Therefore, the balance of the moment, $M^*$ to be carried by the web is:
SAFE Reinforced Concrete Design

\[ M_{uw} = M' - M_{uf} \]

The web is a rectangular section of dimensions \( b_w \) and \( d \), for which the design depth of the compression block is recalculated as:

\[
a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi b_w}}
\]

- If \( a_1 \leq a_{\text{max}} \), the area of tension reinforcement is then given by:

\[
A_{s2} = \frac{M_{uw}}{\phi f_{sy} \left(d - \frac{a_1}{2}\right)}
\]

\[
A_{st} = A_{s1} + A_{s2}
\]

This reinforcement is to be placed at the bottom of the flanged beam.

- If \( a_1 > a_{\text{max}} \), compression reinforcement is required and is calculated as follows:

The compression force in the web concrete alone is given by:

\[
C_w = 0.85 f'_c b_w a_{\text{max}}
\]  

(AS 8.1.2.2)

Therefore the moment resisted by the concrete web and tension reinforcement is:

\[
M_{wc} = C_w \left(d - \frac{a_{\text{max}}}{2}\right) \phi
\]

and the moment resisted by compression and tension reinforcement is:

\[
M_{sw} = M_{uw} - M_{wc}
\]

Therefore, the compression reinforcement is computed as:
Chapter 3 - Design for AS 3600-01

\[ A_{sc} = \frac{M_{sc}}{(f'_s - 0.85f'_c)(d - d''\phi)}, \text{ where} \]

\[ f'_s = 0.003E_s \left[ \frac{c_{max}}{x_{max}} - d' \right] \leq f_{sy} \quad \text{(AS 8.1.2.1, 6.2.2)} \]

The tension reinforcement for balancing compression in the web concrete is:

\[ A_{s2} = \frac{M_{sc}}{f_{sy} \left[ d - a_{max} \right] \phi} \]

and the tension reinforcement for balancing the compression reinforcement is:

\[ A_{s3} = \frac{M_{sc}}{f_{sy} (d - d') \phi} \]

The total tensile reinforcement is \( A_{st} = A_{s1} + A_{s2} + A_{s3} \), and the total compression reinforcement is \( A_{sc} \). \( A_{st} \) is to be placed at the bottom and \( A_{sc} \) is to be placed at the top.

### 3.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following limit:

\[ A_{st,\text{min}} = 0.22 \left( \frac{D}{d} \right)^2 \frac{f'_d}{f_{sy}} bd, \text{ where} \quad \text{(AS 8.1.4.1)} \]

\[ f'_{cf} = 0.6\sqrt{f'_c} \quad \text{(AS 6.1.1.2)} \]

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:
3.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, $V^\ast$.
- Determine the shear force, $V_{uc}$, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

3.5.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

3.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_{uc}$, is calculated as:

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_w d_o \left[ \frac{A_d f'_c}{b_w d_o} \right]^{1/3}$$  \hspace{1cm} (AS 8.2.7.1)

where,

\[
A_s \leq \begin{cases} 
0.04bd & \text{Rectangular beam} \\
0.04b_w d & \text{Flanged beam} \\
0.04bd & \text{Rectangular beam} \\
0.04b_w d & \text{Flanged beam}
\end{cases}
\]
\[ \beta_1 = 1.1 \left( 1.6 - \frac{d_o}{1000} \right) \geq 1.1 \quad (AS \ 8.2.7.1) \]

\[ \beta_2 = 1, \text{ or } (AS \ 8.2.7.1) \]

\[ = 1 - \left( \frac{N^*}{3.5A_g} \right) \geq 0 \text{ for members subject to significant axial tension, or } \]

\[ = 1 + \left( \frac{N^*}{14A_g} \right) \text{ for members subject to significant axial compression.} \]

\[ \beta_3 = 1 \]

3.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to:

\[ V_{u_{\text{min}}} = V_{uc} + 0.6b_v d_o \quad (AS \ 8.2.9) \]

\[ V_{u_{\text{max}}} = 0.2f'_c bd_o \quad (AS \ 8.2.6) \]

Given \( V^* \), \( V_{uc} \), and \( V_{u_{\text{max}}} \), the required shear reinforcement is calculated as follows, where, \( \phi \), the strength reduction factor, is 0.6 by default (AS 2.3).

- If \( V^* \leq \phi V_{uc} / 2 \),

\[ \frac{A_{sv}}{s} = 0, \text{ if } D \leq 750 \text{ mm; otherwise } A_{sv_{\text{min}}} \text{ shall be provided. } \quad (AS \ 8.2.5). \]

- If \( (\phi V_{uc} / 2) < V^* \leq \phi V_{u_{\text{min}}} \),

\[ \frac{A_{sv}}{s} = 0, \text{ if } D < b_w / 2 \text{ or } 250 \text{ mm, whichever is greater (AS 8.2.5(c)(i)); otherwise } A_{sv_{\text{min}}} \text{ shall be provided.} \]

- If \( \phi V_{u_{\text{min}}} < V^* \leq \phi V_{u_{\text{max}}} \),
\[
\frac{A_{sv}}{s} = \frac{(V^{*} - \phi V_{uc})}{\phi f_{sy,f} d_{u} \cot \theta_{v}},
\]

(AS 8.2.10)

and greater than \( A_{sv: \text{min}} \), defined as:

\[
\frac{A_{sv: \text{min}}}{s} = \left( 0.35 \frac{b_{w}}{f_{sy,f}} \right)
\]

(AS 8.2.8)

\( \theta_{v} \) = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when \( V^{*} = \phi V_{uc: \text{min}} \) to 45 degrees when \( V^{*} = \phi V_{uc: \text{max}} \).

- If \( V^{*} > \phi V_{\text{max}} \), a failure condition is declared. (AS 8.2.6)
- If \( V^{*} \) exceeds its maximum permitted value \( \phi V_{\text{max}} \), the concrete section size should be increased (AS 8.2.6).

Note that if torsion design is considered and torsion reinforcement is required, the calculated shear reinforcement is ignored. Closed stirrups are designed for combined shear and torsion according to AS 8.3.4(b).

The maximum of all of the calculated \( A_{sv} / s \) values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

### 3.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, \( T^{*} \).
• Determine special section properties.
• Determine critical torsion capacity.
• Determine the torsion reinforcement required.

3.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design $T^*$ is permitted to be reduced in accordance with the code (AS 8.3.2). However, the program does not automatically redistribute the internal forces and reduce $T^*$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

3.5.3.2 Determine Special Section Properties

For torsion design, special section properties such as $A_t$, $J_t$, and $u_t$ are calculated. These properties are described in the following (AS 8.3).

- $A_t = \text{Area of a polygon with vertices at the center of longitudinal bars at the corners of the cross-section}$
- $u_t = \text{Perimeter of the polygon defined by } A_t$
- $J_t = \text{Torsional modulus}$

In calculating the section properties involving reinforcement, such as $A_{re}/s$ and $A_t$, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm-diameter stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during $T_{uc}$ calculation. With this assumption, the special properties for a rectangular beam section are given as:
where, the section dimensions $b$, $h$ and, $c$ are as shown in Figure 3-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_t = (b_w - 2c)(h - 2c),$$  \hspace{1cm} (AS 8.3.5)

$$u_t = 2(h - 2c) + 2(b_w - 2c),$$  \hspace{1cm} (AS 8.3.6)

$$J_t = 0.4\Sigma x^2y$$  \hspace{1cm} (AS 8.3.3)

where the section dimensions $b_w$, $h$, and $c$ for a flanged beam are as shown in Figure 3-3. The values $x$ and $y$ refer to the smaller and larger dimensions of a component rectangle, respectively.

\[ Figure\ 3-3\ \text{Closed\ stirrup\ and\ section\ dimensions\ for\ torsion\ design} \]
3.5.3.3 Determine Torsion Reinforcement

The torsional strength of the section without torsion reinforcement, $T_{uc}$, is calculated as:

$$T_{uc} = 0.3J_{t}\sqrt{f'_{c}}$$ \hspace{1cm} (AS 8.3.5)

where $J_{t}$ is the torsion modulus of the concrete cross-section as described in detail in the previous section.

Torsion reinforcement also can be ignored if any of the following is satisfied:

$$T^* \leq 0.25\phi T_{uc}$$ \hspace{1cm} (AS 8.3.4(a)(i))

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 0.5$$ \hspace{1cm} (AS 8.3.4(a)(ii))

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1 \text{ and } D \leq \max(250\text{mm}, b/2)$$ \hspace{1cm} (AS 8.3.4(a)(iii))

If the factored torsion $T^*$ alone or in combination with $V^*$ does not satisfy any of the three conditions in the preceding description, torsion reinforcement is needed. It is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (AS 8.3).

- If $T^* > T_{cr}$, the required closed stirrup area per unit spacing, $A_{sw}/s$, is calculated as:

$$A_{sw} = \frac{T^* \tan \theta_{t}}{s \phi 2f_{sy.f}A_{t}}$$ \hspace{1cm} (AS 8.3.5(b))

where, the minimum value of $A_{sw}/s$ is taken as follows:

$$\frac{A_{sw,\text{min}}}{s} = \frac{0.35b_{w}}{f_{sy.f}}$$ \hspace{1cm} (AS 8.2.8)

The value $\theta_{t}$ is the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $T^* = \phi T_{uc}$ to 45 degrees when $T^* = \phi T_{uc,\text{max}}$. 
The following equation shall also be satisfied for combined shear and torsion by adding additional shear stirrups.

\[
\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} \leq 1.0
\]  
(AS 8.3.4(b))

where,

\[
T_{us} = f_{sy.f} \left( \frac{A_{sw}}{s} \right) 2A_i \cot \theta_i
\]  
(AS 8.3.5(b))

\[
V_{us} = \left( A_{sv} f_{sy.f} \frac{d_o}{s} \right) \cot \theta_v
\]  
(AS 8.2.10(a))

The required longitudinal rebar area is calculated as:

\[
A_i = \frac{0.5 f_{sy.f} \left( \frac{A_{sw}}{s} \right) u_i \cot^2 \theta_i}{f_{sy}}
\]  
(AS 8.3.6(a))

An upper limit of the combination of \( V^* \) and \( T^* \) that can be carried by the section is also checked using the equation:

\[
\frac{T^*}{\phi T_{u,\text{max}}} + \frac{V^*}{\phi V_{u,\text{max}}} \leq 1.0
\]  
(AS 8.3.3)

where,

\[
V_{u,\text{max}} = 0.2 f'_c b_w d_o
\]  
(AS 8.2.6)

\[
T_{u,\text{max}} = 0.2 f'_t J_i
\]  
(AS 8.3.5(a))

For rectangular sections, \( b_w \) is replaced with \( b \). If the combination of \( V^* \) and \( T^* \) exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required \((T^* > T_{cr})\), the area of transverse closed stirrups and the area of regular shear stirrups satisfy the following limit.
\[
\left( \frac{A_{sv}}{s} + 2 \frac{A_{sw}}{s} \right) \geq \frac{0.35b}{f_{yv,f}}
\]  

(AS 8.3.7, 8.2.8)

If this equation is not satisfied with the originally calculated \( \frac{A_{sv}}{s} \) and \( \frac{A_{sw}}{s} \), \( \frac{A_{sv}}{s} \) is increased to satisfy this condition. In that case, \( \frac{A_{sv}}{s} \) does not need to satisfy AS Section 8.2.8 independently.

The maximum of all the calculated \( A_l \) and \( \frac{A_{sv}}{s} \) values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 3.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (AS 3600-2001) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

#### 3.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of
those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the following subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

3.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

3.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

3.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (AS 9.1.1):

\[ A_s \geq 0.0025 \, bh \] for flat slabs  
\[ A_s \geq 0.0020 \, bh \] for slabs supported by beams/walls and slab footings
In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

### 3.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

#### 3.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of \(d_{om}/2\) from the face of the support (AS 9.2.1.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (AS 9.2.1.3). Figure 3-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Critical Section for punching shear](image-url)
3.6.2.2 Determine Concrete Capacity

The shear capacity, $f_{cv}$, is calculated based on the minimum of the two expressions from AS 3600-01 equation 11-35, as shown, with the $d_{om}$ and $u$ terms removed to convert force to stress.

$$f_{cv} = \min \left\{ 0.17 \left(1 + \frac{2}{\beta_h} \right) \sqrt{f'_c}, 0.34 \sqrt{f'_c} \right\}$$

(AS 9.2.3(a))

where, $\beta_h$ is the ratio of the longest dimension to the shortest dimension of the critical section.

3.6.2.3 Determine Maximum Shear Stress

The maximum design shear stress is computed along the major and minor axis of column separately using the following equation:

$$v_{max} = \frac{V^*}{ud_{om}} \left[ 1.0 + \frac{uM_v}{8V^*a_d_{om}} \right]$$

(AS 9.2.4(a))

3.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

3.6.3 Design Punching Shear Reinforcement

The design guidelines for shear links or shear studs are not available in AS 3600-2001. SAFE uses the NZS 3101-06 guidelines to design shear studs or shear links.
The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

### 3.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

### 3.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ V_{\text{max}} = 3 \ V_{u,\text{min}} = 3^* V_u \]

(AS 92.2.4(a), (d))

where \( V_u \) is computed from AS 9.2.3 or 9.2.4. Given \( V^* \), \( V_u \), and \( V_{u,\text{max}} \), the required shear reinforcement is calculated as follows, where, \( \phi \) is the strength reduction factor.

\[
\frac{A_{sv}}{s} = \frac{\left( V^* - \phi V_u \right)}{f_{sv} a_{om}},
\]

(AS 8.2.10)

Minimum punching shear reinforcement should be provided such that:

\[ V_s \geq \frac{1}{16} \sqrt{f_{sv} d_{om} u} \]

(NZS 12.7.4.3)

- If \( V^* > \phi V_{\text{max}} \), a failure condition is declared.

- If \( V^* \) exceeds the maximum permitted value of \( \phi V_{\text{max}} \), the concrete section should be increased in size.
3.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 3-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (NZS 12.7.4.4).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

![Figure 3-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone](image)

3.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should
not be less than the minimum cover specified in NZS 3.11 plus half of the
diameter of the flexural reinforcement.

When specifying shear studs, the distance, \( s_o \), between the column face and the
first peripheral line of shear studs should not be smaller than \( 0.5d \). The spacing
between adjacent shear studs, \( g \), at the first peripheral line of studs shall not ex-
ceed \( 2d \) and in the case of studs in a radial pattern, the angle between adjacent
stud rails shall not exceed 60 degrees. The limits of \( s_o \) and the spacing, \( s \), be-
tween the peripheral lines are specified as:

\[
\begin{align*}
  s_o & \leq 0.5d \\
  s & \leq 0.5d \\
  g & \leq 2d
\end{align*}
\]

(NZS 12.7.4.4)
Chapter 4
Design for BS 8110-97

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the British code BS 8110-1997 [BSI 1997] is selected. For light-weight concrete and torsion, reference is made to BS 8110-2:1985 [BSI 1985]. Various notations used in this chapter are listed in Table 4-1. For referencing to the pertinent sections of the British code BS 8110-1997 in this chapter, a prefix “BS” followed by the section number is used.

The design is based on user-specified loading combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

4.1 Notations

<table>
<thead>
<tr>
<th>Table 4-1 List of Symbols Used in the BS 8110-1997 Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_g$</td>
</tr>
<tr>
<td>Symbol</td>
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<tr>
<td>$A_t$</td>
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<tr>
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<td>$h$</td>
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### Table 4-1 List of Symbols Used in the BS 8110-1997 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_f$</td>
<td>Flange thickness, mm</td>
</tr>
<tr>
<td>$h_{min}$</td>
<td>Smaller dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$h_{max}$</td>
<td>Larger dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$K$</td>
<td>Normalized design moment, $\frac{M_u}{bd^2 f_{cu}}$</td>
</tr>
<tr>
<td>$K'$</td>
<td>Maximum $\frac{M_u}{bd^2 f_{cu}}$ for a singly reinforced concrete section, taken as 0.156 by assuming that moment redistribution is limited to 10%.</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Shear strength enhancement factor for support compression</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Concrete shear strength factor, $\left[\frac{f_{cu}}{25}\right]^{1/3}$</td>
</tr>
<tr>
<td>$k_3$</td>
<td>Shear strength reduction factor for light-weight concrete</td>
</tr>
<tr>
<td>$M$</td>
<td>Design moment at a section, N-mm</td>
</tr>
<tr>
<td>$M_{single}$</td>
<td>Limiting moment capacity as singly reinforced beam, N-mm</td>
</tr>
<tr>
<td>$s_v$</td>
<td>Spacing of the links along the length of the beam, mm</td>
</tr>
<tr>
<td>$T$</td>
<td>Design torsion at ultimate design load, N-mm</td>
</tr>
<tr>
<td>$u$</td>
<td>Perimeter of the punching critical section, mm</td>
</tr>
<tr>
<td>$V$</td>
<td>Design shear force at ultimate design load, N</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Design shear stress at a beam cross-section or at a punching critical section, MPa</td>
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<tr>
<td>$\nu_c$</td>
<td>Design concrete shear stress capacity, MPa</td>
</tr>
<tr>
<td>$\nu_{max}$</td>
<td>Maximum permitted design factored shear stress, MPa</td>
</tr>
</tbody>
</table>
4.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, \( \gamma_f \) (BS 2.4.1.3). For BS 8110-1997, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered (BS 2.4.3).

\[
\begin{align*}
1.4D & \\
1.4D + 1.6L & \\
1.4D + 1.6(0.75PL) & \\
1.0D \pm 1.4W & \\
1.4D \pm 1.4W & \\
1.2D + 1.2L \pm 1.2W & (BS 2.4.3)
\end{align*}
\]
1.4D + 1.6L + 1.6S
1.2D + 1.2S ± 1.2W
1.2D + 1.2L + 1.2S ± 1.2W

(BS 2.4.3)

These are also the default design load combinations in SAFE whenever the BS 8110-1997 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

4.3 Limits on Material Strength

The concrete compressive strength, \( f_{cu} \), should not be less than 25 MPa (BS 3.1.7.2). SAFE does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsibility to use the proper strength values while defining the materials.

4.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, \( \gamma_m \). The values of \( \gamma_m \) used in the program are listed in the following table, as taken from BS Table 2.2 (BS 2.4.4.1):

<table>
<thead>
<tr>
<th>Values of ( \gamma_m ) for the Ultimate Limit State</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforcement</td>
<td>1.15</td>
</tr>
<tr>
<td>Concrete in flexure and axial load</td>
<td>1.50</td>
</tr>
<tr>
<td>Concrete shear strength without shear reinforcement</td>
<td>1.25</td>
</tr>
</tbody>
</table>

These factors are already incorporated into the design equations and tables in the code. Note that for reinforcement, the default factor of 1.15 is for Grade 500 reinforcement. If other grades are used, this value should be overwritten as necessary. Changes to the partial safety factors are carried through the design equations where necessary, typically affecting the material strength portions of the equations.
4.5 **Beam Design**

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

### 4.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

#### 4.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top re-
inforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

4.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 4-1 (BS 3.4.4.4). Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., \( \beta \geq 0.9 \); BS 3.4.4.4). The code also places a limitation on the neutral axis depth, \( x/d \leq 0.5 \), to safeguard against non-ductile failures (BS 3.4.4.4). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams) is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed \( (0.1f_{cu}A_g) \) (BS 3.4.4.1); hence, all beams are designed for major direction flexure, shear, and torsion only.

4.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam, \( M_{\text{single}} \), is first calculated for a section. The reinforcement is determined based on \( M \) being greater than, less than, or equal to \( M_{\text{single}} \). See Figure 4-1.

- Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

\[
M_{\text{single}} = K' f_{cu} b d^2, \quad \text{where} \quad (\text{BS 3.4.4.4})
\]

\[
K' = 0.156
\]
If $M \leq M_{\text{single}}$, the area of tension reinforcement, $A_s$, is given by:

$$A_s = \frac{M}{0.87f_yz},$$

where

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right) \leq 0.95d$$

This reinforcement is to be placed at the bottom if $M$ is positive, or at the top if $M$ is negative.

If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$K = \frac{M}{f_{cs}bd^2}$$


\[ A_s' = \frac{M - M_{\text{single}}}{f'_{s} - \frac{0.67f_{cu}}{\gamma_c}}(d - d) \]  

(BS 3.4.4.4)

where \(d'\) is the depth of the compression reinforcement from the concrete compression face, and

\[ f'_{s} = 0.87f_y \text{ if } \frac{d'}{d} \leq \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \]  

(BS 3.4.4.1, 2.5.3, Fig 2.2)

\[ f'_{s} = E_s\epsilon_c\left[1 - \frac{2d'}{d}\right] \text{ if } \frac{d'}{d} > \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \]  

(BS 3.4.4.1, 2.5.3, Fig 2.2)

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

\[ A_s = \frac{M_{\text{single}}}{0.87f_yz} + \frac{M - M_{\text{single}}}{0.87f_y(d - d)}, \text{ where} \]  

(BS 3.4.4.4)

\[ z = d\left(0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right) = 0.777d \]  

(BS 3.4.4.4)

### 4.5.1.2.2 Design of Flanged Beams

#### 4.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, \(M\) (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

#### 4.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located in the flange. Based on this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width \(b_f\).
If the stress block extends beyond the flange depth, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 4-2.

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

$$K = \frac{M}{f_{cu} b_f d^2} \quad \text{(BS 3.4.4.4)}$$

Then the moment arm is computed as:

$$z = d \left\{ 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right\} \leq 0.95d \quad \text{(BS 3.4.4.4)}$$

the depth of the neutral axis is computed as:

$$x = \frac{1}{0.45} (d - z) \quad \text{(BS 3.4.4.4)}$$

and the depth of the compression block is given by:
\[ a = 0.9x \]  

- If \( a \leq h_f \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( K > K' \).

- If \( a > h_f \), when \( M \leq \beta f_c a b d^2 \) and \( h_f \leq 0.45d \), then

\[
A_s = \frac{M + 0.1 f_c a b d \left(0.45d - h_f \right)}{0.87 f_y \left(d - 0.5h_f \right)}, \text{ where} \tag{BS 3.4.4.5}
\]

\[
\beta = 0.45 \frac{h_f}{d} \left(1 - \frac{b_w}{b} \right) \left(1 - \frac{h_f}{2d} \right) + 0.15 \frac{b_w}{b} \tag{BS 3.4.4.5}
\]

Otherwise the calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \), as shown in Figure 4-2.

In that case, the ultimate resistance moment of the flange is given by:

\[
M_f = 0.45 f_c \left(b_f - b_w \right) h_f \left(d - 0.5h_f \right) \tag{BS 3.4.4.5}
\]

The moment taken by the web is computed as:

\[
M_w = M - M_f
\]

and the normalized moment resisted by the web is given by:

\[
K_w = \frac{M_w}{f_c b_w d^2} \tag{BS 3.4.4.4}
\]

- If \( K_w \leq 0.156 \) (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

\[
A_s = \frac{M_f}{0.87 f_y \left(d - 0.5h_f \right)} + \frac{M_w}{0.87 f_y z}, \text{ where}
\]
\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K_w}{0.9}} \right) \leq 0.95d \]

- If \( K_w > K' \) (BS 3.4.4.4), compression reinforcement is required and is calculated as follows:

The ultimate moment of resistance of the web only is given by:

\[ M_{uw} = K' f_{cu} b_w d^2 \quad (\text{BS 3.4.4.4}) \]

The compression reinforcement is required to resist a moment of magnitude \( M_w - M_{uw} \). The compression reinforcement is computed as:

\[ A'_s = \frac{M_w - M_{uw}}{f'_s - \frac{0.67 f_{cu}}{\gamma_c}} \left( d - d' \right) \quad (\text{BS 3.4.4.4}) \]

where, \( d' \) is the depth of the compression reinforcement from the concrete compression face, and

\[ f'_s = 0.87 f_y \quad \text{if} \quad \frac{d'}{d} \leq \frac{1}{2} \left[ 1 - \frac{f_y}{800} \right] \quad (\text{BS 3.4.4.1, 2.5.3, Fig 2.2}) \]

\[ f'_s = E_s \varepsilon_c \left[ 1 - \frac{2d'}{d} \right] \quad \text{if} \quad \frac{d'}{d} > \frac{1}{2} \left[ 1 - \frac{f_y}{800} \right] \quad (\text{BS 3.4.4.1, 2.5.3, Fig 2.2}) \]

The area of tension reinforcement is obtained from equilibrium as:

\[ A_y = \frac{M_f}{0.87 f_y \left( d - 0.5 h_f \right)} + \frac{M_{uw}}{0.87 f_y (0.777d)} + \frac{M_w - M_{uw}}{0.87 f_y \left( d - d' \right)} \]

### 4.5.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from BS Table 3.25 (BS 3.12.5.3) with interpolation for reinforcement of intermediate strength.
### Section 4 - Design for BS 8110-97

#### Definition of percentage

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage $f_y = 250$ MPa</th>
<th>$f_y = 500$ MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td></td>
<td>$100 \frac{A_s}{bh}$</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>T- or L-Beam with web in tension</td>
<td>$b_w/b_f &lt; 0.4$</td>
<td>$100 \frac{A_s}{b_w h}$</td>
<td>0.32</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$b_w/b_f \geq 0.4$</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>T-Beam with web in compression</td>
<td></td>
<td>$100 \frac{A_s}{b_w h}$</td>
<td>0.48</td>
<td>0.26</td>
</tr>
<tr>
<td>L-Beam with web in compression</td>
<td></td>
<td>$100 \frac{A_s}{b_w h}$</td>
<td>0.36</td>
<td>0.20</td>
</tr>
</tbody>
</table>

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from BS Table 3.25 (BS 3.12.5.3).

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td></td>
<td>$100 \frac{A_s}{bh}$</td>
<td>0.20</td>
</tr>
<tr>
<td>T- or L-Beam</td>
<td>Web in tension</td>
<td>$100 \frac{A_s}{b_f h_f}$</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>Web in compression</td>
<td>$100 \frac{A_s}{b_w h}$</td>
<td>0.20</td>
</tr>
</tbody>
</table>

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows (BS 3.12.6.1):
SAFE Reinforced Concrete Design

\[ A_y \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_v d & \text{Flanged beam} \end{cases} \]

\[ A_y' \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_v d & \text{Flanged beam} \end{cases} \]

4.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the shear stress, \( v \).
- Determine the shear stress, \( v_c \), that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

4.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors. The shear stress is then calculated as:

\[ v = \frac{V}{b_v d} \quad \text{(BS 3.4.5.2)} \]

The maximum allowable shear stress, \( v_{\text{max}} \), is defined as:

\[ v_{\text{max}} = \min(0.8 \sqrt{f_{cu}}, 5 \text{ MPa}) \quad \text{(BS 3.4.5.2)} \]

For lightweight concrete, \( v_{\text{max}} \) is defined as:

\[ v_{\text{max}} = \min(0.63 \sqrt{f_{cu}}, 4 \text{ MPa}) \quad \text{(BS 8110-2:1985 5.4)} \]
4.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, $v_c$, is calculated as:

$$v'_c = v_c + 0.6 \frac{NVh}{A_v M} \leq v_c \sqrt{1 + \frac{N}{A_v v_c}} \tag{BS 3.4.5.12}$$

$$v_c = \frac{0.79 k_1 k_2 A_v}{\gamma_m} \left(\frac{100}{bd}\right)^{\frac{V}{3}} \left(\frac{400}{d}\right)^{\frac{V}{4}} \tag{BS 3.4.5.4, Table 3.8}$$

$k_1$ is the enhancement factor for support compression, and is conservatively taken as 1 (BS 3.4.5.8)

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{\frac{V}{3}}, \quad 1 \leq k_2 \leq \left(\frac{40}{25}\right)^{\frac{V}{5}} \tag{BS 3.4.5.4, Table 3.8}$$

$$\gamma_m = 1.25 \tag{BS 2.4.4.1}$$

However, the following limits also apply:

$$0.15 \leq \frac{100 A_v}{bd} \leq 3 \quad \text{(BS 3.4.5.4, Table 3.8)}$$

$$\left(\frac{400}{d}\right)^{\frac{V}{4}} \geq 0.67 \text{ (unreinforced) or } \geq 1 \text{ (reinforced)} \quad \text{(BS 3.4.5.4, Table 3.8)}$$

$$f_{cu} \leq 40 \text{ MPa (for calculation purposes only)} \quad \text{(BS 3.4.5.4)}$$

$$\frac{Vh}{M} \leq 1 \quad \text{(BS 3.4.5.12)}$$

$A_v$ is the area of tension reinforcement.

4.5.2.3 Determine Required Shear Reinforcement

Given $v$, $v_c$, and $v_{\text{max}}$, the required shear reinforcement is calculated as follows (BS Table 3.8, BS 3.4.5.3):

- If $v \leq (v'_c + 0.4)$,
\[
\frac{A_{sv}}{s_v} = \frac{0.4b_w}{0.87f_{yy}} \quad \text{(BS 3.4.5.3, Table 3.7)}
\]

- If \((v' + 0.4) < v \leq v_{\text{max}}\),
  \[
  \frac{A_{sv}}{s_v} = \frac{(v - v'_c)b_w}{0.87f_{yy}} \quad \text{(BS 3.4.5.3, Table 3.7)}
  \]

- If \(v > v_{\text{max}}\), a failure condition is declared. \(\text{(BS 3.4.5.2)}\)

In the preceding expressions, a limit is imposed on \(f_{yy}\) as:

\[
f_{yy} \leq 500 \text{ MPa}. \quad \text{(BS 3.4.5.1)}
\]

The maximum of all of the calculated \(A_{sv}/s_v\) values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

### 4.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, \(v_t\).
- Determine special section properties.
- Determine critical torsion stress.
- Determine the torsion reinforcement required.

Note that this section refers to BS 8110-2:1985 instead of BS 8110-1997 code.
4.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following sections (BS 8110-2:85 3.4.5.13).

The torsional shear stress, \( v_t \), for a rectangular section is computed as:

\[
v_t = \frac{2T}{h_{\text{min}}^3 (h_{\text{max}} - h_{\text{min}} / 3)}
\]

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

\[
T_{\text{seg}} = T \left( \frac{h_{\text{min}}^3 h_{\text{max}}}{\sum (h_{\text{min}}^3 h_{\text{max}})} \right)
\]

\( h_{\text{max}} \) = Larger dimension of a rectangular section

\( h_{\text{min}} \) = Smaller dimension of a rectangular section

If the computed torsional shear stress, \( v_t \), exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

\[
v_t \leq \min(0.8 \sqrt{f_{\text{cu}}}, 5 \text{N/mm}^2) \times \frac{v_t}{550}
\]

(BS 8110-2:85 2.4.5)

4.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, \( v_{t,\text{min}} \), for which the torsion in the section can be ignored is calculated as:
\[ v_{t,\text{min}} = \min \left( 0.067 \sqrt{f_{cu}}, 0.4 \text{N/mm}^2 \right) \]  
(BS 8110-2:85 2.4.6)

where \( f_{cu} \) is the specified concrete compressive strength.

For light-weight concrete, \( v_{t,\text{min}} \) is defined as:

\[ v_{t,\text{min}} = \min \left( 0.067 \sqrt{f_{cu}}, 0.4 \text{N/mm}^2 \right) \times 0.8 \]  
(BS 8110-2:85 5.5)

4.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, \( v_t \) is less than the threshold limit, \( v_{t,\text{min}} \), torsion can be safely ignored (BS 8110-2:85 2.4.6). In that case, the program reports that no torsion reinforcement is required. However, if \( v_t \) exceeds the threshold limit, \( v_{t,\text{min}} \), it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (BS 8110-2:85 2.4.6).

If \( v_t > v_{t,\text{min}} \) the required closed stirrup area per unit spacing, \( A_{sv,t}/s_v \), is calculated as:

\[ A_{sv,t}/s_v = \frac{T}{0.8 x_1 y_1 \left( 0.87 f_{yv} \right)} \]  
(BS 8110-2:85 2.4.7)

and the required longitudinal reinforcement is calculated as:

\[ A_t = \frac{A_{sv,t} f_{yv} (x_1 + y_1)}{s_v f_y} \]  
(BS 8110-2:85 2.4.7)

In the preceding expressions, \( x_1 \) is the smaller center-to-center dimension of the closed link, and \( y_1 \) is the larger center-to-center dimension of the closed link.
An upper limit of the combination of $v$ and $v_t$ that can be carried by the section is also checked using the equation:

$$v + v_t \leq \min\left(0.8\sqrt{f_{cu}}, \frac{5N}{mm^2}\right)$$  \hspace{1cm} (BS 8110-2:85 2.4.5)

For light-weight concrete, $v_{\text{max}}$ is defined as:

$$v_{\text{max}} = \min(0.63 \sqrt{f_{cu}}, 4 \text{ MPa})$$  \hspace{1cm} (BS 8110-2:85 5.4)

If the combination of shear stress, $v$, and torsional shear stress, $v_t$, exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated $A_t$ and $A_{sv}/S_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.
4.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method (BS 8110-97) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

4.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

4.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.
4.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

4.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (BS 3.12.5.3, BS Table 3.25) with interpolation for reinforcement of intermediate strength:

\[
A_t \geq \begin{cases} 
0.0024bh & \text{if } f_y = 250\text{MPa} \\
0.0013bh & \text{if } f_y = 500\text{MPa}
\end{cases}
\]  

(BS 3.12.5.3)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

4.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following subsections.

4.6.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (BS 3.7.6.4) and at a critical section at a distance of 1.5d from the face of the support (BS 3.7.7.6). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point
loads (BS 3.7.7.1). Figure 4-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

\[
\begin{align*}
\text{Interior Column} & \quad 1.5d \\
\text{Edge Column} & \quad 1.5d \\
\text{Corner Column} & \quad 1.5d \\
\text{Circular Column} & \quad 1.5d \\
\text{T-Shape Column} & \quad 1.5d \\
\text{L-Shape Column} & \quad 1.5d
\end{align*}
\]

Figure 4-4 Punching Shear Perimeters

4.6.2.2 Determine Concrete Capacity

The concrete punching shear factored strength is taken as (BS 3.7.7.4, 3.7.7.6):

\[
v_c = \frac{0.79k_1k_2}{\gamma_m} \left( \frac{100A_t}{bd} \right)^{\frac{3}{4}} \left( \frac{400}{d} \right)^{\frac{3}{4}}
\]

(BS 3.4.5.4, Table 3.8)

\(k_1\) is the enhancement factor for support compression, and is conservatively taken as 1 (BS 3.4.5.8)

\[
k_2 = \left( \frac{f_{cu}}{25} \right)^{\frac{3}{4}}, \quad 1 \leq k_2 \leq \left( \frac{40}{25} \right)^{\frac{3}{4}}
\]

(BS 3.4.5.4, Table 3.8)
\[ \gamma_m = 1.25 \]  \hspace{1cm} \text{(BS 3.4.5.2)}

However, the following limitations also apply:

\[ 0.15 \leq \frac{100 \cdot A_t}{bd} \leq 3 \]  \hspace{1cm} \text{(BS 3.4.5.4, Table 3.8)}

\[ \left( \frac{400}{d} \right)^{\frac{1}{4}} \geq 0.67 \text{ (unreinforced) or } \geq 1 \text{ (reinforced)} \]  \hspace{1cm} \text{(BS 3.4.5.4)}

\[ v \leq \min(0.8 \sqrt{f_{cu}}, 5 \text{MPa}) \]  \hspace{1cm} \text{(BS 3.7.6.4)}

For light-weight concrete, \( v_{\text{max}} \) is defined as:

\[ v \leq \min(0.63 \sqrt{f_{cu}}, 4 \text{ MPa}) \]  \hspace{1cm} \text{(BS 8110-2:1985 5.4)}

\[ f_{cu} \leq 40 \text{ MPa (for calculation purpose only)} \]  \hspace{1cm} \text{(BS 3.4.5.4)}

\( A_t = \) area of tension reinforcement, which is taken as the average tension reinforcement of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement is taken as zero in the current implementation.

### 4.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, \( v_{\text{max}} \), is calculated as:

\[ V_{eff,x} = V \left( f + \frac{1.5 M_x}{V_y} \right) \]  \hspace{1cm} \text{(BS 3.7.6.2, 3.7.6.3)}

\[ V_{eff,y} = V \left( f + \frac{1.5 M_y}{V_x} \right) \]  \hspace{1cm} \text{(BS 3.7.6.2, 3.7.6.3)}
SAFE Reinforced Concrete Design

\[ v_{\text{max}} = \max \left\{ \frac{V_{\text{eff},x}}{u d}, \frac{V_{\text{eff},y}}{u d} \right\} \]  

(BS 3.7.7.3)

where,

- \( u \) is the perimeter of the critical section
- \( x \) and \( y \) are the lengths of the sides of the critical section parallel to the axis of bending
- \( M_x \) and \( M_y \) are the design moments transmitted from the slab to the column at the connection
- \( V \) is the total punching shear force
- \( f \) is a factor to consider the eccentricity of punching shear force and is taken as:
  \[ f = \begin{cases} 
  1.00 & \text{for interior columns} \\
  1.25 & \text{for edge columns} \\
  1.25 & \text{for corner columns} 
  \end{cases} \]  

(BS 3.7.6.2, 3.7.6.3)

4.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

4.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (BS 3.7.7.5). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.
The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as explained in the subsections that follow.

4.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

4.6.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

\[ v_{\text{max}} = 2v_c \]  

(BS 3.7.7.5)

Given \( v, v_c, \) and \( v_{\text{max}} \), the required shear reinforcement is calculated as follows (BS 3.7.7.5).

- If \( v \leq 1.6v_c \),
  \[
  \frac{A_v}{s} = \frac{(v - v_c)ud}{0.87 f_{yw}} \geq \frac{0.4ud}{0.87 f_{yw}},
  \]  
  (BS 3.7.7.5)

- If \( 1.6v_c \leq v < 2.0v_c \),
  \[
  \frac{A_v}{s} = \frac{5(0.7v - v_c)ud}{0.87 f_{yw}} \geq \frac{0.4ud}{0.87 f_{yw}},
  \]  
  (BS 3.7.7.5)

- If \( v > v_{\text{max}} \), a failure condition is declared.  
  (BS 3.7.7.5)

If \( v \) exceeds the maximum permitted value of \( v_{\text{max}} \), the concrete section should be increased in size.

4.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 4-5 shows a typical arrangement of
shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $1.5d$ measured in a direction parallel to the column face (BS 3.7.7.6).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

**Figure 4-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone**

### 4.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in BS 3.3 plus half of the diameter of the flexural reinforcement.
Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $1.5d$. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

$$s_o \leq 0.5d$$  \hspace{1cm} (BS 3.7.7.6)

$$s \leq 0.75d$$  \hspace{1cm} (BS 3.7.7.6)

$$g \leq 1.5d$$  \hspace{1cm} (BS 3.7.7.6)
Chapter 5
Design for CSA A23.3-04

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Canadian code CSA A23.3-04 [CSA 04] is selected. Various notations used in this chapter are listed in Table 5-1. For referencing to the pertinent sections of the Canadian code in this chapter, a prefix “CSA” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

5.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>Area enclosed by outside perimeter of concrete cross-section, sq-mm</td>
</tr>
<tr>
<td>$A_{ct}$</td>
<td>Area of concrete on flexural tension side, sq-mm</td>
</tr>
</tbody>
</table>

Table 5-1 List of Symbols Used in the CSA A23.3-04 Code
Table 5-1 List of Symbols Used in the CSA A23.3-04 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_l$</td>
<td>Area of longitudinal reinforcement for torsion, sq-mm</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Gross area enclosed by shear flow path, sq-mm</td>
</tr>
<tr>
<td>$A_{oh}$</td>
<td>Area enclosed by centerline of outermost closed transverse torsional reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_{s\text{(required)}}$</td>
<td>Area of steel required for tension reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_{s/s}$</td>
<td>Area of closed shear reinforcement for torsion per unit length, sq-mm/mm</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Area of shear reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_{v/s}$</td>
<td>Area of shear reinforcement per unit length, sq-mm/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$a_b$</td>
<td>Depth of compression block at balanced condition, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of member, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Effective width of flange (flanged section), mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), mm</td>
</tr>
<tr>
<td>$b_0$</td>
<td>Perimeter of the punching critical section, mm</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Width of the punching critical section in the direction of bending, mm</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Width of the punching critical section perpendicular to the direction of bending, mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, mm</td>
</tr>
<tr>
<td>$c_b$</td>
<td>Depth to neutral axis at balanced conditions, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, mm</td>
</tr>
<tr>
<td>$d_v$</td>
<td>Effective shear depth, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Distance from compression face to compression reinforcement, mm</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Thickness of slab (flanged section), mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
</tbody>
</table>
### Table 5-1 List of Symbols Used in the CSA A23.3-04 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, assumed as 200,000 MPa</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>Specified compressive strength of concrete, MPa</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, psi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Specified yield strength of flexural reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{ys}$</td>
<td>Specified yield strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section, mm</td>
</tr>
<tr>
<td>$I_g$</td>
<td>Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Factored moment at section, N-mm</td>
</tr>
<tr>
<td>$N_f$</td>
<td>Factored axial force at section, N</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Outside perimeter of concrete cross-section, mm</td>
</tr>
<tr>
<td>$p_h$</td>
<td>Perimeter of area $A_{oh}$, mm</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of the shear reinforcement along the length of the beam, mm</td>
</tr>
<tr>
<td>$s_z$</td>
<td>Crack spacing parameter</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Factored torsion at section, N-mm</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear resisted by concrete, N</td>
</tr>
<tr>
<td>$V_{c,max}$</td>
<td>Maximum permitted total factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_f$</td>
<td>Factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Shear force at a section resisted by steel, N</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Ratio of average stress in rectangular stress block to the specified concrete strength</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Factor accounting for shear resistance of cracked concrete</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>Factor for obtaining depth of compression block in concrete</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching critical section</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in concrete</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in reinforcing steel</td>
</tr>
</tbody>
</table>
Table 5-1 List of Symbols Used in the CSA A23.3-04 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_x$</td>
<td>Longitudinal strain at mid-depth of the section</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Strength reduction factor for concrete</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Strength reduction factor for steel</td>
</tr>
<tr>
<td>$\phi_m$</td>
<td>Strength reduction factor for member</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Fraction of unbalanced moment transferred by flexure</td>
</tr>
<tr>
<td>$\gamma_e$</td>
<td>Fraction of unbalanced moment transferred by eccentricity of shear</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of diagonal compressive stresses, degrees</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shear strength factor</td>
</tr>
</tbody>
</table>

5.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For CSA A23.3-04, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (CSA 8.3.2, Table C.1)

1.4D \hspace{1cm} (CSA 8.3.2, Table C.1 Case 1)
1.25D + 1.5L
1.25D + 1.5L + 0.5S
1.25D + 1.5L ± 0.4W
0.9D + 1.5L \hspace{1cm} (CSA 8.3.2, Table C.1 Case 2)
0.9D + 1.5L + 0.5S
0.9D + 1.5L ± 0.4W
1.25D + 1.5(0.75 PL) \hspace{1cm} (CSA 13.8.4.3)
1.25D + 1.5S
1.25D + 1.5S + 0.5L
1.25D + 1.5S ± 0.4W \hspace{1cm} (CSA 8.3.2, Table C.1 Case 3)
0.9D + 1.5S
0.9D + 1.5S + 0.5L
Chapter 5 - Design for CSA A23.3-04

These are also the default design load combinations in SAFE whenever the CSA A23.3-04 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

5.3 **Limits on Material Strength**

The upper and lower limits of $f'_{c}$ are 80 MPa and 20 MPa, respectively, for all framing types (CSA 8.6.1.1).

$$20 \text{ MPa} \leq f'_{c} \leq 80 \text{ MPa}$$ (CSA 8.6.1.1)

The upper limit of $f_{y}$ is 500 MPa for all frames (CSA 8.5.1).

$$f_{y} \leq 500 \text{ MPa}$$ (CSA 8.5.1)

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

5.4 **Strength Reduction Factors**

The strength reduction factors, $\phi$, are material dependent and defined as:
\[ \phi_c = 0.65 \text{ for concrete} \quad \text{(CSA 8.4.2)} \]
\[ \phi_s = 0.85 \text{ for reinforcement} \quad \text{(CSA 8.4.3a)} \]

These values can be overwritten; however, caution is advised.

5.5 **Beam Design**

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

5.5.1 **Design Flexural Reinforcement**

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

5.5.1.1 **Determine Factored Moments**

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by
factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom reinforcement. In such cases the beam may be designed as a rectangular or flanged beam. Negative beam moments produce top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

### 5.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 5-1 (CSA 10.1.7). Furthermore, it is assumed that the compression carried by the concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed \(0.1 f'c A_g\), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than \(0.1 f'c A_g\) and axial tensions are always included in flexural and shear design.

#### 5.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, \(M_f\) (i.e., designing top or bottom reinforcement), the depth of the compression block is given by \(a\) (see Figure 5-1), where,
Figure 5-1 Rectangular Beam Design

\[
a = d - \sqrt{d^2 - \frac{2|M_f|}{\alpha_i f'c \phi b}}
\]  
(CSA 10.1)

where the value of \( \phi_c \) is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters \( \alpha_i, \beta_i, \) and \( c_b \) are calculated as:

\[
\alpha_i = 0.85 - 0.0015f'c \geq 0.67,
\]  
(CSA 10.1.7)

\[
\beta_i = 0.97 - 0.0025f'c \geq 0.67,
\]  
(CSA 10.1.7)

\[
c_b = \frac{700}{700 + f_y} d
\]  
(CSA 10.5.2)

The balanced depth of the compression block is given by:

\[
a_b = \beta_i c_b
\]  
(CSA 10.1.7)
• If $a \leq a_b$ (CSA 10.5.2), the area of tension reinforcement is given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2}\right)}$$

This reinforcement is to be placed at the bottom if $M_f$ is positive, or at the top if $M_f$ is negative.

• If $a > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The factored compressive force developed in the concrete alone is given by:

$$C = \phi_c \alpha_1 f'_c b a_b$$

and the factored moment resisted by concrete compression and tension reinforcement is:

$$M_{fc} = C \left(d - \frac{a_b}{2}\right)$$

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{fs} = M_f - M_{fc}$$

The required compression reinforcement is given by:

$$A'_c = \frac{M_{fs}}{(\phi_c f'_c - \phi_s \alpha_1 f'_y)(d - d')}$$

where

$$f'_y = 0.0035 E_c \left(\frac{c - d'}{c}\right) \leq f_y$$

The required tension reinforcement for balancing the compression in the concrete is:
and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A_{s2} = \frac{M_{fs}}{f_y (d - d') \phi_s} \]

Therefore, the total tension reinforcement, \( A_s = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A's \). \( A_s \) is to be placed at the bottom and \( A's \) is to be placed at the top if \( M_f \) is positive, and vice versa if \( M_f \) is negative.

### 5.5.1.2.2 Design of Flanged Beams

#### 5.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, \( M_f \) (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

#### 5.5.1.2.2.2 Flanged Beam Under Positive Moment

- If \( M_f > 0 \), the depth of the compression block is given by:

\[ a = d - \sqrt{d^2 - \frac{2M_f}{\alpha f'_c \phi_c f_f}} \quad \text{(CSA 10.1)} \]

where, the value of \( \phi_c \) is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters \( \alpha, \beta, \) and \( c_b \) are calculated as:

\[ \alpha_i = 0.85 - 0.0015 f'_c \geq 0.67, \quad \text{(CSA 10.1.7)} \]

\[ \beta_i = 0.97 - 0.0025 f'_c \geq 0.67, \quad \text{(CSA 10.1.7)} \]

\[ c_b = \frac{700}{700 + f_y} d \quad \text{(CSA 10.5.2)} \]
The balanced depth of the compression block is given by:

\[ a_b = \beta c_b \]  

(CSA 10.1.4, 10.1.7)

- If \( a \leq h_s \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( a > a_b \).

- If \( a > h_s \), calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \) as shown in Figure 5-2. \( C_f \) is given by:

\[ C_f = \alpha f'c_y (b_f - b_w) \min(h_s, a_b) \]  

(CSA 10.1.7)

Therefore, \( A_{sl} = \frac{C_f \phi_c}{f_y \phi} \) and the portion of \( M_f \) that is resisted by the flange is given by:
\[ M_{ff} = C_f \left( d - \frac{\min(h_z, a_b)}{2}\right) \phi_c \]

Therefore, the balance of the moment, \( M_f \) to be carried by the web is:

\[ M_{fw} = M_f - M_{ff} \]

The web is a rectangular section with dimensions \( b_w \) and \( d \), for which the design depth of the compression block is recalculated as:

\[ a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f_c' \phi_c b_w}} \]  \hspace{1cm} (CSA 10.1)

• If \( a_1 \leq a_b \) (CSA 10.5.2), the area of tension reinforcement is then given by:

\[ A_{s2} = \frac{M_{fw}}{\phi_s f_y \left( d - \frac{a_1}{2} \right)} \], and

\[ A_s = A_{s1} + A_{s2} \]

This reinforcement is to be placed at the bottom of the flanged beam.

• If \( a_1 > a_b \) (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The compressive force in the web concrete alone is given by:

\[ C = \phi_c \alpha_1 f_c' b_w a_b \]  \hspace{1cm} (CSA 10.1.7)

Therefore the moment resisted by the concrete web and tension reinforcement is:

\[ M_{fc} = C \left( d - \frac{a_b}{2} \right) \]

and the moment resisted by compression and tension reinforcement is:

\[ M_{fs} = M_{fw} - M_{fc} \]
Therefore, the compression reinforcement is computed as:

\[ A_c' = \frac{M_{fs}}{(\phi f'_{c} - \phi \alpha f'_{c}) (d - d')} \]

where

\[ f'_{c} = \epsilon_c E_c \left[ \frac{c - d'}{c} \right] \leq f_y \]

(CSA 10.1.2, 10.1.3)

The tension reinforcement for balancing compression in the web concrete is:

\[ A_{s2} = \frac{M_{fc}}{f_y (d - \frac{a_b}{2}) \phi_s} \]

and the tension reinforcement for balancing the compression reinforcement is:

\[ A_{s3} = \frac{M_{fs}}{f_y (d - d') \phi_s} \]

The total tension reinforcement is \( A_s = A_{s1} + A_{s2} + A_{s3} \), and the total compression reinforcement is \( A'_s \). \( A_s \) is to be placed at the bottom and \( A'_s \) is to be placed at the top.

**5.5.1.3 Minimum and Maximum Reinforcement**

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the following two limits:

\[ A_s \geq 0.2 \sqrt{f'_{c}} \frac{b_w h}{f_y} \]  
(CSA 10.5.1.2)

\[ A_s \geq \frac{4}{3} A_s(\text{required}) \]  
(CSA 10.5.1.3)

In addition, the minimum flexural tension reinforcement provided in a flanged beam with the flange under tension in an ordinary moment resisting frame is given by the limit:
\[A_s \geq 0.004 \left( b - b_w \right) h_s \]  
(CSA 10.5.3.1)

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

\[
A_s \leq \begin{cases} 
0.04bd & \text{Rectangular beam} \\
0.04b_wd & \text{Flanged beam} 
\end{cases}
\]

\[
A'_t \leq \begin{cases} 
0.04bd & \text{Rectangular beam} \\
0.04b_wd & \text{Flanged beam} 
\end{cases}
\]

### 5.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, \(V_f\).
- Determine the shear force, \(V_c\), that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

#### 5.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

#### 5.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, \(V_c\), is calculated as:

\[V_c = \phi_c \lambda \beta \sqrt{f_y b_w d_v} \]  
(CSA 11.3.4)
\[ \sqrt{f'_c} \leq 8 \text{ MPa} \quad (\text{CSA 11.3.4}) \]

\( \phi_c \) is the resistance factor for concrete. By default it is taken as 0.65 (CSA 8.4.2).

\( \lambda \) is the strength reduction factor to account for low density concrete (CSA 2.2). For normal density concrete, its value is 1 (CSA 8.6.5), which is taken by the program as the default value. For concrete using lower density aggregate, the user can change the value of \( \lambda \) in the material property data. The recommended value for \( \lambda \) is as follows (CSA 8.6.5):

\[
\lambda = \begin{cases} 
1.00, & \text{for normal density concrete,} \\
0.85, & \text{for semi-low-density concrete} \\
0.75, & \text{in which all of the fine aggregate is natural sand,} \\
0.75, & \text{in which none of the fine aggregate is natural sand.}
\end{cases} \quad (\text{CSA 8.6.5})
\]

\( \beta \) is the factor for accounting for the shear resistance of cracked concrete (CSA 2.2). Its value is normally between 0.1 and 0.4. It is determined according to CSA 11.3.6, and described further in the following sections.

\( b_w \) is the effective web width. For rectangular beams, it is the width of the beam. For flanged beams, it is the width of the web of the beam.

\( d_v \) is the effective shear depth. It is taken as the greater of 0.9\( d \) or 0.72\( h \)(CSA 2.3), where \( d \) is the distance from the extreme compression fiber to the centroid of the tension reinforcement, and \( h \) is the overall depth of the cross-section in the direction of the shear force (CSA 2.3).

The value of \( \beta \) is preferably taken as the special value (CSA 11.3.6.2) or it is determined using the simplified method (CSA 11.3.6.3), if applicable. When the conditions of the special value or simplified method do not apply, the general method is used (CSA 11.3.6.4).

If the overall beam depth, \( h \), is less than 250 mm or if the depth of a flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, \( \beta \) is taken as 0.21 (CSA 11.3.6.2).
When the specified yield strength of the longitudinal reinforcing \( f_y \) does not exceed 400 MPa, the specified concrete strength \( f'_{c} \) does not exceed 60 MPa, and the tensile force is negligible, \( \beta \) is determined in accordance with the simplified method, as follows (CSA 11.3.6.3):

- When the section contains at least the minimum transverse reinforcement, \( \beta \) is taken as 0.18 (CSA 11.6.3.3a).

\[
\beta = 0.18 \quad \text{(CSA 11.3.6.3(a))}
\]

- When the section contains no transverse reinforcement, \( \beta \) is determined based on the specified maximum nominal size of coarse aggregate, \( a_{g} \).

For a maximum size of coarse aggregate not less than 20 mm, \( \beta \) is taken as:

\[
\beta = \frac{230}{1000 + d_{v}} \quad \text{(CSA 11.3.6.3(b))}
\]

where \( d_{v} \) is the effective shear depth expressed in millimeters.

For a maximum size of coarse aggregate less than 20 mm, \( \beta \) is taken as:

\[
\beta = \frac{230}{1000 + s_{ze}} \quad \text{(CSA 11.3.6.3 (c))}
\]

where, \( s_{ze} = \frac{35s_{s}}{15 + a_{g}} \geq 0.85s_{s} \)  \quad \text{(CSA 11.3.6.3.c)}

In the preceding expression, the crack spacing parameter, \( s_{ze} \), shall be taken as the minimum of \( d_{v} \) and the maximum distance between layers of distributed longitudinal reinforcement. However, \( s_{ze} \) is conservatively taken as equal to \( d_{v} \).

In summary, for simplified cases, \( \beta \) can be expressed as follows:
0.18, if minimum transverse reinforcement is provided, 230, if no transverse reinforcement is provided, and \( a_g \geq 20 \text{mm} \), \[
\beta = \begin{cases} 
0.18, & \text{if minimum transverse reinforcement is provided}, \\
\frac{230}{1000 + d_v}, & \text{if no transverse reinforcement is provided, and } a_g \geq 20 \text{mm}, \\
\frac{230}{1000 + S_{ze}}, & \text{if no transverse reinforcement is provided, and } a_g < 20 \text{mm}.
\end{cases}
\]

- When the specified yield strength of the longitudinal reinforcing \( f_y \) is greater than 400 MPa, the specified concrete strength \( f'_c \) is greater than 60 MPa, or tension is not negligible, \( \beta \) is determined in accordance with the general method as follows (CSA 11.3.6.1, 11.3.6.4):

\[
\beta = \frac{0.40}{(1+1500\varepsilon_x)} \cdot \frac{1300}{(1000 + S_{ze})}
\]

(CSA 11.3.6.4)

In the preceding expression, the equivalent crack spacing parameter, \( s_{ze} \) is taken equal to 300 mm if minimum transverse reinforcement is provided (CSA 11.3.6.4). Otherwise it is determined as stated in the simplified method.

\[
S_{ze} = \begin{cases} 
300 & \text{if minimum transverse reinforcement is provided}, \\
\frac{35}{15 + a_g} S_z \geq 0.85 S_z & \text{otherwise}.
\end{cases}
\]

(CSA 11.3.6.3, 11.3.6.4)

The value of \( a_g \) in the preceding equations is taken as the maximum aggregate size for \( f'_c \) of 60 MPa, is taken as zero for \( f'_c \) of 70 MPa, and linearly interpolated between these values (CSA 11.3.6.4).

The longitudinal strain, \( \varepsilon_x \) at mid-depth of the cross-section is computed from the following equation:

\[
\varepsilon_x = \frac{M_f/d_v + V_f + 0.5 N_f}{2(E_s A_s)}
\]

(CSA 11.3.6.4)

In evaluating \( \varepsilon_x \) the following conditions apply:

- \( \varepsilon_x \) is positive for tensile action.
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- $V_f$ and $M_f$ are taken as positive quantities. (CSA 11.3.6.4(a))
- $M_f$ is taken as a minimum of $V_f d_v$. (CSA 11.3.6.4(a))
- $N_f$ is taken as positive for tension. (CSA 2.3)

$A_s$ is taken as the total area of longitudinal reinforcement in the beam. It is taken as the envelope of the reinforcement required for all design load combinations. The actual provided reinforcement might be slightly higher than this quantity. The reinforcement should be developed to achieve full strength (CSA 11.3.6.3(b)).

If the value of $\varepsilon_x$ is negative, it is recalculated with the following equation, in which $A_{ct}$ is the area of concrete in the flexural tensile side of the beam, taken as half of the total area.

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5 N_f}{2(E_s A_s + E_c A_{ct})}$$

(CSA 11.3.6.4(c))

$E_s = 200,000$ MPa

(CSA 8.5.4.1)

$E_c = 4500\sqrt{f'c}$ MPa

(CSA 8.6.2.3)

If the axial tension is large enough to induce tensile stress in the section, the value of $\varepsilon_x$ is doubled (CSA 11.3.6.4(e)).

For sections closer than $d_v$ from the face of the support, $\varepsilon_x$ is calculated based on $M_f$ and $V_f$ at a section at a distance $d_v$ from the face of the support (CSA 11.3.6.4(d)). This condition currently is not checked by SAFE.

An upper limit on $\varepsilon_x$ is imposed as:

$$\varepsilon_x \leq 0.003$$

(CSA 11.3.6.4(f))

In both the simplified and general methods, the shear strength of the section due to concrete, $v_c$ depends on whether the minimum transverse reinforcement is provided. To check this condition, the program performs the design in two passes. In the first pass, it assumes that no transverse shear reinforcement is needed. When the program determines that shear reinforcement is required, the
program performs the second pass assuming that at least minimum shear reinforcement is provided.

5.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to $V_{r,\text{max}}$ where:

$$V_{r,\text{max}} = 0.25\phi f' c b w d$$  \hspace{1cm} (CSA 11.3.3)

Given $V_f$, $V_c$, and $V_{r,\text{max}}$, the required shear reinforcement is calculated as follows:

- If $V_f \leq V_c$,
  $$\frac{A_x}{s} = 0$$ \hspace{1cm} \text{(CSA 11.3.5.1)}

- If $V_c < V_f \leq V_{r,\text{max}}$,
  $$\frac{A_x}{s} = \frac{(V_f - V_c)\tan\theta}{\phi f' c d_y}$$ \hspace{1cm} \text{(CSA 11.3.3, 11.3.5.1)}

- If $V_f > V_{r,\text{max}}$, a failure condition is declared. \hspace{1cm} \text{(CSA 11.3.3)}

A minimum area of shear reinforcement is provided in the following regions (CSA 11.2.8.1):

(a) in regions of flexural members where the factored shear force $V_f$ exceeds $V_c$

(b) in regions of beams with an overall depth greater than 750 mm

(c) in regions of beams where the factored torsion $T_f$ exceeds $0.25T_{cr}$.

Where the minimum shear reinforcement is required by CSA 11.2.8.1, or by calculation, the minimum area of shear reinforcement per unit spacing is taken as:
\[
\frac{A_v}{s} \geq 0.06 \sqrt{\frac{f_{c}'}{f_{ys}}} b_w
\]  
(CSA 11.2.8.2)

In the preceding equations, the term \( \theta \) is used, where \( \theta \) is the angle of inclination of the diagonal compressive stresses with respect to the longitudinal axis of the member (CSA 2.3). The \( \theta \) value is normally between 22 and 44 degrees. It is determined according to CSA 11.3.6.

Similar to the \( \beta \) factor, which was described previously, the value of \( \theta \) is preferably taken as the special value (CSA 11.3.6.2), or it is determined using the simplified method (CSA 11.3.6.3), whenever applicable. The program uses the general method when conditions for the simplified method are not satisfied (CSA 11.3.6.4).

- If the overall beam depth, \( h \), is less than 250 mm or if the depth of the flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, \( \theta \) is taken as 42 degrees (CSA 11.3.6.2).

- If the specified yield strength of the longitudinal reinforcing \( f_y \) does not exceed 400 MPa, or the specified concrete strength \( f'c \) does not exceed 60 MPa, \( \theta \) is taken to be 35 degree (CSA 11.3.6.3).

\[
\theta = 35^\circ \text{ for } P_f \leq 0 \text{ or } f_y \leq 400 \text{ MPa or } f'c \leq 60 \text{ MPa} \]  
(CSA 11.3.6.3)

- If the axial force is tensile, the specified yield strength of the longitudinal reinforcing \( f_y > 400 \text{ MPa} \), and the specified concrete strength \( f'c > 60 \text{ MPa} \), \( \theta \) is determined using the general method as follows (CSA 11.3.6.4),

\[
\theta = 29 + 7000 \varepsilon_x \text{ for } P_f < 0, \ f_y > 400 \text{ MPa, } f'c \leq 60 \text{ MPa} \]  
(CSA 11.3.6.4)

where \( \varepsilon_x \) is the longitudinal strain at the mid-depth of the cross-section for the factored load. The calculation procedure is described in preceding sections.

The maximum of all of the calculated \( A_v/s \) values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup require-
ments to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

5.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, \( T_f \).
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

5.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design \( T_f \) is permitted to be reduced in accordance with the code (CSA 11.2.9.2). However, the program does not automatically redistribute the internal forces and reduce \( T_f \). If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

5.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as \( A_c, A_{oh}, A_o, p_c, \) and \( p_h \), are calculated. These properties are described in the following (CSA 2.3).

\[
A_c = \text{Area enclosed by outside perimeter of concrete cross-section}
\]
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\[ A_{oh} = \text{Area enclosed by centerline of the outermost closed transverse torsional reinforcement} \]

\[ A_o = \text{Gross area enclosed by shear flow path} \]

\[ p_c = \text{Outside perimeter of concrete cross-section} \]

\[ p_h = \text{Perimeter of centerline of outermost closed transverse torsional reinforcement} \]

In calculating the section properties involving reinforcement, such as \( A_{oh}, A_o, \) and \( p_h \), it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 millimeters. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during \( T_{cr} \) calculation. With this assumption, the special properties for a rectangular beam section are given as follows:

\[ A_c = bh \]  
\[ A_{oh} = (b - 2c)(h - 2c) \]  
\[ A_o = 0.85 A_{oh} \]  
\[ p_c = 2b + 2h \]  
\[ p_h = 2(b - 2c) + 2(h - 2c) \]

where, the section dimensions \( b, h, \) and \( c \) are shown in Figure 5-3. Similarly, the special section properties for a flanged beam section are given as follows:

\[ A_c = b_w h + (b_f - b_w) h_s \]  
\[ A_{oh} = (b_w - 2c)(h - 2c) \]  
\[ A_o = 0.85 A_{oh} \]  
\[ p_c = 2b_f + 2h \]  
\[ p_h = 2(h - 2c) + 2(b_w - 2c) \]

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where the section dimensions \( b_f, b_w, h, h_f, \) and \( c \) for a flanged beam are shown in Figure 5-3. Note that the flange width on either side of the beam web is limited to the smaller of \( 6h_f \) or \( 1/12 \) the span length (CSA 10.3.4).

5.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, \( T_{cr} \), for which the torsion in the section can be ignored, is calculated as:

\[
T_{cr} = \frac{0.38 \lambda \phi c \sqrt{f'c} \left( \frac{A^2}{p_c} \right)}{4}
\]

where \( A_{cp} \) and \( p_c \) are the area and perimeter of the concrete cross-section as described in the previous section, \( \lambda \) is a factor to account for low-density concrete, \( \phi_c \) is the strength reduction factor for concrete, which is equal to 0.65, and \( f'c \) is the specified concrete compressive strength.

5.5.3.4 Determine Torsion Reinforcement

If the factored torsion \( T_f \) is less than the threshold limit, \( T_{cr} \), torsion can be safely ignored (CSA 11.2.9.1). In that case, the program reports that no torsion reinforcement is required. However, if \( T_f \) exceeds the threshold limit, \( T_{cr} \), it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CSA 11.3).

- If \( T_f > T_{cr} \), the required closed stirrup area per unit spacing, \( A_t/s \), is calculated as:

\[
\frac{A_t}{s} = \frac{T_f \tan \theta}{\phi_c 2A_o f_y}
\]

and the required longitudinal reinforcement is calculated as:

\[
A_y = \frac{M_f}{d_v} + 0.5N_f + \sqrt{\left( V_f - 0.5V_y \right)^2 + \left( \frac{0.45 p_b T_f}{2A_o} \right)^2} \cot \theta
\]

\[
\phi_c f_y
\]

(CSA 11.3.10.6, 11.3.9)
Figure 5-3 Closed stirrup and section dimensions for torsion design

In the preceding expressions, $\theta$ is computed as previously described for shear, except that if the general method is being used, the value $\varepsilon_s$, calculated as specified in CSA 11.3.6.4, is replaced by:

$$\varepsilon_s = \frac{\frac{M_f}{d_v} + \sqrt{V_f^2 + \left(\frac{0.9 p_h T_f}{2 A_o}\right)^2} + 0.5 N_f}{2(E_s A_s)}$$

(CSA 11.3.10.5)

An upper limit of the combination of $V_u$ and $T_u$ that can be carried by the section also is checked using the equation:

$$\sqrt{\left(\frac{V_f}{b_v d_v}\right)^2 + \left(\frac{T_f p_h}{1.7 A_{oh}^2}\right)} \leq 0.25 \phi_c f'_c$$

(CSA 11.3.10.4(b))

For rectangular sections, $b_v$ is replaced with $b$. If the combination of $V_f$ and $T_f$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.
When torsional reinforcement is required \( (T_f > T_{cr}) \), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

\[
\left( \frac{A_v}{s} + 2 \frac{A_t}{s} \right) \geq 0.06 \sqrt{f'c} \frac{b_w}{f_{yt}}
\]  

(CSA 11.2.8.2)

If this equation is not satisfied with the originally calculated \( A_v/s \) and \( A_t/s \), \( A_v/s \) is increased to satisfy this condition.

The maximum of all of the calculated \( A_t \) and \( A_t/s \) values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 5.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is performed based on the ultimate strength design method (CSA A23.3-04) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

### 5.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.
The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

### 5.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

### 5.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

### 5.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limit (CSA 13.10.1):
\[ A_s \geq 0.002 \, bh \]  \hspace{1cm} (CSA 7.8.1)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

### 5.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

#### 5.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of \( d/2 \) from the face of the support (CSA 13.3.3.1 and CSA 13.3.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.3.3.3). Figure 5-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 5-4 Punching Shear Perimeters](image_url)

*Figure 5-4 Punching Shear Perimeters*
5.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be \( \gamma_f M_u \) and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be \( \gamma_v M_u \), where

\[
\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right) \sqrt{b_1/b_2}}, \quad \text{and} \quad (CSA 13.10.2)
\]

\[
\gamma_v = 1 - \frac{1}{1 + \left(\frac{2}{3}\right) \sqrt{b_1/b_2}}, \quad (CSA 13.3.5.3)
\]

where \( b_1 \) is the width of the critical section measured in the direction of the span, and \( b_2 \) is the width of the critical section measured in the direction perpendicular to the span.

5.6.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

\[
v_c = \min \left\{ \frac{\phi_t \left[ 1 + \frac{2}{\beta_c} \right] 0.19 \lambda \sqrt{f'_c}}{\phi_t \left[ 0.19 + \frac{\alpha_s d}{b_0} \right] \lambda \sqrt{f'_c}}, \phi_t 0.38 \lambda \sqrt{f'_c} \right\} \quad (CSA 13.3.4.1)
\]

where, \( \beta_c \) is the ratio of the minimum to the maximum dimensions of the critical section, \( b_0 \) is the perimeter of the critical section, and \( \alpha_s \) is a scale factor based on the location of the critical section.

\[
\alpha_s = \begin{cases} 
4, & \text{for interior columns} \\
3, & \text{for edge columns, and} \\
2, & \text{for corner columns.} \quad (CSA 13.3.4.1(b))
\end{cases}
\]
The value of \( f_{vc}' \) is limited to 8 MPa for the calculation of the concrete shear capacity (CSA 13.3.4.2).

If the effective depth, \( d \), exceeds 300 mm, the value of \( v_c \) is reduced by a factor equal to \( 1300/(1000 + d) \) (CSA 13.3.4.3).

### 5.6.2.4 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

\[
v_f = \frac{V_f}{bod} + \frac{\gamma v_s [M_{f2} - V_f (y_3 - y_1)] [I_{33} (y_4 - y_3) - I_{23} (x_4 - x_3)]}{I_{22} I_{33} - I_{23}^2} \]

Eq. 1

\[
I_{22} = \sum_{sides=1}^{n} T_{22}, \quad \text{where "sides" refers to the sides of the critical section for punching shear} \]

Eq. 2

\[
I_{33} = \sum_{sides=1}^{n} T_{33}, \quad \text{where "sides" refers to the sides of the critical section for punching shear} \]

Eq. 3

\[
I_{23} = \sum_{sides=1}^{n} T_{23}, \quad \text{where "sides" refers to the sides of the critical section for punching shear} \]

Eq. 4

The equations for \( T_{22}, T_{33}, \) and \( T_{23} \) are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 5-5.
Figure 5-5 Shear Stress Calculations at Critical Sections

\[ T_{22} = Ld(y_2 - y_3)^2, \text{ for side of critical section parallel to 2-axis} \]  
Eq. 5a

\[ T_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2, \text{ for side of critical section parallel to 3-axis} \]  
Eq. 5b

\[ T_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2, \text{ for side of critical section parallel to 2-axis} \]  
Eq. 6a

\[ T_{33} = Ld(x_2 - x_3)^2, \text{ for side of critical section parallel to 3-axis} \]  
Eq. 6b
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\[ T_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \]

**Eq. 7**

**NOTE:** \( T_{23} \) is explicitly set to zero for corner condition.

where,

- \( b_0 \): Perimeter of the critical section for punching shear
- \( d \): Effective depth at the critical section for punching shear based on the average of \( d \) for 2 direction and \( d \) for 3 direction
- \( I_{22} \): Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis
- \( I_{33} \): Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis
- \( I_{23} \): Product of inertia of the critical section for punching shear with respect to the 2 and 3 planes
- \( L \): Length of the side of the critical section for punching shear currently being considered
- \( M_{f2} \): Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)
- \( M_{f3} \): Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)
- \( V_f \): Punching shear stress
- \( V_f \): Shear at the center of the column (positive upward)
- \( x_1, y_1 \): Coordinates of the column centroid
- \( x_2, y_2 \): Coordinates of the center of one side of the critical section for punching shear
- \( x_3, y_3 \): Coordinates of the centroid of the critical section for punching shear
- \( x_4, y_4 \): Coordinates of the location where stress is being calculated
5.6.2.5 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

5.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CSA 13.2.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed, and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as explained in the subsections that follow.

5.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is taken as:

\[ v_c = 0.28 \lambda \phi_c \sqrt{f'_c} \]

for shear studs \hspace{1cm} (CSA 13.3.8.3)

\[ v_c = 0.19 \lambda \phi_c \sqrt{f'_c} \]

for shear stirrups \hspace{1cm} (CSA 13.3.9.3)

5.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of \[ v_{r,\text{max}} \], where
Given \( v_f, v_c, \) and \( v_{r,\text{max}} \), the required shear reinforcement is calculated as follows, where \( \phi_s \) is the strength reduction factor.

- If \( v_f > v_{r,\text{max}} \),
  \[
  A_v = \frac{(v_f - v_c)}{\phi_s f_{yv}} \cdot b_o
  \]
  (CSA 13.3.8.5, 13.3.9.4)

- If \( v_f > v_{r,\text{max}} \), a failure condition is declared. (CSA 13.3.8.2)

- If \( v_f \) exceeds the maximum permitted value of \( v_{r,\text{max}} \), the concrete section should be increased in size.

### 5.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 5-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.
The distance between the column face and the first line of shear reinforcement shall not exceed

\[
0.4d \text{ for shear studs} \quad \text{(CSA 13.3.8.6)}
\]

\[
0.25d \text{ for shear stirrups} \quad \text{(CSA 13.3.8.6)}
\]

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

### 5.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CSA 7.9 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, \( s_o \), between the column face and the first peripheral line of shear studs should not be smaller than 0.4\( d \). The limits of \( s_o \) and the spacing, \( s \), between the peripheral lines are specified as:

\[
 s_o \leq 0.4d \quad \text{(CSA 13.3.8.6)}
\]

\[
 s \leq \begin{cases} 
 0.75d & \nu_f \leq 0.56 \phi_p \sqrt{f'_c} \\
 0.50d & \nu_f > 0.56 \phi_p \sqrt{f'_c} 
\end{cases} \quad \text{(CSA 13.3.8.6)}
\]

For shear stirrups,

\[
 s_o \leq 0.25d \quad \text{(CSA 13.3.9.5)}
\]

\[
 s \leq 0.25d \quad \text{(CSA 13.3.9.5)}
\]

The minimum depth for reinforcement should be limited to 300 mm (CSA 13.3.9.1).
Chapter 6
Design for Eurocode 2-2004

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the European code Eurocode 2-2004 [EN 1992-1-1:2004] is selected. For the load combinations, reference is also made to Eurocode 0 [EN 1990], which is identified with the prefix “EC0.” Various notations used in this chapter are listed in Table 6-1. For referencing to the pertinent sections of the Eurocode in this chapter, a prefix “EC2” followed by the section number is used. It also should be noted that this section describes the implementation of the CEN Default version of Eurocode 2-2004, without a country specific National Annex. Where Nationally Determined Parameters [NDPs] are to be considered, this is highlighted in the respective section by the notation [NDP].

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.
6.1 Notations

Table 6-1 List of Symbols Used in the Eurocode 2-2004

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>Area of concrete section, mm$^2$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sl}$</td>
<td>Area of longitudinal reinforcement for torsion, mm$^2$</td>
</tr>
<tr>
<td>$A_{sw}$</td>
<td>Total cross-sectional area of links at the neutral axis, mm$^2$</td>
</tr>
<tr>
<td>$A_{sw}/s$</td>
<td>Area of shear reinforcement per unit length, mm$^2$/mm</td>
</tr>
<tr>
<td>$A_t/s$</td>
<td>Area of transverse reinforcement per unit length for torsion, mm$^2$/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width or effective width of the section in the compression zone, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Width or effective width of flange, mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Average web width of a flanged beam, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Effective depth of compression reinforcement, mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement</td>
</tr>
<tr>
<td>$f_{cd}$</td>
<td>Design concrete strength = $\alpha_{cc} f_{ck} / \gamma_c$, MPa</td>
</tr>
<tr>
<td>$f_{ck}$</td>
<td>Characteristic compressive concrete cylinder strength at 28 days, MPa</td>
</tr>
<tr>
<td>$f_{cwm}$</td>
<td>Mean value of concrete axial tensile strength, MPa</td>
</tr>
<tr>
<td>$f_{cwd}$</td>
<td>Design concrete compressive strength for shear design = $\alpha_{cc} f_{cwk} / \gamma_c$, MPa</td>
</tr>
<tr>
<td>$f_{cwk}$</td>
<td>Characteristic compressive cylinder strength for shear design, MPa</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Compressive stress in compression reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{yd}$</td>
<td>Design yield strength of reinforcement = $f_{yk} / \gamma_s$, MPa</td>
</tr>
</tbody>
</table>
### Table 6-1 List of Symbols Used in the Eurocode 2-2004

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{yk}$</td>
<td>Characteristic strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{ywd}$</td>
<td>Design strength of shear reinforcement = $f_{yk} / \gamma_s$, MPa</td>
</tr>
<tr>
<td>$f_{ywk}$</td>
<td>Characteristic strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of section, mm</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Flange thickness, mm</td>
</tr>
<tr>
<td>$M_{Ed}$</td>
<td>Design moment at a section, N-mm</td>
</tr>
<tr>
<td>$m$</td>
<td>Normalized design moment, $M/bd^2/\gamma_{fcd}$</td>
</tr>
<tr>
<td>$m_{lim}$</td>
<td>Limiting normalized moment capacity as a singly reinforced beam</td>
</tr>
<tr>
<td>$s_v$</td>
<td>Spacing of the shear reinforcement, mm</td>
</tr>
<tr>
<td>$T_{Ed}$</td>
<td>Torsion at ultimate design load, N-mm</td>
</tr>
<tr>
<td>$T_{Rdc}$</td>
<td>Torsional cracking moment, N-mm</td>
</tr>
<tr>
<td>$T_{Rd,max}$</td>
<td>Design torsional resistance moment, N-mm</td>
</tr>
<tr>
<td>$u$</td>
<td>Perimeter of the punch critical section, mm</td>
</tr>
<tr>
<td>$V_{Rdc}$</td>
<td>Design shear resistance from concrete alone, N</td>
</tr>
<tr>
<td>$V_{Rd,max}$</td>
<td>Design limiting shear resistance of a cross-section, N</td>
</tr>
<tr>
<td>$V_{Ed}$</td>
<td>Shear force at ultimate design load, N</td>
</tr>
<tr>
<td>$x$</td>
<td>Depth of neutral axis, mm</td>
</tr>
<tr>
<td>$x_{lim}$</td>
<td>Limiting depth of neutral axis, mm</td>
</tr>
<tr>
<td>$z$</td>
<td>Lever arm, mm</td>
</tr>
<tr>
<td>$\alpha_{cc}$</td>
<td>Coefficient accounting for long-term effects on the concrete compressive strength</td>
</tr>
<tr>
<td>$\alpha_{cw}$</td>
<td>Coefficient accounting for the state of stress in the compression chord</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Redistribution factor</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Concrete strain</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in tension reinforcement</td>
</tr>
<tr>
<td>$\varepsilon_s'$</td>
<td>Strain in compression steel</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>Partial safety factor for concrete strength</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Partial safety factor for reinforcement strength</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Factor defining the effective depth of the compression zone</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Effectiveness factor for shear resistance without concrete crushing</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Concrete strength reduction factor for sustained loading and stress block</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>Tension reinforcement ratio</td>
</tr>
<tr>
<td>$\sigma_{cp}$</td>
<td>Axial stress in the concrete, MPa</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of the concrete compression strut</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Normalized tension reinforcement ratio</td>
</tr>
<tr>
<td>$\omega'$</td>
<td>Normalized compression reinforcement ratio</td>
</tr>
<tr>
<td>$\omega_{lim}$</td>
<td>Normalized limiting tension reinforcement ratio</td>
</tr>
</tbody>
</table>

### 6.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be checked. Eurocode 0-2002 allows load combinations to be defined based on EC0 Equation 6.10 or the less favorable of EC0 Equations 6.10a and 6.10b [NDP].

\[
\sum_{j=1}^{j} \gamma_{G,j}G_{k,j} + \gamma_pP + \gamma_{Q,1}Q_{k,1} + \sum_{i=1}^{i} \gamma_{Q,i}\psi_{0,i}Q_{k,j} = (EC0 Eq. 6.10)
\]

\[
\sum_{j=1}^{j} \gamma_{G,j}G_{k,j} + \gamma_pP + \gamma_{Q,1}\psi_{0,1}Q_{k,1} + \sum_{i=1}^{i} \gamma_{Q,i}\psi_{0,i}Q_{k,j} = (EC0 Eq. 6.10a)
\]

\[
\sum_{j=1}^{j} \xi_{j}\gamma_{G,j}G_{k,j} + \gamma_pP + \gamma_{Q,1}Q_{k,1} + \sum_{i=1}^{i} \gamma_{Q,i}\psi_{0,i}Q_{k,j} = (EC0 Eq. 6.10b)
\]

Load combinations considering seismic loading are automatically generated based on EC0 Equation 6.12b.

6 - 4 Design Load Combinations
\[ \sum_{j=1}^{n} G_{k,j} + P + A_{Ed} + \sum_{j=1}^{n} \psi_{2,j} Q_{k,j} \]  

(EC0 Eq. 6.12b)

For this code, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations need to be considered if equation 6.10 is specified for generation of the load combinations (EC0 6.4.3):

\[ \gamma_{Gj,\text{sup}} D \]

(EC0 Eq. 6.10)

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} L \]

(EC0 Eq. 6.10)

\[ \gamma_{Gj,\text{sup}} D + (0.75) \gamma_{Q,1} \text{PL} \]

(EC0 Eq. 6.10)

\[ \gamma_{Gj,\text{inf}} D \pm \gamma_{Q,1} W \]

\[ \gamma_{Gj,\text{sup}} D \pm \gamma_{Q,1} W \]

(EC0 Eq. 6.10)

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} L \pm \gamma_{Q,1} \psi_{0,1} W \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} L + \gamma_{Q,1} \psi_{0,1} S \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} W + \gamma_{Q,1} \psi_{0,1} L \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} W + \gamma_{Q,1} \psi_{0,1} S \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} S \pm \gamma_{Q,1} \psi_{0,1} W \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} S + \gamma_{Q,1} \psi_{0,1} L \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} L + \gamma_{Q,1} \psi_{0,1} W + \gamma_{Q,1} \psi_{0,1} W \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} L + \gamma_{Q,1} \psi_{0,1} S + \gamma_{Q,1} \psi_{0,1} S \]

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} S + \gamma_{Q,1} \psi_{0,1} W + \gamma_{Q,1} \psi_{0,1} L \]

\[ D \pm 1.0E \]

\[ D \pm 1.0E + \psi_{2,1} L \]

\[ D \pm 1.0E + \psi_{2,1} L + \gamma_{Q,1} \psi_{0,1} S \]

(EC0 Eq. 6.12b)

If the load combinations are specified to be generated from the max of EC0 Equations 6.10a and 6.10b, the following load combinations from both equations are considered in the program.

\[ \gamma_{Gj,\text{sup}} D \]

(EC0 Eq. 6.10a)

\[ \xi \gamma_{Gj,\text{sup}} D \]

(EC0 Eq. 6.10b)

\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} \psi_{0,1} L \]

(EC0 Eq. 6.10a)
SAFE Reinforced Concrete Design

\[ \xi \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} L \]  
\[ \gamma_{Gj,\text{sup}} D + (0.75)\gamma_{Q,1} \psi_{0,1} PL \]  
\[ \xi \gamma_{Gj,\text{sup}} D + (0.75)\gamma_{Q,1} PL \]  
\[ \gamma_{Gj,\text{inf}} D \pm \gamma_{Q,1} \psi_{0,1} W \]  
\[ \gamma_{Gj,\text{sup}} D \pm \gamma_{Q,1} \psi_{0,1} W \]  
\[ \gamma_{Gj,\text{inf}} D \pm \gamma_{Q,1} W \]  
\[ \xi \gamma_{Gj,\text{sup}} D \pm \gamma_{Q,1} W \]  
\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} \psi_{0,1} L \pm \gamma_{Q,i} \psi_{0,i} W \]  
\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} \psi_{0,1} L + \gamma_{Q,i} \psi_{0,i} S \]  
\[ \gamma_{Gj,\text{sup}} D \pm \gamma_{Q,1} \psi_{0,1} W + \gamma_{Q,i} \psi_{0,i} L \]  
\[ \gamma_{Gj,\text{sup}} D \pm \gamma_{Q,1} \psi_{0,1} W + \gamma_{Q,i} \psi_{0,i} S \]  
\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} \psi_{0,1} S + \gamma_{Q,i} \psi_{0,i} L \]  
\[ \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} \psi_{0,1} S \pm \gamma_{Q,i} \psi_{0,i} W \]  
\[ \xi \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} \psi_{0,1} L \pm \gamma_{Q,i} \psi_{0,i} W \]  
\[ \xi \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} S \pm \gamma_{Q,i} \psi_{0,i} \]  
\[ \xi \gamma_{Gj,\text{sup}} D + \gamma_{Q,1} S + \gamma_{Q,i} \psi_{0,i} L \]  
\[ \gamma_{Gj,\text{inf}} D \pm \gamma_{Q,1} W + \gamma_{Q,i} \psi_{0,i} L \]  
\[ \gamma_{Gj,\text{inf}} D \pm \gamma_{Q,1} W \pm \gamma_{Q,i} \psi_{0,i} S \]  
\[ D \pm 1.0E \]  
\[ D \pm 1.0E + \psi_{2,i} L \]  
\[ D \pm 1.0E + \psi_{2,i} L + \psi_{2,i} S \]  

For both sets of load combinations, the variable values for the CEN Default version of the load combinations are defined in the list that follows [NDP].

\[ \gamma_{Gj,\text{sup}} = 1.35 \]  
\[ \gamma_{Gj,\text{inf}} = 1.00 \]  
\[ \gamma_{Q,1} = 1.5 \]  
\[ \gamma_{Q,i} = 1.5 \]  
\[ \psi_{0,i} = 0.7 \text{ (live load, assumed not to be storage)} \]  
\[ \psi_{0,i} = 0.6 \text{ (wind load)} \]  

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\[ \psi_{0,i} = 0.5 \text{ (snow load, assumed } H \leq 1,000 \text{ m)} } \quad (\text{EC0 Table A1.1}) \\
\xi = 0.85 \quad (\text{EC0 Table A1.2(B)}) \\
\psi_{2,i} = 0.3 \text{ (live, assumed office/residential space) } \quad (\text{EC0 Table A1.1}) \\
\psi_{2,i} = 0 \text{ (snow, assumed } H \leq 1,000 \text{ m)} } \quad (\text{EC0 Table A1.1})

These are also the default design load combinations in SAFE whenever the Eurocode 2-2004 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

6.3 Limits on Material Strength

The concrete compressive strength, \( f_{ck} \), should not be greater than 90 MPa (EC2 3.1.2(2)). The lower and upper limits of the reinforcement yield strength, \( f_{yk} \), should be 400 and 600 MPa, respectively (EC2 3.2.2(3)).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. It is the user's responsibility to ensure that the minimum strength is satisfied.

6.4 Partial Safety Factors

The design strengths for concrete and steel are obtained by dividing the characteristic strengths of the materials by the partial safety factors, \( \gamma_s \) and \( \gamma_c \) as shown here \([NDP]\).

\[
\begin{align*}
    f_{cd} &= \alpha_{cc} f_{ck} / \gamma_c \quad \text{(EC2 3.1.6(1))} \\
    f_{yd} &= f_{yk} / \gamma_s \quad \text{(EC2 3.2.7(2))} \\
    f_{ywd} &= f_{ywk} / \gamma_s \quad \text{(EC2 3.2.7(2))}
\end{align*}
\]
\( \alpha_{cc} \) is the coefficient taking account of long term effects on the compressive strength. \( \alpha_{cc} \) is taken as 1.0 by default and can be overwritten by the user (EC2 3.1.6(1)).

The partial safety factors for the materials and the design strengths of concrete and reinforcement are given in the text that follows (EC2 2.4.2.4(1), Table 2.1N):

- Partial safety factor for reinforcement, \( \gamma_{s} = 1.15 \)
- Partial safety factor for concrete, \( \gamma_{c} = 1.5 \)

These values are recommended by the code to give an acceptable level of safety for normal structures under regular design situations (EC2 2.4.2.4). For accidental and earthquake situations, the recommended values are less than the tabulated values. The user should consider those separately.

These values can be overwritten; however, caution is advised.

### 6.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement
6.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

6.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

6.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 6-1 (EC2 3.1.7(3)). The area of the stress block and the depth of the compressive block is taken as:

\[ F_c = \eta f_{cd} \cdot ab \]  

\[ a = \lambda x \]  

(EC2 3.1.7(3), Fig 3.5)
where \( x \) is the depth of the neutral axis. The factor \( \lambda \) defining the effective height of the compression zone and the factor \( \eta \) defining the effective strength are given as:

\[
\lambda = 0.8 \quad \text{for} \quad f_{ck} \leq 50 \, \text{MPa} \quad \text{(EC2 3.1.7(3))}
\]

\[
\lambda = 0.8 \left( \frac{f_{ck} - 50}{400} \right) \quad \text{for} \quad 50 < f_{ck} \leq 90 \, \text{MPa} \quad \text{(EC2 3.1.7(3))}
\]

\[
\eta = 1.0 \quad \text{for} \quad f_{ck} \leq 50 \, \text{MPa} \quad \text{(EC2 3.1.7(3))}
\]

\[
\eta = 1.0 - \left( \frac{f_{ck} - 50}{200} \right) \quad \text{for} \quad 50 < f_{ck} \leq 90 \, \text{MPa} \quad \text{(EC2 3.1.7(3))}
\]

Furthermore, it is assumed that moment redistribution in the member does not exceed the code-specified limiting value. The code also places a limitation on the neutral axis depth, to safeguard against non-ductile failures (EC2 5.5(4)). When the applied moment exceeds the limiting moment capacity as a singly reinforced beam, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, \( \left( \frac{x}{d} \right)_{\text{lim}} \), is expressed as a function of the ratio of the redistributed moment to the moment before redistribution, \( \delta \), as follows:

\[
\left( \frac{x}{d} \right)_{\text{lim}} = \frac{\delta - k_1}{k_2} \quad \text{for} \quad f_{ck} \leq 50 \, \text{MPa} \quad \text{(EC2 5.5(4))}
\]

\[
\left( \frac{x}{d} \right)_{\text{lim}} = \frac{\delta - k_3}{k_4} \quad \text{for} \quad f_{ck} > 50 \, \text{MPa} \quad \text{(EC2 5.5(4))}
\]

For reinforcement with \( f_{yk} \leq 500 \, \text{MPa} \), the following values are used:

\[
k_1 = 0.44 \, [NDP] \quad \text{(EC 5.5(4))}
\]

\[
k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{\text{cu2}}) \, [NDP] \quad \text{(EC 5.5(4))}
\]

\[
k_3 = 0.54 \, [NDP] \quad \text{(EC 5.5(4))}
\]
\( \delta \) is assumed to be 1

where the ultimate strain, \( \varepsilon_{cu2} \) \([NDP]\), is determined from EC2 Table 3.1 as:

\[
\varepsilon_{cu2} = 0.0035 \text{ for } f_{ck} < 50 \text{ MPa} \quad \text{(EC2 Table 3.1)}
\]

\[
\varepsilon_{cu2} = 2.6 + 35 \left( \frac{90 - f_{ck}}{100} \right)^4 \text{ for } f_{ck} \geq 50 \text{ MPa} \quad \text{(EC2 Table 3.1)}
\]

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow.

**Figure 6-1 Rectangular Beam Design**

### 6.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the normalized moment, \( m \), and the normalized section capacity as a singly reinforce beam, \( m_{lim} \), are obtained first. The reinforcement area is determined based on whether \( m \) is greater than, less than, or equal to \( m_{lim} \).
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- The normalized design moment, $m$, is calculated as:

$$m = \frac{M}{bd^2\eta_{cd}}$$

- The normalized concrete moment capacity as a singly reinforced beam, $m_{lim}$, is calculated as:

$$m_{lim} = \lambda \left( \frac{x}{d} \right)_{lim} \left[ 1 - \frac{\lambda}{2} \left( \frac{x}{d} \right)_{lim} \right]$$

- If $m \leq m_{lim}$, a singly reinforced beam is designed. The normalized reinforcement ratio is calculated as:

$$\omega = 1 - \sqrt{1 - 2m}$$

The area of tension reinforcement, $A_s$, is then given by:

$$A_s = \omega \left( \frac{\eta_{cd} b d}{f_{yd}} \right)$$

This reinforcement is to be placed at the bottom if $M_{Ed}$ is positive, or at the top if $M_{Ed}$ is negative.

- If $m > m_{lim}$, both tension and compression reinforcement is designed as follows:

The normalized steel ratios $\omega'$, $\omega_{lim}$, and $\omega$ are calculated as:

$$\omega_{lim} = \lambda \left( \frac{x}{d} \right)_{lim} = 1 - \sqrt{1 - 2m_{lim}}$$

$$\omega' = \frac{m - m_{lim}}{1 - d'/d}$$

$$\omega = \omega_{lim} + \omega'$$

where, $d'$ is the depth to the compression reinforcement from the concrete compression face.

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The area of compression and tension reinforcement, $A'_s$ and $A_s$, are given by:

$$A'_s = \omega \left[ \frac{\eta f_{cd} b d}{f'_s - \eta f_{cd}} \right]$$

$$A_s = \omega \left[ \frac{\eta f'_{cd} b d}{f'_{yd}} \right]$$

where, $f'_s$ is the stress in the compression reinforcement, and is given by:

$$f'_s = E_s \varepsilon_{cu_3} \left[ 1 - \frac{d'}{x_{lim}} \right] \leq f'_{yd} \quad \text{(EC2 6.1, 3.2.7(4), Fig 3.8)}$$

### 6.5.1.2.2 Design of Flanged Beams

#### 6.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M_{Ed}$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

#### 6.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located within the flange. Based on this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange, additional calculation is required. See Figure 6-2.

- The normalized design moment, $m$, is calculated as:

$$m = \frac{M}{b d^2 \eta f_{cd}} \quad \text{(EC2 6.1, 3.1.7(3))}$$

- The limiting values are calculated as:
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\[ m_{\text{lim}} = \lambda \left( \frac{x}{d} \right)_{\text{lim}} \left[ 1 - \frac{\lambda}{2} \left( \frac{x}{d} \right)_{\text{lim}} \right] \]  

(EC2 5.5(4), 3.1.7(3))

\[ \omega_{\text{lim}} = \lambda \left( \frac{x}{d} \right)_{\text{lim}} \]

\[ a_{\text{max}} = \omega_{\text{lim}} d \]

- The values \( \omega \) and \( a \) are calculated as:

\[ \omega = 1 - \sqrt{1 - 2m} \]

\[ a = \omega d \]

[Figures 6-2 T-Beam Design]

- If \( a \leq h_f \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( m > m_{\text{lim}} \).

- If \( a > h_f \), the calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, and the second part is for balancing the compressive force from the web, as shown in Figure 6-2. The reinforcement area required for balancing the flange compression, \( A_{s2} \) is given as:

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\[ A_{s2} = \frac{(b_f - b_w)h_f \eta f_{cd}}{f_{yd}} \]

and the corresponding resistive moment is given by

\[ M_2 = A_{s2} f_{yd} \left( d - \frac{h_f}{2} \right) \]

The reinforcement required for balancing the compressive force from the web, considering a rectangular section of width \( b_w \) to resist the moment, \( M_1 = M - M_2 \), is determined as follows:

\[ m_1 = \frac{M_1}{b_w d^2 \eta f_{cd}} \]

- If \( m_1 \leq m_{\text{lim}} \),

\[ \omega_h = 1 - \sqrt{1 - 2m_1} \]

\[ A_{s1} = \omega_h \left[ \frac{\eta f_{cd} b_w d}{f_{yd}} \right] \]

- If \( m_1 > m_{\text{lim}} \),

\[ \omega' = \frac{m_1 - m_{\text{lim}}}{1 - d'/d} \]

\[ \omega_{\text{lim}} = \lambda \left( \frac{x}{d} \right)_{\text{lim}} \]

\[ \omega_h = \omega_{\text{lim}} + \omega' \]

\[ A'_{s} = \omega' \left[ \frac{\eta f_{cd} bd}{f'_s - \eta f_{cd}} \right] \]

\[ A_{s1} = \omega_h \left[ \frac{\eta f_{cd} b_w d}{f_{yd}} \right] \]
where, $f'_s$ is given by:

$$f'_s = E_s E_{cm3} \left[ 1 - \frac{d'}{x_{lim}} \right] \leq f_{yd} \quad \text{(EC2 6.1, 3.2.7(4), Fig 3.8)}$$

The total tension reinforcement is $A_s = A_s1 + A_s2$, and the total compression reinforcement is $A's$. $A_s$ is to be placed at the bottom and $A's$ is to be placed at the top.

### 6.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement, $A_{s,\min}$ [NDP], required in a beam section is given by the maximum of the following two limits:

$$A_{s,\min} = 0.26 \frac{f_{cm}}{f_{yk}} bd \quad \text{(EC2 9.2.1.1(1))}$$

$$A_{s,\min} = 0.0013bd \quad \text{(EC2 9.2.1.1(1))}$$

where $f_{cm}$ is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{cm} = 0.30 f_{ck}^{(2/3)} \text{ for } f_{ck} \leq 50 \text{ MPa} \quad \text{(EC2 3.12, Table 3.1)}$$

$$f_{cm} = 2.12 \ln \left(1 + \frac{f_{cm}}{10}\right) \text{ for } f_{ck} > 50 \text{ MPa} \quad \text{(EC2 3.12, Table 3.1)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \quad \text{(EC2 3.12, Table 3.1)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement, $A_{s,\max}$ [NDP], has been imposed to be 0.04 times the gross cross-sectional area (EC 9.2.1.1(3)).

### 6.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a partic-
ular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (EC2 6.2):

- Determine the factored shear force, $V_{Ed}$.
- Determine the shear force, $V_{rd,c}$, that can be resisted by the concrete.
- Determine the shear reinforcement required.

The following three sections describe in detail the algorithms associated with these steps.

### 6.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

### 6.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_{rd,c}$, is calculated as:

$$V_{rd,c} = \left[ C_{rd,c}k \left( 100 \rho_t f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] b_w d$$  \hspace{1cm} (EC2 6.2.2(1))

with a minimum of:

$$V_{rd,c} = \left( v_{min} + k_1 \sigma_{cp} \right) b_w d$$  \hspace{1cm} (EC2 6.2.2(1))

where

$f_{ck}$ is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm}$$  \hspace{1cm} (EC2 6.2.2(1))

$$\rho_t = \text{tension reinforcement ratio} = \frac{A_{s1}}{b_w d} \leq 0.02$$  \hspace{1cm} (EC2 6.2.2(1))

$$A_{s1} = \text{area of tension reinforcement}$$  \hspace{1cm} (EC2 6.2.2(1))
\[ \sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{cd} \text{ MPa} \] (EC2 6.2.2(1))

The value of \( C_{Rd,c} \), \( v_{\text{min}} \) and \( k_1 \) for use in a country may be found in its National Annex. The program default values for \( C_{Rd,c} \), \( v_{\text{min}} \), and \( k_1 \) are given as follows (EC2 6.2.2(1)):

\[ C_{Rd,c} = 0.18/\gamma_c \] (EC2 6.2.2(1))
\[ v_{\text{min}} = 0.035 \ k^{3/2} f_{ck}^{1/2} \] (EC2 6.2.2(1))
\[ k_1 = 0.15. \] (EC2 6.2.2(1))

For light-weight concrete:

\[ C_{Rd,c} = 0.18/\gamma_c \] (EC2 11.6.1(1))
\[ v_{\text{min}} = 0.03 \ k^{3/2} f_{ck}^{1/2} \] (EC2 11.6.1(1))
\[ k_1 = 0.15. \] (EC2 11.6.1(1))

6.5.2.3 **Determine Required Shear Reinforcement**

The shear force is limited to a maximum of:

\[ V_{Rd,\text{max}} = \frac{\alpha_{cw} b_z v_1 f_{cd}}{\cot \theta + \tan \theta}, \text{ where} \]

\( \alpha_{cw} \) \([NDP]\) is conservatively taken as 1. (EC2 6.2.3(3))

The strength reduction factor for concrete cracked in shear, \( v_1 \) \([NDP]\), is defined as:

\[ v_1 = 0.6 \left(1 - \frac{f_{ck}}{250}\right) \] (EC2 6.2.2(6))

\[ z = 0.9d \] (EC2 6.2.3(1))

\( \theta \) is optimized by the program and is set to 45° for combinations including seismic loading (EC2 6.2.3(2)).

Given \( V_{Ed} \), \( V_{Rdc} \), and \( V_{Rd,\text{max}} \), the required shear reinforcement is calculated as follows:
- If $V_{Ed} \leq V_{Rd, c}$,
  \[ \frac{A_{sw}}{s} = \frac{A_{sw, min}}{s} \]

- If $V_{R, dc} < V_{Ed} \leq V_{Rd, max}$
  \[ \frac{A_{sw}}{s} = \frac{V_{Ed}}{zf_{yw} \cot \theta} \geq \frac{A_{sw, min}}{s} \]  \hspace{1cm} (EC2 6.2.3(3))

- If $V_{Ed} > V_{Rd, max}$, a failure condition is declared.  \hspace{1cm} (EC2 6.2.3(3))

The minimum shear reinforcement is defined as:

\[ \frac{A_{sw, min}}{s} = 0.08 \sqrt{\frac{f_{ck}}{f_{yk}}} b_w \]  \hspace{1cm} (EC2 9.2.2(5))

The maximum of all of the calculated $A_{sw}/s$ values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

### 6.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, $T_{Ed}$.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.
6.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design $T_{Ed}$ is permitted to be reduced in accordance with the code (EC2 6.3.1(2)). However, the program does not automatically redistribute the internal forces and reduce $T_{Ed}$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

6.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as $A_k$, $t_{ef}$, $u$, $u_k$, and $z_i$ are calculated. These properties are described as follows (EC2 6.3.2).

$$A = \text{Area enclosed by the outside perimeter of the cross-section}$$

$$A_k = \text{Area enclosed by centerlines of the connecting walls, where the centerline is located a distance of } t_{ef}/2 \text{ from the outer surface}$$

$$t_{ef} = \text{Effective wall thickness, } A/u. \text{ It is taken as at least twice the distance between the edge and center of the longitudinal rebar.}$$

$$u = \text{Outer perimeter of the cross-section}$$

$$u_k = \text{Perimeter of the area } A_k$$

$$z_i = \text{Side length of wall } i, \text{ defined as the distance between the intersection points of the wall centerlines}$$

In calculating the section properties involving reinforcement, such as $A_k$ and $u_k$, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during calculation of torsion.
section properties. With this assumption, the special properties for a rectangular beam section are given as:

\[ A = bh \]  
\[ A_k = (b - t_{\text{ef}})(h - t_{\text{ef}}) \]  
\[ u = 2b + 2h \]  
\[ u_k = 2(b - t_{\text{ef}}) + 2(h - t_{\text{ef}}) \]  
\[ t_{\text{ef}} = \frac{A}{u} \geq 2 \times \text{cover to center} \]

where, the section dimensions \( b, h, \) and \( c \) are shown in Figure 6-3. Similarly, the special section properties for a flanged beam section are given as:

\[ A = b_w h \]  
\[ A_k = (b_w - t_{\text{ef}})(h - t_{\text{ef}}) \]  
\[ u = 2b_w + 2h \]  
\[ u_k = 2(h - t_{\text{ef}}) + 2(b_w - t_{\text{ef}}) \]

where the section dimensions \( b_f, b_w, h, h_f, \) and \( c \) for a flanged beam are shown in Figure 6-3.

6.5.3.3 Determine Critical Torsion Capacity

The torsion in the section can be ignored, with only minimum shear reinforcement (EC2 9.2.1.1) required, if the following condition is satisfied:

\[ \frac{T_{\text{Ed},c}}{T_{\text{Ed},E}} + \frac{V_{\text{Ed},c}}{V_{\text{Ed},E}} \leq 1.0 \]  
\[ \text{(EC2 6.3.2(5))} \]

where \( V_{\text{Ed},c} \) is as defined in the previous section and \( T_{\text{Ed},c} \) is the torsional cracking moment, calculated as:

\[ T_{\text{Ed},c} = f_{\text{cd}} t_{\text{ef}} \frac{2A_k}{T_{\text{Ed},c}} \]  
\[ \text{(EC2 6.3.2(1), 6.3.2(5))} \]

where \( t_{\text{ef}}, \) and \( f_{\text{cd}}, \) the design tensile strength, are defined as:
Figure 6-3 Closed stirrup and section dimensions for torsion design

\[ t_{ef} = A/u \]  
\[ f_{cdt} = \alpha_{ct} f_{c0.05} / \gamma_c \]  

where \( A \) is the gross cross-section area, \( u \) is the outer circumference of the cross-section, \( \alpha_{ct} \) \([NDP]\) is a coefficient, taken as 1.0, taking account of long term effects on the tensile strength, and \( f_{c0.05} \) is defined as:

\[ f_{c0.05} = 0.7 f_{ctm} \]  

6.5.3.4 Determine Torsion Reinforcement

If the expression in the previous subsection is satisfied, torsion can be safely ignored (EC2 6.3.2(5)), with only minimum shear reinforcement required. In that case, the program reports that no torsion reinforcement is required. However, if the equation is not satisfied, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals.

If torsion reinforcement in the form of closed stirrups is required, the shear due to this torsion, \( V_t \), is first calculated, followed by the required stirrup area, as:
The required longitudinal reinforcement for torsion is defined as:

\[ A_{sl} = \frac{V_t}{2f_{yd} \cot \theta} \]

where \( \theta \) is the angle of the compression struts, as previously defined for beam shear. In the preceding expressions, \( \theta \) is taken as 45 degrees. The code allows any value between 21.8 and 45 degrees (EC2 6.2.3(2)), while the program assumes the conservative value of 45 degrees.

When torsional reinforcement is required, an upper limit on the combination of \( V_{Ed} \) and \( T_{Ed} \) that can be carried by the section without exceeding the capacity of the concrete struts also is checked using:

\[ \frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1.0 \]

where \( T_{Rd,max} \), the design torsional resistance moment is defined as:

\[ T_{Rd,max} = 2\nu\alpha_{cw} f_{cd} A_{kt} t_{ef} \sin \theta \cos \theta \]

If this equation is not satisfied, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated \( A_{sl} \) and \( A_t/s \) values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or
longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

6.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (Eurocode 2-2004) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

6.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.
6.6.1.1 Determine Factored Moments for Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

6.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

6.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (EC2 9.3.1.1) \([NDP]\):

\[
A_{s,\min} = 0.26 \frac{f_{cm}}{f_{yk}} bd
\]

\[
A_{s,\min} = 0.0013 bd
\]

where \(f_{cm}\) is the mean value of axial tensile strength of the concrete and is computed as:

\[
f_{cm} = 0.30 f_{ck}^{(2/3)} \text{ for } f_{ck} \leq 50 \text{ MPa}
\]

\[
f_{cm} = 2.12 \ln(1 + f_{cm}/10) \text{ for } f_{ck} > 50 \text{ MPa}
\]

\[
f_{cm} = f_{ck} + 8 \text{ MPa}
\]

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.
An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC 9.2.1.1(3)).

### 6.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

#### 6.6.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (EC 2.6.4.1(4)) and at a critical section at a distance of $2.0d$ from the face of the support (EC 2.6.4.2(1)). The perimeter of the critical section should be constructed such that its length is minimized. Figure 6-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 6-4 Punching Shear Perimeters](image)

---

6 - 26 Slab Design
6.6.2.2 Determination of Concrete Capacity

The concrete punching shear stress capacity is taken as:

\[ V_{rd,c} = \left[ C_{rd,c} \cdot k \left( 100 \rho_1 f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] \]  

(EC2 6.4.4(1))

with a minimum of:

\[ V_{rd,c} = \left( \nu_{\text{min}} + k_1 \sigma_{cp} \right) \]  

(EC2 6.4.4(1))

where \( f_{ck} \) is in MPa and

\[ k = 1 + \frac{200}{d} \leq 2.0 \text{ with } d \text{ in mm} \]  

(EC2 6.4.4(1))

\[ \rho_i = \sqrt{\rho_{ix} \rho_{iy}} \leq 0.02 \]  

(EC2 6.4.4(1))

where \( \rho_{ix} \) and \( \rho_{iy} \) are the reinforcement ratios in the x and y directions respectively, which is taken as the average tension reinforcement ratios of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement ratio is taken as zero in the current implementation, and

\[ \sigma_{cp} = \frac{\sigma_{cx} + \sigma_{cy}}{2} \]  

(EC2 6.4.4(1))

where \( \sigma_{cx} \) and \( \sigma_{cy} \) are the normal concrete stresses in the critical section in the x and y directions respectively, conservatively taken as zeros.

\[ C_{rd,c} = 0.18/\gamma_c \text{ [NDP]} \]  

(EC2 6.4.4(1))

\[ \nu_{\text{min}} = 0.035 k^{3/2} f_{ck}^{1/2} \text{ [NDP]} \]  

(EC2 6.4.4(1))

\[ k_1 = 0.15 \text{ [NDP]} \]  

(EC2 6.4.4(1))

6.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, \( \nu_{\text{Ed}} \), is calculated as:

\[ \nu_{\text{Ed}} = \left( \frac{1}{\gamma_c} \cdot \frac{1}{2} C_{rd,c} k \nu_{\text{min}} \right) \]  

where \( \gamma_c \) is the concrete strength reduction factor.
where (EC2 6.4.4(2))

\[ v_{Ed} = \frac{V_{Ed}}{ud} \left[ 1 + k \frac{M_{Ed,2}u_1}{V_{Ed}W_{1,2}} + k \frac{M_{Ed,3}u_1}{V_{Ed}W_{1,3}} \right], \]

where \( k \) is the function of the aspect ratio of the loaded area in Table 6.1 of EN 1992-1-1.

\( u_1 \) is the effective perimeter of the critical section.

\( d \) is the mean effective depth of the slab.

\( M_{Ed} \) is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3.

\( V_{Ed} \) is the total punching shear force.

\( W_1 \) accounts for the distribution of shear based on the control perimeter along bending axis 2 and 3.

### 6.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

### 6.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as described in the subsections that follow.
6.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

6.6.3.2 Determine Required Shear Reinforcement

The shear is limited to a maximum of $V_{Rd,max}$ calculated in the same manner as explained previously for beams.

Given $V_{Ed}$, $V_{Rd,c}$, and $V_{Rd,max}$, the required shear reinforcement is calculated as follows (EC2 6.4.5).

- If $V_{R,d,c} < V_{Ed} \leq V_{Rd,max}$

$$A_{sw} = \frac{(V_{Ed} - 0.75V_{Rd,c})}{1.5f_yd\sigma_f}$$

(EC2 6.4.5)

- If $V_{Ed} > V_{Rd,max}$, a failure condition is declared. (EC2 6.2.3(3))

- If $V_{Ed}$ exceeds the maximum permitted value of $V_{Rd,max}$, the concrete section should be increased in size.

6.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 6-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.
The distance between the column face and the first line of shear reinforcement shall not exceed 2d. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5d measured in a direction parallel to the column face (EC2 9.4.3(1)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

### 6.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in EC2 4.4.1 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than 0.3d. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not ex-
ceed 1.5\(d\) and should not exceed 2\(d\) at additional perimeters. The limits of \(s_o\) and the spacing, \(s\), between the peripheral lines are specified as:

\[
0.3d \leq s_o \leq 2d \quad \text{(EC2 9.4.3(1))}
\]
\[
s \leq 0.75d \quad \text{(EC2 9.4.3(1))}
\]
\[
g \leq 1.5d \quad \text{(first perimeter)} \quad \text{(EC2 9.4.3(1))}
\]
\[
g \leq 2d \quad \text{(additional perimeters)} \quad \text{(EC2 9.4.3(1))}
\]

### 6.7 Nationally Determined Parameters (NDPs)

The Comité Européen de Normalisation (CEN) version of Eurocode 2-2004 specifies a set of clauses in the design code, for which Nationally Determined Parameters [NDPs] are permitted to be adjusted by each member country within their National Annex. Variations in these parameters between countries are considered in the program by choosing the desired country from the Options menu > Preferences > Concrete Frame Design command. This appendix lists the NDPs as adopted in the program for the CEN Default version of the design code. Additional tables are provided that list the NDPs that differ from the CEN Default values for each country supported in the program.

**Table 6-2 CEN Default NDPs**

<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_c)</td>
<td>2.4.2.4(1)</td>
<td>1.5</td>
</tr>
<tr>
<td>(\gamma_s)</td>
<td>2.4.2.4(1)</td>
<td>1.15</td>
</tr>
<tr>
<td>(\alpha_{cc})</td>
<td>3.1.6(1)</td>
<td>1.0</td>
</tr>
<tr>
<td>(\alpha_{ct})</td>
<td>3.1.6(2)</td>
<td>1.0</td>
</tr>
<tr>
<td>(\max f_{sk})</td>
<td>3.2.2(3)</td>
<td>600MPa</td>
</tr>
<tr>
<td>Load Combinations</td>
<td>5.1.3(1)</td>
<td>Combinations from Eq. 6.10</td>
</tr>
<tr>
<td>NDP</td>
<td>Clause</td>
<td>Value</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>$\theta$</td>
<td>5.2(5)</td>
<td>0.005</td>
</tr>
<tr>
<td>$k_1$</td>
<td>5.5(4)</td>
<td>0.44</td>
</tr>
<tr>
<td>$k_2$</td>
<td>5.5(4)</td>
<td>$1.25(0.6 + 0.0014/\varepsilon_{cu}^2)$</td>
</tr>
<tr>
<td>$k_3$</td>
<td>5.5(4)</td>
<td>0.54</td>
</tr>
<tr>
<td>$k_4$</td>
<td>5.5(4)</td>
<td>$1.25(0.6 + 0.0014/\varepsilon_{cu}^2)$</td>
</tr>
<tr>
<td>$\lambda_{lim}$</td>
<td>5.8.3.1(1)</td>
<td>$20 \cdot A \cdot B \cdot C/\sqrt{n}$</td>
</tr>
<tr>
<td>$C_{bd,c}$</td>
<td>6.2.2(1)</td>
<td>$0.18/\gamma_c$</td>
</tr>
<tr>
<td>$v_{min}$</td>
<td>6.2.2(1)</td>
<td>$0.035k^{1/2}f_{ck}1/2$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>6.2.2(1)</td>
<td>0.15</td>
</tr>
<tr>
<td>$\theta$</td>
<td>6.2.3(2)</td>
<td>45 degrees</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>6.2.3(3)</td>
<td>$0.6\left[1 - \frac{f_{ck}}{250}\right]$</td>
</tr>
<tr>
<td>$\alpha_{cw}$</td>
<td>6.2.3(3)</td>
<td>1.0</td>
</tr>
<tr>
<td>Beam $A_{s,\text{min}}$</td>
<td>9.2.1.1(1)</td>
<td>$0.26\frac{f_{im}}{f_{ck}}b_d \geq 0.0013b_d$</td>
</tr>
<tr>
<td>Beam $A_{s,\text{max}}$</td>
<td>9.2.1.1(3)</td>
<td>$0.04A_c$</td>
</tr>
<tr>
<td>Beam $\rho_{s,\text{min}}$</td>
<td>9.2.2(5)</td>
<td>$\left(0.08\sqrt{f_{ck}}\right)/f_{ck}$</td>
</tr>
<tr>
<td>$\alpha_{cc}$</td>
<td>11.3.5(1)</td>
<td>0.85</td>
</tr>
<tr>
<td>$\alpha_{ct}$</td>
<td>11.3.5(2)</td>
<td>0.85</td>
</tr>
</tbody>
</table>
### Table 6-2 CEN Default NDPs

<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{nd,c}$</td>
<td>11.6.1(1)</td>
<td>$0.15/\gamma_c$</td>
</tr>
<tr>
<td>$v_{i,\min}$</td>
<td>11.6.1(1)</td>
<td>$0.30k^{3/2}f_{ck}^{1/2}$</td>
</tr>
<tr>
<td>$k_i$</td>
<td>11.6.1(1)</td>
<td>0.15</td>
</tr>
<tr>
<td>$v_i$</td>
<td>11.6.2(1)</td>
<td>$0.5\eta(1 - f_{ck}/250)$</td>
</tr>
</tbody>
</table>

### Table 6-3 Denmark NDPs

<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_c$</td>
<td>2.4.2.4(1)</td>
<td>1.45</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>2.4.2.4(1)</td>
<td>1.20</td>
</tr>
<tr>
<td>$\text{Max } f_{ck}$</td>
<td>3.2.2(3)</td>
<td>650MPa</td>
</tr>
<tr>
<td>Load Combinations</td>
<td>5.1.3(1)</td>
<td>Combinations from Eq. 6.10a/b</td>
</tr>
<tr>
<td>$\lambda_{lim}$</td>
<td>5.8.3.1(1)</td>
<td>$20\sqrt{\frac{A_{f_{ck}}}{N_{ld}}}$</td>
</tr>
<tr>
<td>Beam $\rho_{w,\min}$</td>
<td>9.2.2(5)</td>
<td>$\left(0.063\sqrt{f_{ck}}\right)/f_{ck}$</td>
</tr>
<tr>
<td>$\alpha_{0,c}$</td>
<td>11.3.5(1)</td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha_{0,d}$</td>
<td>11.3.5(2)</td>
<td>1.0</td>
</tr>
<tr>
<td>$v_{i,\min}$</td>
<td>11.6.1(1)</td>
<td>$0.03k^{2/3}f_{ck}^{1/2}$</td>
</tr>
</tbody>
</table>

### Table 6-4 Finland NDPs
<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{cc}$</td>
<td>3.1.6(1)</td>
<td>0.85</td>
</tr>
<tr>
<td>Max $f_{yk}$</td>
<td>3.2.2(3)</td>
<td>700MPa</td>
</tr>
<tr>
<td>Load Combinations</td>
<td>5.1.3(1)</td>
<td>Combinations from Eq. 6.10a/b</td>
</tr>
<tr>
<td>$k_2$</td>
<td>5.5(4)</td>
<td>1.10</td>
</tr>
<tr>
<td>Beam $A_{s,max}$</td>
<td>9.2.1.1(3)</td>
<td>Unlimited</td>
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</table>

**Table 6-5 Norway NDPs**

<table>
<thead>
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<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{cc}$</td>
<td>3.1.6(1)</td>
<td>0.85</td>
</tr>
<tr>
<td>$\alpha_{ct}$</td>
<td>3.1.6(2)</td>
<td>0.85</td>
</tr>
<tr>
<td>$\lambda_{lim}$</td>
<td>5.8.3.1(1)</td>
<td>13(2 − $r_m$)$A_f$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>6.2.2(1)</td>
<td>0.15 for compression 0.3 for tension</td>
</tr>
<tr>
<td>$v_{\min}$</td>
<td>6.2.2(1)</td>
<td>0.035$k^{3/2}f_{ck}^{1/2}$</td>
</tr>
<tr>
<td>Beam $\rho_w,\min$</td>
<td>9.2.2(5)</td>
<td>$\left(0.1\sqrt{f_{ck}}\right)/f_{ck}$</td>
</tr>
<tr>
<td>$v_{l,\min}$</td>
<td>11.6.1(1)</td>
<td>0.03$k^{2/3}f_{ck}^{1/2}$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>11.6.1(1)</td>
<td>0.15 for compression 0.3 for tension</td>
</tr>
<tr>
<td>$v_1$</td>
<td>11.6.2(1)</td>
<td>0.5(1 − $f_{ub}/250$)</td>
</tr>
</tbody>
</table>

**Table 6-6 Singapore NDPs**

6 - 34 Nationally Determined Parameters (NDPs)
### Table 6-7 Slovenia NDPS

<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{cc}$</td>
<td>3.1.6(1)</td>
<td>0.85</td>
</tr>
<tr>
<td>$k_1$</td>
<td>5.5(4)</td>
<td>0.4</td>
</tr>
<tr>
<td>$k_2$</td>
<td>5.5(4)</td>
<td>$0.6 + 0.0014/\varepsilon_{cu}$</td>
</tr>
<tr>
<td>$k_3$</td>
<td>5.5(4)</td>
<td>0.54</td>
</tr>
<tr>
<td>$k_4$</td>
<td>5.5(4)</td>
<td>$0.6 + 0.0014/\varepsilon_{cu}$</td>
</tr>
<tr>
<td>$v_{lim}$</td>
<td>5.8.3.1(1)</td>
<td>$0.30k_3^{1/2}/f_{ck}^{1/2}$</td>
</tr>
</tbody>
</table>

### Table 6-8 Sweden NDPS

<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam $A_{s,max}$</td>
<td>9.2.1.1(3)</td>
<td>Unlimited</td>
</tr>
<tr>
<td>$\alpha_{cc}$</td>
<td>11.3.5(1)</td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha_{lt}$</td>
<td>11.3.5(2)</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Table 6-9 United Kingdom NDPS

<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{0,i}$ (wind load)</td>
<td>EC0 Combos</td>
<td>0.5</td>
</tr>
</tbody>
</table>
### Table 6-9 United Kingdom NDPs

<table>
<thead>
<tr>
<th>NDP</th>
<th>Clause</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{cc} )</td>
<td>3.1.6(1)</td>
<td>0.85</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>5.5(4)</td>
<td>0.4</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>5.5(4)</td>
<td>0.6 + 0.0014/( \mu_{cu2} )</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>5.5(4)</td>
<td>0.4</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>5.5(4)</td>
<td>0.6 + 0.0014/( \mu_{cu2} )</td>
</tr>
<tr>
<td>( v_{l,\text{min}} )</td>
<td>11.6.1(1)</td>
<td>0.30( k^{3/2} f_{sk}^{1/2} )</td>
</tr>
</tbody>
</table>
This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Hong Kong limit state code CP-04 [CP 04], which also incorporates Amendment 1 published in June 2007, is selected. The various notations used in this chapter are listed in Table 7-1. For referencing to the pertinent sections of the Hong Kong code in this chapter, a prefix “CP” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

### 7.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_g$</td>
<td>Gross area of cross-section, mm$^2$</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Area of longitudinal reinforcement for torsion, mm$^2$</td>
</tr>
</tbody>
</table>
Table 7-1 List of Symbols Used in the CP-04 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A'_t$</td>
<td>Area of compression reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}$</td>
<td>Total cross-sectional area of links at the neutral axis, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv,t}$</td>
<td>Total cross-sectional area of closed links for torsion, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}/S_v$</td>
<td>Area of shear reinforcement per unit length, mm$^2$/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width or effective width of the section in the compression zone, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Width or effective width of flange, mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Average web width of a flanged beam, mm</td>
</tr>
<tr>
<td>$C$</td>
<td>Torsional constant, mm$^4$</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Depth to center of compression reinforcement, mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, N/mm$^2$</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, assumed as 200,000 N/mm$^2$</td>
</tr>
<tr>
<td>$f$</td>
<td>Punching shear factor considering column location</td>
</tr>
<tr>
<td>$f_{cu}$</td>
<td>Characteristic cube strength, N/mm$^2$</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, N/mm$^2$</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Characteristic strength of reinforcement, N/mm$^2$</td>
</tr>
<tr>
<td>$f_{yy}$</td>
<td>Characteristic strength of shear reinforcement, N/mm$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section in the plane of bending, mm</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Flange thickness, mm</td>
</tr>
<tr>
<td>$h_{min}$</td>
<td>Smaller dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$h_{max}$</td>
<td>Larger dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$K$</td>
<td>Normalized design moment, $\frac{M_u}{b d^2 f_{cu}}$</td>
</tr>
<tr>
<td>$K'$</td>
<td>Maximum $\frac{M_u}{b d^2 f_{cu}}$ for a singly reinforced concrete section</td>
</tr>
</tbody>
</table>
Table 7-1 List of Symbols Used in the CP-04 Code

- \( k_1 \): Shear strength enhancement factor for support compression
- \( k_2 \): Concrete shear strength factor, \( \left[ \frac{f_{cu}}{25} \right]^{\beta} \)
- \( M \): Design moment at a section, N-mm
- \( M_{\text{single}} \): Limiting moment capacity as singly reinforced beam, N-mm
- \( s_v \): Spacing of the links along the length of the beam, mm
- \( T \): Design torsion at ultimate design load, N-mm
- \( u \): Perimeter of the punch critical section, mm
- \( V \): Design shear force at ultimate design load, N
- \( v \): Design shear stress at a beam cross-section or at a punching critical section, N/mm²
- \( v_c \): Design concrete shear stress capacity, N/mm²
- \( v_{\text{max}} \): Maximum permitted design factored shear stress, N/mm²
- \( v_t \): Torsional shear stress, N/mm²
- \( x \): Neutral axis depth, mm
- \( x_{\text{bal}} \): Depth of neutral axis in a balanced section, mm
- \( z \): Lever arm, mm
- \( \beta \): Torsional stiffness constant
- \( \beta_h \): Moment redistribution factor in a member
- \( \gamma_f \): Partial safety factor for load
- \( \gamma_m \): Partial safety factor for material strength
- \( \varepsilon_c \): Maximum concrete strain
- \( \varepsilon_s \): Strain in tension reinforcement
- \( \varepsilon'_s \): Strain in compression reinforcement

### 7.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are
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obtained by multiplying the characteristic loads by appropriate partial factors of safety, $\gamma_f$ (CP 2.3.1.3). For CP-04, if a structure is subjected to dead (D), live (L), pattern live (PL), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered. (CP 2.3.2.1, Table 2.1).

1.4D  
1.4D + 1.6L  
1.4D + 1.6(0.75PL)  
1.0D ± 1.4W  
1.4D ± 1.4W  
1.2D + 1.2L ± 1.2W  

These are also the default design load combinations in SAFE whenever the CP-04 code is used. If roof live load is separately treated or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

7.3 Limits on Material Strength

The concrete compressive strength, $f_{cu}$, should not be less than 20 N/mm² (CP 3.1.3). The program does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsibility to use the proper strength values while defining the materials.

7.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, $\gamma_m$. The values of $\gamma_m$ used in the program are listed in the following table, as taken from CP Table 2.2 (CP 2.4.3.2):

7 - 4 Limits on Material Strength
Values of $\gamma_m$ for the Ultimate Limit State

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete in flexure and axial load</td>
<td>1.50</td>
</tr>
<tr>
<td>Concrete shear strength without shear reinforcement</td>
<td>1.25</td>
</tr>
</tbody>
</table>

These factors are incorporated into the design equations and tables in the code, but can be overwritten.

### 7.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the sections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

#### 7.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the length of the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement
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7.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

7.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 7-1 (CP 6.1.2.4(a)), where $\varepsilon_{c,max}$ is defined as:

$$\varepsilon_{c,max} = \begin{cases} 0.0035 & \text{if } f'_{cu} \leq 60\text{N/mm}^2 \\ 0.0035 - 0.00006 (f'_{cu} - 60)^{1/2} & \text{if } f'_{cu} > 60\text{N/mm}^2 \end{cases}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$; CP 6.1.2.4(b)). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \leq \begin{cases} 0.5 & \text{for } f'_{cu} \leq 45\text{N/mm}^2 \\ 0.4 & \text{for } 45 < f'_{cu} \leq 70\text{N/mm}^2 \\ 0.33 & \text{for } 70 < f'_{cu} \leq 100\text{N/mm}^2 \end{cases}$$

(CP 6.1.2.4(b))
to safeguard against non-ductile failures (CP 6.1.2.4(b)). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The depth of the compression block is given by:

\[
a = \begin{cases} 
  0.9x & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\
  0.8x & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\
  0.72x & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 
\end{cases} \quad (\text{CP 6.1.2.4(a)}, \text{Fig 6.1})
\]

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed \(0.1f_{cu}A_g\) (CP 6.1.2.4(a)), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than \(0.1f_{cu}A_g\) and axial tensions are always included in flexural and shear design.

### 7.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam, \(M_{\text{single}}\), is obtained first for a section. The reinforcing is determined based on whether \(M\) is greater than, less than, or equal to \(M_{\text{single}}\). See Figure 7-1

Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

\[
M_{\text{single}} = K' f_{cu} bd^2, \quad \text{where} \quad (\text{CP 6.1.2.4(c)})
\]

\[
K' = \begin{cases} 
  0.156 & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\
  0.120 & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\
  0.094 & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 
\end{cases}
\]

- If \(M \leq M_{\text{single}}\), the area of tension reinforcement, \(A_s\), is obtained from:

\[
A_s = \frac{M}{0.87 f_{y}z}, \quad \text{where} \quad (\text{CP 6.1.2.4(c)})
\]
This reinforcement is to be placed at the bottom if $M$ is positive, or at the top if $M$ is negative.

- If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$A'_s = \frac{M - M_{\text{single}}}{f'_s - \frac{0.67 f_{cu}}{\gamma_c}} \left(d - d'\right)$$

(CP 6.1.2.4(c))
where $d'$ is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = E_s \varepsilon_c \left(1 - \frac{d'}{x}\right) \leq 0.87 f_y,$$  \hspace{1cm} (CP 6.1.2.4(c), 3.2.6, Fig. 3.9)

$$x = \begin{cases} 
\frac{d-z}{0.45} & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\
\frac{d-z}{0.40} & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\
\frac{d-z}{0.36} & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 
\end{cases}$$  \hspace{1cm} (CP 6.1.2.4(a), Fig 6.1)

$$z = d \left[0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right] \leq 0.95d$$  \hspace{1cm} (CP 6.1.2.4(c))

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$A_t = \frac{M_{\text{single}}}{0.87f_y z} + \frac{M - M_{\text{single}}}{0.87f'_s (d - d')}$$  \hspace{1cm} (CP 6.1.2.4(c))

### 7.5.1.2.2 Design of Flanged Beams

#### 7.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

#### 7.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located in the flange. On the basis of this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange depth, the contribution
of the web to the flexural strength of the beam is taken into account. See Figure 7-2.

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

\[ K = \frac{M}{f_{cu} b_f d^2}. \]  

(CP 6.1.2.4(c))

Then the moment arm is computed as:

\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right) \leq 0.95d, \]  

(CP 6.1.2.4(c))

the depth of the neutral axis is computed as:

\[ x = \begin{cases} 
\frac{d - z}{0.45}, & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\
\frac{d - z}{0.40}, & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\
\frac{d - z}{0.36}, & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 
\end{cases} \]  

(CP 6.1.2.4(c), Fig 6.1)

and the depth of the compression block is given by:

\[ a = \begin{cases} 
0.9x & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\
0.8x & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\
0.72x & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 
\end{cases} \]  

(CP 6.1.2.4(a), Fig 6.1)

- If \( a \leq h_f \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( K > K' \).

- If \( a > h_f \), the calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \), as shown in Figure 7-2.
In that case, the ultimate resistance moment of the flange is given by:

$$M_f = \frac{0.67}{\gamma_c} f_{cu} (b_f - b_w) h_f (d - 0.5h_f)$$

The moment taken by the web is computed as:

$$M_w = M - M_f$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2}$$

- If $K_w \leq K'$ (CP 6.1.2.4(c)), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.
\[ A_s = \frac{M_f}{0.87 f_y (d - 0.5 h_f)} + \frac{M_w}{0.87 f_y z}, \text{ where} \]
\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K_w}{0.9}} \right) \leq 0.95 d \]

- If \( K_w > K' \), compression reinforcement is required and is calculated as follows:

The ultimate moment of resistance of the web only is given by:

\[ M_{uw} = K_f c_w b_w d^2 \]

The compression reinforcement is required to resist a moment of magnitude \( M_w - M_{uw} \). The compression reinforcement is computed as:

\[ A'_s = \frac{M_w - M_{uw}}{\left( f'_s - \frac{0.67 f_{cu}}{\gamma_c} \right) (d - d')} \]

where, \( d' \) is the depth of the compression reinforcement from the concrete compression face, and

\[ f_s = E_s c_e \left( 1 - \frac{d}{x} \right) \leq 0.87 f_y \quad \text{(CP 6.1.2.4(c), 3.2.6, Fig 3.9)} \]

The area of tension reinforcement is obtained from equilibrium as:

\[ A_s = \frac{1}{0.87 f_y} \left[ \frac{M_f}{d - 0.5 h_f} + \frac{M_{uw}}{z} + \frac{M_w - M_{uw}}{d - d'} \right] \]
\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \right) \leq 0.95 d \]
7.5.1.2.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from CP Table 9.1 (CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>—</td>
<td>100 ( \frac{A_s}{bh} )</td>
<td>0.24 0.13</td>
</tr>
<tr>
<td>T or L-Beam with web in tension</td>
<td>( \frac{b_w}{b_f} &lt; 0.4 )</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.32 0.18</td>
</tr>
<tr>
<td></td>
<td>( \frac{b_w}{b_f} \geq 0.4 )</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.24 0.13</td>
</tr>
<tr>
<td>T-Beam with web in compression</td>
<td>—</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.48 0.26</td>
</tr>
<tr>
<td>L-Beam with web in compression</td>
<td>—</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.36 0.20</td>
</tr>
</tbody>
</table>

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from CP Table 9.1 (CP 9.2.1.1).

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>—</td>
<td>100 ( \frac{A_s}{bh} )</td>
<td>0.20</td>
</tr>
<tr>
<td>T or L-Beam</td>
<td>Web in tension</td>
<td>100 ( \frac{A_s}{b_f h_f} )</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>Web in compression</td>
<td>100 ( \frac{A_s}{b_j h} )</td>
<td>0.20</td>
</tr>
</tbody>
</table>

An upper limit of 0.04 times the gross cross-sectional area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows (CP 9.2.1.3):
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\[
\begin{align*}
A_j &\leq \begin{cases} 
0.04bd & \text{Rectangular beam} \\
0.04b_n d & \text{Flanged beam}
\end{cases} & (CP 9.2.1.3) \\
A'_j &\leq \begin{cases} 
0.04bd & \text{Rectangular beam} \\
0.04b_n d & \text{Flanged beam}
\end{cases}
\end{align*}
\]

7.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (CP 6.1.2.5):

- Determine the shear stress, \( v \).
- Determine the shear stress, \( v_c \), that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

7.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors. The shear stress is then calculated as:

\[
v = \frac{V}{bd}
\]

(CP 6.1.2.5(a))

The maximum allowable shear stress, \( v_{\max} \) is defined as:

\[
v_{\max} = \min(0.8\sqrt{f_{cu}}, \ 7 \text{ MPa})
\]

(CP 6.1.2.5(a))

7.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, \( v_c \), is calculated as:

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\[ v'_c = v_c + 0.6 \frac{NVh}{A_c M} \leq v_c \sqrt{1 + \frac{N}{A_c v_c}} \]  
(CP 6.1.2.5(k))

\[ v_c = \frac{0.79k_1 k_2}{\gamma_m} \left( 100 \frac{A_c}{bd} \right)^{\frac{1}{3}} \left( \frac{400}{d} \right)^{\frac{1}{4}} \]  
(CP 6.1.2.5(c), Table 6.3)

\( k_1 \) is the enhancement factor for support compression, and is conservatively taken as 1  
(CP 6.1.2.5(g))

\[ k_2 = \left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}}, \quad 1 \leq k_2 \leq \left( \frac{80}{25} \right)^{\frac{1}{3}} \]  
(CP 6.1.2.5(c), Table 6.3)

\( \gamma_m = 1.25 \)  
(CP 6.1.2.5(c), Table 6.3)

However, the following limitations also apply:

\[ 0.15 \leq \frac{100A_c}{bd} \leq 3, \]  
(CP 6.1.2.5(c), Table 6.3)

\[ \left( \frac{400}{d} \right)^{\frac{1}{4}} \geq 0.67, \quad \text{Members without shear reinforcement} \]  
\[ \left( \frac{400}{d} \right)^{\frac{1}{4}} \geq 1.00, \quad \text{Members with shear reinforcement} \]  
(CP 6.1.2.5(c), Table 6.3)

\[ \frac{Vh}{M} \leq 1 \]  
(CP 6.1.2.5(k))

### 7.5.2.3 Determine Required Shear Reinforcement

Given \( v, v_c, \) and \( v_{\max} \), the required shear reinforcement is calculated as follows (CP Table 6.2, CP 6.1.2.5(b)):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, \( v_r \), as:
\[ v_r = \begin{cases} 
0.4 & \text{if } f_{cu} \leq 40 \text{ MPa} \\
0.4 \left( \frac{f_{cu}}{40} \right)^{\frac{3}{5}} & \text{if } 40 < f_{cu} \leq 80 \text{ MPa} \\
0.4 \left( \frac{80}{40} \right)^{\frac{3}{5}} & \text{if } f_{cu} > 80 \text{ MPa}
\end{cases} \] (CP 6.1.2.5(b), Table 6.2)

- If \( v \leq v'_c + v_r \), minimum reinforcement is required:
  \[ \frac{A_s}{s_v} = \frac{v_b}{0.87 f_{vyv}} , \] (CP 6.1.2.5(b))

- If \( v > v'_c + v_r \),
  \[ \frac{A_{sr}}{s_v} = \frac{(v - v'_c)b}{0.87 f_{vy}} \] (CP 6.1.2.5(b))

- If \( v > v_{max} \), a failure condition is declared. (CP 6.1.2.5(b))

The maximum of all the calculated \( A_{sv}/s_v \) values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

### 7.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, \( v_t \).
- Determine special section properties.
• Determine critical torsion stress.
• Determine the torsion reinforcement required.

7.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given, as follows (CP 6.3.1).

The torsional shear stress, \( v_t \), for a rectangular section is computed as:

\[
v_t = \frac{2T}{h_{\text{min}}^2 (h_{\text{max}} - h_{\text{min}} / 3)} \quad \text{(CP 6.3.3(a))}
\]

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

\[
T_{\text{seg}} = T \left( \frac{h_{\text{min}}^3 h_{\text{max}}}{\sum(h_{\text{min}}^3 h_{\text{max}})} \right) \quad \text{(CP 6.3.3(b))}
\]

\[h_{\text{max}} = \text{Larger dimension of a rectangular section}\]
\[h_{\text{min}} = \text{Smaller dimension of a rectangular section}\]

If the computed torsional shear stress, \( v_t \), exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

\[
v_t \leq \min(0.8 \sqrt{f_{\text{cu}}}, 7 \text{ N/mm}^2) \times \frac{v_t}{550} \quad \text{(CP 6.3.4, Table 6.17)}
\]
7.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\text{min}}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\text{min}} = \min\left(0.067\sqrt{f_{cu}}, 0.6 \text{ N/mm}^2\right)$$

(CP 6.3.4, Table 6.17)

where $f_{cu}$ is the specified concrete compressive strength.

7.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, $v_t$, is less than the threshold limit, $v_{t,\text{min}}$, torsion can be safely ignored (CP 6.3.5). In that case, the program reports that no torsion reinforcement is required. However, if $v_t$ exceeds the threshold limit, $v_{t,\text{min}}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP 6.3.5).

- If $v_t > v_{t,\text{min}}$, the required closed stirrup area per unit spacing, $A_{sv,t}/s_r$, is calculated as:

![Figure 7-3 Closed stirrup and section dimensions for torsion design](image-url)
and the required longitudinal reinforcement is calculated as:

\[
A_{f} = \frac{A_{sv,t}f_{yv}(x_{1} + y_{1})}{s_{y}f_{y}} \quad \text{(CP 6.3.6)}
\]

In the preceding expressions, \(x_{1}\) is the smaller center-to-center dimension of the closed link, and \(y_{1}\) is the larger center-to-center dimension of the closed link.

An upper limit of the combination of \(v\) and \(v_{t}\) that can be carried by the section also is checked using the equation:

\[
v + v_{t} \leq \min\left(0.8\sqrt{f_{cu}}, 7 \text{ N/mm}^2\right) \quad \text{(CP 6.3.4)}
\]

If the combination of \(v\) and \(v_{t}\) exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated \(A_{f}\) and \(A_{sv,t}/s_{y}\) values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 7.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is performed based on the ultimate strength design method (CP-04) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.
7.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

7.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

7.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the
design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

### 7.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required in each direction of a slab is given by the following limits (CP 9.3.1.1), with interpolation for reinforcement of intermediate strength:

\[
A_{t} \geq \begin{cases} 
0.0024bh & \text{if } f_{y} \leq 250 \text{ MPa} \\
0.0013bh & \text{if } f_{y} \geq 460 \text{ MPa}
\end{cases} 
\]  

(CP 9.3.1.1(a))

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 9.2.1.3).

### 7.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

#### 7.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5\(d\) from the face of the support (CP 6.1.5.7(d)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CP 6.1.5.7). Figure 7-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.
7.6.2.2 Determine Concrete Capacity

The concrete punching shear factored strength is taken as (CP 6.1.5.7(d), Table 6.3):

\[ v_c = \frac{0.79 k_1 k_2}{\gamma_m} \left( \frac{100 A_v}{bd} \right)^{\frac{1}{3}} \left( \frac{400}{d} \right)^{\frac{1}{8}} \]  

(CP 6.1.2.5(d), Table 6.3)

- \( k_1 \) is the enhancement factor for support compression, and is conservatively taken as 1 (CP 6.1.2.5(g), 6.1.5.7(d))

\[ k_2 = \left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}} \leq k_2 \leq \left( \frac{80}{25} \right)^{\frac{1}{3}} \]  

(CP 6.1.2.5(c), Table 6.3)

\[ \gamma_m = 1.25 \]  

(CP 2.4.3.2, Table 2.2)

However, the following limitations also apply:
\[ 0.15 \leq \frac{100A_s}{bd} \leq 3, \]  
\[ \left( \frac{400}{d} \right)^{\frac{1}{4}} \geq \begin{cases} 0.67, & \text{Members without shear reinforcement} \\ 1.00, & \text{Members with shear reinforcement} \end{cases} \]  
\[ \text{(CP 6.1.2.5(c), Table 6.3)} \]

\[ A_s = \text{area of tension reinforcement, which is taken as the average tension reinforcement of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement is taken as zero in the current implementation.} \]

\[ v \leq \min(0.8 \sqrt{f_{cu}}, 7 \text{ MPa}) \]  
\[ f_{cu} \leq 80 \text{ MPa (for calculation purpose only)} \]  
\[ \text{(CP 6.1.5.7(b))} \]

### 7.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, \( v_{\text{max}} \), is calculated as:

\[ V_{\text{eff},x} = V \left( f + \frac{1.5M_x}{V_y} \right) \]  
\[ \text{(CP 6.1.5.6(b), 6.1.5.6(c))} \]

\[ V_{\text{eff},y} = V \left( f + \frac{1.5M_y}{V_x} \right) \]  
\[ \text{(CP 6.1.5.6(b), 6.1.5.6(c))} \]

\[ v_{\text{max}} = \max \left\{ \frac{V_{\text{eff},x}}{ud}, \frac{V_{\text{eff},y}}{ud} \right\} \]  
\[ \text{(CP 6.1.5.7)} \]

where,
SAFE Reinforced Concrete Design

\[ u \] is the perimeter of the critical section,

\[ x \] and \( y \) are the lengths of the sides of the critical section parallel to the axis of bending,

\[ M_x \] and \( M_y \) are the design moments transmitted from the slab to the column at the connection,

\( V \) is the total punching shear force, and

\( f \) is a factor to consider the eccentricity of punching shear force and is taken as

\[
    f = \begin{cases} 
    1.00 & \text{for interior columns} \\
    1.25 & \text{for edge columns} \\
    1.25 & \text{for corner columns}
    \end{cases} \quad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}
\]

### 7.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

### 7.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 6.1.5.7(e)). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.
7.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

7.6.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

$$v_{\text{max}} = 2v_c$$  \hspace{1cm} (CP 6.1.5.7(e))

Given \(v\), \(v_c\), and \(v_{\text{max}}\), the required shear reinforcement is calculated as follows (CP 6.1.5.7(e)).

- If \(v \leq 1.6v_c\),

$$A_v = \frac{(v - v_c)ud}{0.87 f_{yr}} \geq \frac{v_ud}{0.87 f_{yr}},$$  \hspace{1cm} (CP 6.1.5.7(e))

- If \(1.6v_c \leq v < 2.0v_c\),

$$A_v = \frac{5(0.7v - v_c)ud}{0.87 f_{yr}} \geq \frac{v_ud}{0.87 f_{yr}},$$  \hspace{1cm} (CP 6.1.5.7(e))

$$v_r = \begin{cases} 0.4 \\ 0.4 \left( \frac{f_{cu}}{40} \right)^{2/3} \text{ N/mm}^2 \end{cases}$$  \hspace{1cm} (CP 6.1.5.7, Table 6.2)

- If \(v > 2.0v_c\), a failure condition is declared.  \hspace{1cm} (CP 6.1.5.7(e))

If \(v\) exceeds the maximum permitted value of \(v_{\text{max}}\), the concrete section should be increased in size.

7.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 7-5 shows a typical arrangement of
shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \). The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5\( d \) measured in a direction parallel to the column face (CP 6.1.5.7(f)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

![Typical Stud Rail](image)

**Figure 7-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone**

### 7.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 4.2.4 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.
When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $1.5d$. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

$$s_o \leq 0.5d \quad \text{(CP 6.1.5.7(f))}$$

$$s \leq 0.75d \quad \text{(CP 6.1.5.7(f))}$$

$$g \leq 1.5d \quad \text{(CP 6.1.5.7(f))}$$

Stirrups are only permitted when slab thickness is greater than 200 mm (CP 6.1.5.7(e)).
This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Indian Code IS 456-2000 [IS 2000] is selected. Various notations used in this chapter are listed in Table 8-1. For referencing to the pertinent sections of the Indian code in this chapter, a prefix “IS” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

### 8.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>Area of concrete, mm$^2$</td>
</tr>
<tr>
<td>$A_{cv}$</td>
<td>Area of section for shear resistance, mm$^2$</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Gross cross-sectional area of a frame member, mm$^2$</td>
</tr>
</tbody>
</table>
### Table 8-1 List of Symbols Used in the IS 456-2000 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sV}$</td>
<td>Total cross-sectional area of links at the neutral axis, mm$^2$</td>
</tr>
<tr>
<td>$A_{sV}/s_V$</td>
<td>Area of shear reinforcement per unit length, mm$^2$/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth to the center of the compression block, mm</td>
</tr>
<tr>
<td>$a_1$</td>
<td>Width of the punching critical section in the direction of bending, mm</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Width of the punching critical section perpendicular to the direction of bending, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width or effective width of the section in the compression zone, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Width or effective width of flange, mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Average web width of a flanged beam, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Effective depth of compression reinforcement, mm</td>
</tr>
<tr>
<td>$D$</td>
<td>Overall depth of a beam or slab, mm</td>
</tr>
<tr>
<td>$D_f$</td>
<td>Flange thickness in a flanged beam, mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, assumed as 200,000 MPa</td>
</tr>
<tr>
<td>$f_{cd}$</td>
<td>Design concrete strength = $f_{ck}/\gamma_c$, MPa</td>
</tr>
<tr>
<td>$f_{ck}$</td>
<td>Characteristic compressive strength of concrete, MPa</td>
</tr>
<tr>
<td>$f_{sc}$</td>
<td>Compressive stress in beam compression steel, MPa</td>
</tr>
<tr>
<td>$f_{yd}$</td>
<td>Design yield strength of reinforcement = $f_y/\gamma_s$, MPa</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Characteristic strength of reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{ys}$</td>
<td>Characteristic strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$k$</td>
<td>Enhancement factor of shear strength for depth of the beam</td>
</tr>
<tr>
<td>$M_{\text{single}}$</td>
<td>Design moment resistance of a section as a singly reinforced section, N-mm</td>
</tr>
<tr>
<td>$M_u$</td>
<td>Ultimate factored design moment at a section, N-mm</td>
</tr>
</tbody>
</table>
Table 8-1 List of Symbols Used in the IS 456-2000 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_t$</td>
<td>Equivalent factored bending moment due to torsion at a section, N-mm</td>
</tr>
<tr>
<td>$M_{e1}$</td>
<td>Equivalent factored moment including moment and torsion effects ($M_{e1} = M_u + M_t$) at a section, N-mm</td>
</tr>
<tr>
<td>$M_{e2}$</td>
<td>Residual factored moment when $M_t &gt; M_u$ at a section applied in the opposite sense of $M_{e1}$ at a section, N-mm</td>
</tr>
<tr>
<td>$m$</td>
<td>Normalized design moment, $M/\beta bd^2 \alpha f_{ck}$</td>
</tr>
<tr>
<td>$s_v$</td>
<td>Spacing of the shear reinforcement along the length of the beam, mm</td>
</tr>
<tr>
<td>$T_u$</td>
<td>Factored torsional moment at a section, N-mm</td>
</tr>
<tr>
<td>$V_u$</td>
<td>Factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_e$</td>
<td>Equivalent factored shear force including torsion effects, N</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Allowable shear stress in punching shear mode, N</td>
</tr>
<tr>
<td>$x_u$</td>
<td>Depth of neutral axis, mm</td>
</tr>
<tr>
<td>$x_{u,\text{max}}$</td>
<td>Maximum permitted depth of neutral axis, mm</td>
</tr>
<tr>
<td>$z$</td>
<td>Lever arm, mm</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Concrete strength reduction factor for sustained loading, as well as reinforcement over strength factor for computing capacity moment at a section</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Factor for the depth of compressive force resultant of the concrete stress block</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Ratio of the minimum to maximum dimensions of the punching critical section</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>Partial safety factor for concrete strength</td>
</tr>
<tr>
<td>$\gamma_\gamma$</td>
<td>Partial safety factor for load, and fraction of unbalanced moment transferred by flexure</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>Partial safety factor for material strength</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>Partial safety factor for reinforcement strength</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Enhancement factor of shear strength for compression</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>ε&lt;sub&gt;c, max&lt;/sub&gt;</td>
<td>Maximum concrete strain in the beam and slab (= 0.0035)</td>
</tr>
<tr>
<td>ε&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Strain in tension steel</td>
</tr>
<tr>
<td>ε&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Strain in compression steel</td>
</tr>
<tr>
<td>τ&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Average design shear stress resisted by concrete, MPa</td>
</tr>
<tr>
<td>τ&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Basic design shear stress resisted by concrete, MPa</td>
</tr>
<tr>
<td>τ&lt;sub&gt;c, max&lt;/sub&gt;</td>
<td>Maximum possible design shear stress permitted at a section, MPa</td>
</tr>
<tr>
<td>τ&lt;sub&gt;cd&lt;/sub&gt;</td>
<td>Design shear stress resisted by concrete, MPa</td>
</tr>
</tbody>
</table>

### 8.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For IS 456-2000, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (IS 36.4, Table 18):

1.5D
1.5D + 1.5L (IS 36.4.1)
1.5D + 1.5S (IS 36.4.1)
1.5D + 1.5(0.75 PL) (IS 31.5.2.3)
1.5D ± 1.5W
0.9D ± 1.5W (IS 36.4.1)
1.2D + 1.2L ± 1.2W
1.5D + 1.5L ± 1.0W
1.5D ± 1.5E
0.9D ± 1.5E
1.2D + 1.2L ± 1.2E (IS 36.4.1)
1.5D + 1.5L ± 1.0E

---

8 - 4 Design Load Combinations
These are also the default design load combinations in SAFE whenever the IS 456-2000 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

### 8.3 Partial Safety Factors

The design strength for concrete and reinforcement is obtained by dividing the characteristic strength of the material by a partial safety factor, \( \gamma_m \). The values of \( \gamma_m \) used in the program are as follows:

- Partial safety factor for reinforcement, \( \gamma_r = 1.15 \) (IS 36.4.2.1)
- Partial safety factor for concrete, \( \gamma_c = 1.5 \) (IS 36.4.2.1)

These factors are already incorporated into the design equations and tables in the code. These values can be overwritten; however, caution is advised.

### 8.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

#### 8.4.1 Effects of Torsion

IS 456, 14.1 states that wherever torsion is required to maintain equilibrium, beams must be designed for torsion. However, torsion can be ignored for indeterminate structures where torsion develops primarily because of compatibility...
of deformations. However, the program does not automatically redistribute the internal forces and reduce torsion. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

Note that the torsion design can be turned off by choosing not to consider torsion in the Design Preferences.

The beam design procedure involves the following steps:

- Determine design bending moments and shears
- Design flexural reinforcement
- Design shear reinforcement

8.4.1.1 Determine Design Bending Moments and Shears

IS 456 uses a simplified approach and does not require the calculation of shear stresses produced by torsion separately. Rather, torsion and bending shear are combined as an equivalent shear $V_e$, and bending moment and torsion are combined as an equivalent bending moment $M_e$. The beam is checked for adequacy and then designed for the equivalent moment and shear. If the shear stress caused by equivalent shear is less than the concrete shear capacity, torsion is ignored completely and only required minimum shear links are computed. If the shear stress caused by equivalent shear is more than the concrete shear capacity, additional longitudinal reinforcement and shear links are computed, as detailed in the subsections that follow.

8.4.1.2 Determine Factored Moments when Torsion is Excluded

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.
8.4.1.3 Determine Factored Moments when Torsion is Included

In the design of flexural reinforcement of concrete beams, the factored moments and torsion for each load combination at a particular beam station are obtained by factoring the corresponding moments and torsion for different load cases with the corresponding load factors.

The equivalent moment at a particular station is computed as described in the text that follows. The beam is then designed for the maximum positive and maximum negative factored moments obtained from all the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

The equivalent moment is calculated from the following equation:

\[ M_{e1} = M_u + M_t \]

where

\[ M_t = T_u \left( \frac{1 + D/b}{1.7} \right) \]

and \( D \) and \( b \) are the overall depth and width of the beam, respectively.

If \( M_t \) exceeds \( M_u \), additional reinforcement will be computed for the moment \( M_{e2} \) applied in the opposite sense of \( M_u \). Effectively, this will result in additional longitudinal reinforcement on the compression face of the beam because the moment sign is reversed. The additional moment \( M_{e2} \) is computed as:

\[ M_{e2} = M_t - M_u \]

8.4.1.4 Determine Factored Shears when Torsion is Excluded

In the design of the beam shear reinforcement, the factored shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.
8.4.1.5 Determine Factored Shears when Torsion is Included

In the design of beam shear reinforcement, the factored shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

The equivalent shear at a particular station is computed as described in the text that follows. The beam is then designed for the equivalent shear at the station.

When a torsional moment is to be included, the equivalent shear $V_e$ is calculated from the following equation:

$$V_e = V_u + 1.6 \left( \frac{T_u}{b} \right)$$

(IS 41.3.1)

where $b$ is the width of the beam web.

8.4.2 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam.

8.4.2.1 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified parabolic stress block shown in Figure 8-1 (IS 38.1). The area of the stress block, $c$, and the depth of the center of the compressive force from the extreme compression fiber, $a$, are taken as

$$c = \alpha f_{ck} x_u$$

(IS 38.1)

$$a = \beta x_u$$

(IS 38.1)
Figure 8-1 Rectangular Beam Design

where $x_u$ is the depth of the neutral axis, and $\alpha$ and $\beta$ are taken as:

$$\alpha = 0.36 \quad \text{(IS 38.1)}$$
$$\beta = 0.42 \quad \text{(IS 38.1)}$$

where $\alpha$ is the reduction factor to account for sustained compression and the partial safety factor for concrete and is generally taken to be 0.36 for the assumed parabolic stress block (IS 38.1). The $\beta$ factor considers the depth to the center of the compressive force.

Furthermore, it is assumed that moment redistribution in the member does not exceed the code-specified limiting value. The code also places a limitation on the neutral axis depth, as shown in the following table, to safeguard against non-ductile failures (IS 38.1). SAFE uses interpolation between these three values.

<table>
<thead>
<tr>
<th>$f_y$ (MPa)</th>
<th>$x_{u,max}/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.53</td>
</tr>
<tr>
<td>415</td>
<td>0.48</td>
</tr>
</tbody>
</table>
When the applied moment exceeds the moment capacity of the beam as a singly reinforced beam, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value. The maximum fiber compression is taken as:

\[ \varepsilon_{c,\text{max}} = 0.0035 \]  
(IS 38.1)

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force can be neglected; hence all beams are designed for major direction flexure, shear, and torsion only.

### 8.4.2.2 Design of Rectangular Beams

For rectangular beams, the limiting depth of the neutral axis, \( x_{u,\text{max}} \), and the moment capacity as a singly reinforced beam, \( M_{\text{single}} \), are obtained first. The reinforcement area is determined based on whether \( M_u \) is greater than, less than, or equal to \( M_{\text{single}} \).

- **Calculate the limiting depth of the neutral axis.**

\[
\frac{x_{u,\text{max}}}{d} = \begin{cases} 
0.53 & \text{if } f_r \leq 250 \text{ MPa} \\
0.53 - 0.05 \frac{f_r - 250}{165} & \text{if } 250 < f_r \leq 415 \text{ MPa} \\
0.48 - 0.02 \frac{f_r - 415}{85} & \text{if } 415 < f_r \leq 500 \text{ MPa} \\
0.46 & \text{if } f_r \geq 500 \text{ MPa} 
\end{cases}  
\]  
(IS 38.1)

- **Calculate the limiting ultimate moment of resistance as a singly reinforced beam.**

\[
M_{\text{single}} = \alpha \frac{x_{u,\text{max}}}{d} \left( 1 - \beta \frac{x_{u,\text{max}}}{d} \right) bd^2 f_{ck}  
\]  
(IS G-1.1)

- **Calculate the depth of the neutral axis as:**

\[
\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4 \beta m}}{2 \beta}  
\]
where the normalized design moment, \( m \), is given by

\[
m = \frac{M_u}{bd^2\alpha f_{ck}}
\]

- If \( M_u \leq M_{\text{single}} \) the area of tension reinforcement, \( A_s \), is obtained from

\[
A_s = \frac{M_u}{(f_y / \gamma_s)z}, \text{ where } \quad z = d\left\{1 - \beta \frac{x_u}{d}\right\}.
\]

This reinforcement is to be placed at the bottom if \( M_u \) is positive, or at the top if \( M_u \) is negative.

- If \( M_u > M_{\text{single}} \), the area of compression reinforcement, \( A'_{s} \), is given by:

\[
A'_{s} = \frac{M_u - M_{\text{single}}}{(f_{sc} - 0.67 f_{ck})/\gamma_m (d - d')} \quad \text{(IS G-1.2)}
\]

where \( d' \) is the depth of the compression reinforcement from the concrete compression face, and

\[
f_{sc} = e_{c,\max} E_s \left[1 - \frac{d'}{x_{u,\max}}\right] \leq \frac{f_y}{\gamma_s} \quad \text{(IS G-1.2)}
\]

The required tension reinforcement is calculated as:

\[
A_s = \frac{M_{\text{single}}}{(f_y / \gamma_s)z} + \frac{M_u - M_{\text{single}}}{(f_y / \gamma_s)(d - d')}, \text{ where } \quad z = d\left\{1 - \beta \frac{x_{u,\max}}{d}\right\}
\]

\[
A_s \text{ is to be placed at the bottom and } A'_{s} \text{ is to be placed at the top if } M_u \text{ is positive, and vice versa if } M_u \text{ is negative.}
\]
8.4.2.3  Design of Flanged Beams

8.4.2.3.1  Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M_u$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

8.4.2.3.2  Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located within the flange. On the basis of this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange depth, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 8-2.

Assuming the neutral axis lies in the flange, the depth of the neutral axis is calculated as:
where the normalized design moment, $m$, is given by

$$m = \frac{M_u}{b_f d^2 \alpha f_{ck}}$$

- If $\left(\frac{x_u}{d}\right) \leq \left(\frac{D_f}{d}\right)$, the neutral axis lies within the flange and the subsequent calculations for $A_s$ are exactly the same as previously defined for the rectangular beam design (IS G-2.1). However, in this case, the width of the beam is taken as $b_f$. Compression reinforcement is required when $M_u > M_{single}$.

- If $\left(\frac{x_u}{d}\right) > \left(\frac{D_f}{d}\right)$, the neutral axis lies below the flange and the calculation for $A_s$ has two parts. The first part is for balancing the compressive force from the flange, $C_f$, and the second part is for balancing the compressive force from the web, $C_w$, as shown in Figure 8-2.

  - Calculate the ultimate resistance moment of the flange as:

    $$M_f = 0.45 f_{ck} \left( b_f - b_w \right) \gamma_f \left( d - \frac{\gamma_f}{2} \right)$$

    (IS G-2.2)

    where $\gamma$ is taken as:

    $$\gamma_f = \begin{cases} 2 & \text{if } D_f \leq 0.2d \\ 0.15 x_u + 0.65 D_f & \text{if } D_f > 0.2d \end{cases}$$

    (IS G-2.2)

  - Calculate the moment taken by the web as

    $$M_w = M_u - M_f$$

  - Calculate the limiting ultimate moment of resistance of the web for tension reinforcement as:
\[ M_{w,\text{single}} = \alpha f_{y} b_{w} d^{2} \frac{x_{u,\max}}{d} \left[ 1 - \beta \frac{x_{u,\max}}{d} \right] \]

where

\[ x_{u,\max} = \begin{cases} 0.53 & \text{if } f_{y} \leq 250 \text{ MPa} \\ 0.53 - 0.05 \frac{f_{y} - 250}{165} & \text{if } 250 < f_{y} \leq 415 \text{ MPa} \\ 0.48 - 0.02 \frac{f_{y} - 415}{85} & \text{if } 415 < f_{y} \leq 500 \text{ MPa} \\ 0.46 & \text{if } f_{y} \geq 500 \text{ MPa} \end{cases} \]

- If \( M_{w} \leq M_{w,\text{single}} \), the beam is designed as a singly reinforced concrete beam. The area of reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

\[ A_{s} = \frac{M_{f}}{(f_{y} / \gamma_{s})(d - 0.5 y_{f})} + \frac{M_{w}}{(f_{y} / \gamma_{s}) z} \]

\[ z = d \left[ 1 - \beta \frac{x_{u}}{d} \right] \]

\[ \frac{x_{u}}{d} = 1 - \sqrt{1 - 4 \beta m} \]

\[ m = \frac{M_{w}}{b_{w} d^{2} \alpha f_{ck}} \]

- If \( M_{w} > M_{w,\text{single}} \), the area of compression reinforcement, \( A'_{s} \), is given by:

\[ A'_{s} = \frac{M_{w} - M_{w,\text{single}}}{\left( f'_{s} - 0.67 f_{ck} / \gamma_{m} \right) (d - d')} \]

where \( d' \) is the depth of the compression reinforcement from the concrete compression face, and
The required tension reinforcement is calculated as:

\[ A_s = \frac{M_f}{(f_y/\gamma_s)(d - 0.5\gamma_f)} + \frac{M_{w,single}}{(f_y/\gamma_s)z} + \frac{M_w - M_{w,single}}{(f_y/\gamma_s)(d - d')} \]

where

\[ z = d \left\{ 1 - \beta \frac{x_{u,max}}{d} \right\} \]

8.4.2.4 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given as (IS 26.5.1.1):

\[ A_s \geq \frac{0.85}{f_y}bd \]  

(IS 26.5.1.1)

An upper limit of 0.04 times the gross web area on both the tension reinforcement (IS 26.5.1.1) and the compression reinforcement (IS 26.5.1.2) is imposed upon request as follows:

\[ A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_wd & \text{Flanged beam} \end{cases} \]  

(IS 26.5.1.1)

\[ A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_wd & \text{Flanged beam} \end{cases} \]  

(IS 26.5.1.2)

8.4.3 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station, the following steps are involved (IS 40.1):

- Determine the design shear stress
- Determine the shear stress that can be resisted by the concrete
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- Determine the shear reinforcement required to carry the balance

### 8.4.3.1 Design for Shear when Torsion is Excluded

Determine the design nominal shear stress as follows.

- For prismatic sections

  \[ \tau_v = \frac{V_u}{bd} \]  
  \hspace{1cm} (IS 40.1)

- For non-prismatic sections (beams with varying depth)

  \[ \tau_v = \frac{V_u \pm \frac{M_u}{d} \tan \beta}{bd}, \text{ where} \]

  \[ \beta = \text{angle between the top and bottom edges of the beam} \]

  \[ M_u \] is the moment at the section, and the negative sign is considered when the numerical value of the moment increases in the same direction as the depth, \( d \), and the positive sign is considered when the numerical value of the moment decreases in the same direction as the depth increases.

  \[ \tau_v \leq \tau_{c,\text{max}} \]  
  \hspace{1cm} (IS 40.2.3, Table 20)

The maximum nominal shear stress, \( \tau_{c,\text{max}} \) is given in IS Table 20 as follows:

<table>
<thead>
<tr>
<th>Maximum Shear Stress, ( \tau_{c,\text{max}} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(IS 40.2.3, IS Table 20)</td>
</tr>
<tr>
<td>Concrete Grade</td>
</tr>
<tr>
<td>( \tau_{c,\text{max}} ) (MPa)</td>
</tr>
</tbody>
</table>

The maximum nominal shear stress, \( \tau_{c,\text{max}} \), is computed using linear interpolation for concrete grades between those indicated in IS Table 20.

Determine the design shear stress that can be carried by the concrete, as:

\[ \tau_{cd} = k \delta \lambda \tau_c \]  
\hspace{1cm} (IS 40.2)
where \( k \) is the enhancement factor for the depth of the section, taken as 1.0 for beams, and is computed as follows for other slabs:

\[
k = 1 \quad \text{(IS 40.2.1.1)}
\]

\( \delta \) is the enhancement factor for compression and is given as:

\[
\delta = \begin{cases} 
1 + 3 \frac{P_u}{A_y f_{ck}} & \text{if } P_u > 0, \text{ Under Compression} \\
1 & \text{if } P_u \leq 0, \text{ Under Tension}
\end{cases} \quad \text{(IS 40.2.2)}
\]

\( \delta \) is always taken as 1

\( \lambda \) is the factor for light-weight concrete, and

\( \tau_c \) is the basic design shear strength for concrete, which is given by:

\[
\tau_c = 0.64 \left( \frac{100 A_s}{bd} \right)^{\frac{1}{3}} \left( \frac{f_{ck}}{25} \right)^{\frac{1}{4}} \quad \text{(IS 40.2.1)}
\]

The preceding expression approximates IS Table 19. It should be noted that the value of \( \gamma_c \) has already been incorporated in IS Table 19 (see note in IS 36.4.2.1). The following limitations are enforced in the determination of the design shear strength as is the case in the Table.

\[
0.15 \leq \frac{100 A_s}{bd} \leq 3 \quad \text{(IS 40.2.1, Table 19)}
\]

\( f_{ck} \leq 40 \text{ MPa (for calculation purpose only)} \quad \text{(IS 40.2.1, Table 19)}
\]

Determine required shear reinforcement:

- If \( \tau_v \leq \tau_{cd} + 0.4 \),

\[
\frac{A_{yw}}{s_v} = \frac{0.4 b}{0.87 f_y} \quad \text{(IS 40.3, 26.5.1.6)}
\]

- If \( \tau_{cd} + 0.4 < \tau_v \leq \tau_{c,\text{max}} \),
\[
\frac{A_{sv}}{s_v} = \frac{(\tau_v - \tau_{cd})b}{0.87 f_y}
\]  
(IS 40.4(a))

\[
\frac{A_{sv}}{s_v} \geq \frac{0.4b}{0.87 f_y}
\]  
(IS 40.4(a))

- If \( \tau_v > \tau_{c,\text{max}} \), a failure condition is declared.  
(IS 40.2.3)

In calculating the shear reinforcement, a limit is imposed on the \( f_y \) as:

\[
f_y \leq 415 \text{ MPa} \quad (\text{IS 40.4})
\]

8.4.3.2 Design for Shear when Torsion is Included

Determine the design nominal shear stress as:

\[
\tau_{ve} = \frac{V_e}{bd}
\]  
(IS 40.1)

\[
\tau_{ve} \leq \tau_{c,\text{max}} \quad (\text{IS 40.2.3})
\]

The maximum nominal shear stress, \( \tau_{c,\text{max}} \) is determined as defined in the last section.

Determine required shear reinforcement:

- If \( \tau_{ve} \leq \tau_{cd} \),

\[
\frac{A_{sv}}{s_v} = \frac{0.4b}{0.87 f_y}
\]  
(IS 41.3, 26.5.1.6)

- If \( \tau_{ve} \geq \tau_{cd} \), provide 2-legged closed stirrups, taken as the maximum of:

\[
\frac{A_{sv}}{s_v} = \frac{T_u}{b_1 d_1 (0.87 f_y)} + \frac{V_u}{2.5 d_1 (0.87 f_y)} \quad \text{and} \quad (\text{IS 41.4.3})
\]

\[
\frac{A_{sv}}{s_v} = \frac{(\tau_{ve} - \tau_v)b}{0.87 f_y}
\]  
(IS 41.4.3)
The maximum of all of the calculated \( A_{sv}/s_{v} \) values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

### 8.5 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the limit state of collapse (IS 456-2000) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

#### 8.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and
bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

8.5.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

8.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

8.5.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (IS 26.5.2):

\[
A_t \leq \begin{cases} 
0.0015bD & \text{if } f_y < 415 \text{ MPa} \\
0.0012bD & \text{if } f_y \geq 415 \text{ MPa} 
\end{cases}
\]  
(IS 26.5.2.1)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (IS 26.5.1.1).

8.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following sections.
8.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of \( d/2 \) from the face of the support (IS 31.6.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (IS 31.6.1). Figure 8-4 shows the auto-punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner), and the punching perimeter may be overwritten using the Punching Check Overwrites.

\[ \alpha = \frac{1}{1 + (2/3)\sqrt{a_1/a_2}} \]  
(IS 31.3.3)
and $a_1$ is the width of the critical section measured in the direction of the span and $a_2$ is the width of the critical section measured in the direction perpendicular to the span.

### 8.5.2.3 Determine Concrete Capacity

The concrete punching shear factored strength is taken as:

$$v_c = k_s \tau_c$$  \hspace{1cm} (IS 31.6.3.1)

$$k_s = 0.5 + \beta_c \leq 1.0$$  \hspace{1cm} (IS 31.6.3.1)

$$\tau_c = 0.25 \sqrt{f'_{ck}}$$  \hspace{1cm} (IS 31.6.3.1)

$\beta_c$ = ratio of the minimum to the maximum dimensions of the support section.

### 8.5.2.4 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

$$v_U = \frac{V_U}{b_d} + \frac{\sqrt{V_2}[M_{U2} - V_U(y_3 - y_4)][I_{23}(x_4 - x_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^2}$$  \hspace{1cm} Eq. 1

$$I_{22} = \sum_{sides = 1}^{n} \bar{T}_{22}$$, where "sides" refers to the sides of the critical section for punching shear  \hspace{1cm} Eq. 2

$$I_{33} = \sum_{sides = 1}^{n} \bar{T}_{33}$$, where "sides" refers to the sides of the critical section for punching shear  \hspace{1cm} Eq. 3

$$I_{23} = \sum_{sides = 1}^{n} \bar{T}_{23}$$, where "sides" refers to the sides of the critical section for punching shear  \hspace{1cm} Eq. 4
Figure 8-5 Shear Stress Calculations at Critical Sections

The equations for $T_{22}$, $T_{33}$ and $T_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 8-5.

$$T_{22} = Ld(y_2 - y_3)^2$$, for side of critical section parallel to 2-axis  \hspace{1cm} \text{Eq. 5a}

$$T_{22} = \frac{Ld^3}{12} + \frac{dl^3}{12} + Ld(y_2 - y_3)^2$$, for side of critical section parallel to 3-axis  \hspace{1cm} \text{Eq. 5b}

$$T_{33} = \frac{Ld^3}{12} + \frac{dl^3}{12} + Ld(x_2 - x_3)^2$$, for side of critical section parallel to 2-axis  \hspace{1cm} \text{Eq. 6a}
\[ T_{33} = Ld(x_2 - x_3)^2, \text{ for side of critical section parallel to 3-axis} \quad \text{Eq. 6b} \]

\[ T_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \quad \text{Eq. 7} \]

**NOTE:** \( T_{23} \) is explicitly set to zero for corner condition.

where,

- \( b_0 \) = Perimeter of critical section for punching shear
- \( d \) = Effective depth at critical section for punching shear based on the average of \( d \) for the 2 direction and \( d \) for the 3 direction
- \( I_{22} \) = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 2-axis
- \( I_{33} \) = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 3-axis
- \( I_{23} \) = Product of inertia of critical section for punching shear with respect to the 2 and 3 planes
- \( L \) = Length of side of critical section for punching shear currently being considered
- \( M_{U2} \) = Moment about line parallel to 2-axis at center of column (positive in accordance with the right-hand rule)
- \( M_{U3} \) = Moment about line parallel to 3-axis at center of column (positive in accordance with the right-hand rule)
- \( v_U \) = Punching shear stress
- \( V_U \) = Shear at center of column (positive upward)
- \( x_1, y_1 \) = Coordinates of column centroid
- \( x_2, y_2 \) = Coordinates of center of one side of critical section for punching shear
- \( x_3, y_3 \) = Coordinates of centroid of critical section for punching shear
$x_4, y_4 =$ Coordinates of location where you are calculating stress

$\gamma_2 =$ Percent of $M_{U2}$ resisted by shear

$\gamma_3 =$ Percent of $M_{U3}$ resisted by shear

### 8.5.2.5 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

### 8.5.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is completed as described in the subsections that follow.

#### 8.5.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined, but limited to:

\[ v_c \leq 1.5 \tau_c \]  \hspace{1cm} (IS 31.6.3.2)

#### 8.5.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ V_{\text{max}} = 0.5 \tau_c b_s d \]  \hspace{1cm} (IS 31.6.3.2)

Given $V_u$, $V_c$, and $V_{\text{max}}$, the required shear reinforcement is calculated as follows (IS 31.6.3.2).

\[ A_v = \frac{(V_u - 0.5V_c)S}{0.87 f_y d} \]  \hspace{1cm} (IS 31.6.3.2, 40.4(a))
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- If $V_u > V_{\text{max}}$, a failure condition is declared. (IS 31.6.3.2)
- If $V_u$ exceeds the maximum permitted value of $V_{\text{max}}$, the concrete section should be increased in size.

8.5.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 8-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

![Diagram of typical arrangement of shear studs](image)

Figure 8-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face.

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.
8.5.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in IS 26.4 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $2d$. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

\[
\begin{align*}
    s_o &\leq 0.5d \\
    s &\leq 0.5d \\
    g &\leq 2d
\end{align*}
\]
This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the New Zealand code NZS 3101-06 [NZS 06] is selected. Various notations used in this chapter are listed in Table 9-1. For referencing to the pertinent sections of the New Zealand code in this chapter, a prefix “NZS” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

### 9.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{co}$</td>
<td>Area enclosed by perimeter of the section, sq-mm</td>
<td></td>
</tr>
<tr>
<td>$A_{cv}$</td>
<td>Area of concrete used to determine shear stress, sq-mm</td>
<td></td>
</tr>
<tr>
<td>$A_g$</td>
<td>Gross area of concrete, sq-mm</td>
<td></td>
</tr>
</tbody>
</table>
### Table 9-1 List of Symbols Used in the NZS 3101-06 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_l$</td>
<td>Area of longitudinal reinforcement for torsion, sq-mm</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Gross area enclosed by shear flow path, sq-mm</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A's$</td>
<td>Area of compression reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_s$(required)</td>
<td>Area of steel required for tension reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_l/s$</td>
<td>Area of closed shear reinforcement per unit length for torsion, sq-mm/mm</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Area of shear reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_v/s$</td>
<td>Area of shear reinforcement per unit length, sq-mm/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$a_b$</td>
<td>Depth of compression block at balanced condition, mm</td>
</tr>
<tr>
<td>$a_{max}$</td>
<td>Maximum allowed depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of member, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Effective width of flange (flanged section), mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), mm</td>
</tr>
<tr>
<td>$b_0$</td>
<td>Perimeter of the punching critical section, mm</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Width of the punching critical section in the direction of bending, mm</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Width of the punching critical section perpendicular to the direction of bending, mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Distance from extreme compression fiber to the neutral axis, mm</td>
</tr>
<tr>
<td>$c_b$</td>
<td>Distance from extreme compression fiber to neutral axis at balanced condition, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from extreme compression fiber to tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Distance from extreme compression fiber to compression reinforcement, mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, assumed as 200,000 MPa</td>
</tr>
</tbody>
</table>
### Table 9-1 List of Symbols Used in the NZS 3101-06 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_c$</td>
<td>Specified compressive strength of concrete, MPa</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, psi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Specified yield strength of flexural reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{st}$</td>
<td>Specified yield strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of sections, mm</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Thickness of slab or flange, mm</td>
</tr>
<tr>
<td>$k_a$</td>
<td>Factor accounting for influence of aggregate size on shear strength</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Factor accounting for influence of member depth on shear strength</td>
</tr>
<tr>
<td>$M^*$</td>
<td>Factored design moment at a section, N-mm</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Outside perimeter of concrete section, mm</td>
</tr>
<tr>
<td>$p_o$</td>
<td>Perimeter of area $A_o$, mm</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of shear reinforcement along the length, mm</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Factored design torsion at a section, N-mm</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Assumed wall thickness of an equivalent tube for the gross section, mm</td>
</tr>
<tr>
<td>$t_o$</td>
<td>Assumed wall thickness of an equivalent tube for the area enclosed by the shear flow path, mm</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear force resisted by concrete, N</td>
</tr>
<tr>
<td>$V^*$</td>
<td>Factored shear force at a section, N</td>
</tr>
<tr>
<td>$v^*$</td>
<td>Average design shear stress at a section, MPa</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Design shear stress resisted by concrete, MPa</td>
</tr>
<tr>
<td>$v_{max}$</td>
<td>Maximum design shear stress permitted at a section, MPa</td>
</tr>
<tr>
<td>$v_{tn}$</td>
<td>Shear stress due to torsion, MPa</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>Punching shear factor accounting for column location</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>Concrete strength factor to account for sustained loading and equivalent stress block</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>Factor for obtaining depth of compression block in concrete</td>
</tr>
</tbody>
</table>
**SAFE Reinforced Concrete Design**

**Table 9-1 List of Symbols Used in the NZS 3101-06 Code**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_c$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching critical section</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in concrete</td>
</tr>
<tr>
<td>$\varepsilon_{c,max}$</td>
<td>Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)</td>
</tr>
<tr>
<td>$\varepsilon_t$</td>
<td>Strain in reinforcement</td>
</tr>
<tr>
<td>$\phi_b$</td>
<td>Strength reduction factor for bending</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Strength reduction factor for shear and torsion</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Fraction of unbalanced moment transferred by flexure</td>
</tr>
<tr>
<td>$\gamma_e$</td>
<td>Fraction of unbalanced moment transferred by eccentricity of shear</td>
</tr>
</tbody>
</table>

**9.2 Design Load Combinations**

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For NZS 3101-06, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (AS/NZS 1170.0, 4.2.2):

1. $1.35D$ (AS/NZS 1170.0, 4.2.2(a))
2. $1.2D + 1.5L$ (AS/NZS 1170.0, 4.2.2(b))
3. $1.2D + 1.5(0.75 PL)$ (AS/NZS 1170.0, 4.2.2(b))
4. $1.2D + 0.4L + 1.0S$ (AS/NZS 1170.0, 4.2.2(g))
5. $1.2D \pm 1.0W$ (AS/NZS 1170.0, 4.2.2(d))
6. $0.9D \pm 1.0W$ (AS/NZS 1170.0, 4.2.2(e))
7. $1.2D + 0.4L \pm 1.0W$ (AS/NZS 1170.0, 4.2.2(d))
8. $1.0D \pm 1.0E$ (AS/NZS 1170.0, 4.2.2(f))
9. $1.0D + 0.4L \pm 1.0E$ (AS/NZS 1170.0, 4.2.2(f))
Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in SAFE whenever the NZS 3101-06 code is used. If roof live load is treated separately or if other types of loads are present, other appropriate load combinations should be used.

### 9.3 Limits on Material Strength

The upper and lower limits of $f'_c$ shall be as follows:

$$25 \leq f'_c \leq 100 \text{ MPa} \quad \text{(NZS 5.2.1)}$$

The lower characteristic yield strength of longitudinal reinforcement, $f_y$, should be equal to or less than 500 MPa for all frames (NZS 5.3.3). The lower characteristic yield strength of transverse (stirrup) reinforcement, $f_{yt}$, should not be greater than 500 MPa for shear or 800 MPa for confinement (NZS 5.3.3).

The code allows use of $f'_c$ and $f_y$ beyond the given limits, provided special study is conducted (NZS 5.2.1).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

### 9.4 Strength Reduction Factors

The strength reduction factors, $\phi$, are applied to the specified strength to obtain the design strength provided by a member. The $\phi$ factors for flexure, shear, and torsion are as follows:

$$\phi_b = 0.85 \text{ for flexure} \quad \text{(NZS 2.3.2.2)}$$

$$\phi_s = 0.75 \text{ for shear and torsion} \quad \text{(NZS 2.3.2.2)}$$

These values can be overwitten; however, caution is advised.
9.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

9.5.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

9.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top
reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

9.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 9-1 (NZS 7.4.2.7). Furthermore, it is assumed that the compression carried by the concrete is 0.75 times that which can be carried at the balanced condition (NZS 9.3.8.1). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

Figure 9-1 Rectangular Beam Design
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The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed \((0.1 f'_c A_g)\), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than \(0.1 f'_c A_g\) and axial tensions are always included in flexural and shear design.

### 9.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive \(M^*\) (i.e., designing top or bottom reinforcement), the depth of the compression block is given by \(a\) (see Figure 9-1), where,

\[
a = d - \sqrt{d^2 - \frac{2|M^*|}{\alpha_i f'_c \phi_b b}}
\]

where the default value of \(\phi_b\) is 0.85 (NZS 2.3.2.2) in the preceding and following equations. The factor \(\alpha_i\) is calculated as follows (NZS 7.4.2.7):

\[
\alpha_i = 0.85 \quad \text{for } f'_c \leq 55\text{MPa}
\]

\[
\alpha_i = 0.85 - 0.004(f'_c - 55) \quad \text{for } f'_c \geq 55\text{MPa}, \quad 0.75 \leq \alpha_i \leq 0.85
\]

The value \(\beta_i\) and \(c_b\) are calculated as follows:

\[
\beta_i = 0.85 \quad \text{for } f'_c \leq 30,
\]

\[
\beta_i = 0.85 - 0.008(f'_c - 30), \quad 0.65 \leq \beta_i \leq 0.85
\]

\[
c_b = \frac{\varepsilon_c}{\varepsilon_c + f'_c/E_s}d
\]

The maximum allowed depth of the rectangular compression block, \(a_{\text{max}}\), is given by:

\[
a_{\text{max}} = 0.75\beta_i c_b
\]

- If \(a \leq a_{\text{max}}\) (NZS 9.3.8.1), the area of tension reinforcement is given by:
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\[ A_s = \frac{M^*}{\phi_b f_y \left( d - \frac{a}{2} \right)} \]

The reinforcement is to be placed at the bottom if \( M^* \) is positive, or at the top if \( M^* \) is negative.

- If \( a > a_{\text{max}} \) (NZS 9.3.8.1), compression reinforcement is required (NZS 7.4.2.9) and is calculated as follows:

The compressive force developed in the concrete alone is given by:

\[ C = \alpha_1 f'_c b a_{\text{max}} \quad \text{(NZS 7.4.2.7)} \]

and the moment resisted by concrete compression and tension reinforcement is:

\[ M^* = C \left( d - \frac{a_{\text{max}}}{2} \right) \phi_b \]

Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

\[ M^*_s = M^* - M^*_c \]

The required compression reinforcement is given by:

\[ A'_s = \frac{M^*_s}{(f'_s - \alpha_1 f'_c)(d - d')\phi_b}, \quad \text{where} \]

\[ f'_s = E_{c,\text{max}} E_s \left[ \frac{c - d'}{c} \right] \leq f_y \quad \text{(NZS 7.4.2.2, 7.4.2.4)} \]

The required tension reinforcement for balancing the compression in the concrete is:

\[ A_{st} = \frac{M^*_c}{f_y \left( d - \frac{a_{\text{max}}}{2} \right) \phi_b} \]
and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A_{s2} = \frac{M_s^*}{f_y(d - d')\phi_b} \]

Therefore, the total tension reinforcement, \( A_s = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A_s' \). \( A_s \) is to be placed at the bottom and \( A_s' \) is to be placed at the top if \( M^* \) is positive, and vice versa if \( M^* \) is negative.

### 9.5.1.2.2 Design of Flanged Beams

#### 9.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, \( M^* \) (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

#### 9.5.1.2.2.2 Flanged Beam Under Positive Moment

If \( M^* > 0 \), the depth of the compression block is given by:

\[ a = d - \sqrt{d^2 - \frac{2|M^*|}{\alpha_i f_c' \phi_b b_f}} \]  

(NZS 7.4.2)

The maximum allowable depth of the rectangular compression block, \( a_{\text{max}} \), is given by:

\[ a_{\text{max}} = 0.75 \beta_c c_b \]  

(NZS 7.4.2.7, 9.3.8.1)

- If \( a \leq h_f \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( a > a_{\text{max}} \).

- If \( a > h_f \), calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \), as shown in Figure 9-2.
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Figure 9-2 Design of a T-Beam Section

$C_f$ is given by:

$$C_f = \alpha f' (b_f - b_w) h_f$$

(NZS 7.4.2.7)

Therefore, $A_{sf} = \frac{C_f}{f_y}$ and the portion of $M^*$ that is resisted by the flange is given by:

$$M^*_f = C_f \left(d - \frac{d_s}{2}\right) \phi_h$$

Therefore, the balance of the moment, $M^*$, to be carried by the web is:

$$M^*_w = M^* - M^*_f$$

The web is a rectangular section with dimensions $b_w$ and $d$, for which the depth of the compression block is recalculated as:

$$a_i = d - \sqrt{d^2 - \frac{2M^*_w}{\alpha f'_s \phi_h b_w}}$$

(NZS 7.4.2 )
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- If \( a_1 \leq a_{\text{max}} \) (NZS 9.3.8.1), the area of tension reinforcement is then given by:

\[
A_{s2} = \frac{M_w^*}{\phi_b f_y' \left( d - \frac{a_1}{2} \right)}, \text{ and}
\]

\[
A_s = A_{s1} + A_{s2}
\]

This reinforcement is to be placed at the bottom of the flanged beam.

- If \( a_1 > a_{\text{max}} \) (NZS 9.3.8.1), compression reinforcement is required and is calculated as follows:

The compressive force in the web concrete alone is given by:

\[
C_w = \alpha_1 f'_c b_w a_{\text{max}} \quad \text{(NZS 7.4.2.7)}
\]

and the moment resisted by the concrete web and tension reinforcement is:

\[
M_w^* = C_w \left( d - \frac{a_{\text{max}}}{2} \right) \phi_b
\]

The moment resisted by compression and tension reinforcement is:

\[
M_s^* = M_w^* - M_c^*
\]

Therefore, the compression reinforcement is computed as:

\[
A'_s = \frac{M_s^*}{(f_s' - \alpha_1 f'_c)(d - d') \phi_b}, \text{ where}
\]

\[
f'_s = c_{c,\text{max}} E_s \left[ \frac{c - d'}{c} \right] \leq f_y \quad \text{(NZS 7.4.2.2, 7.4.2.4)}
\]

The tension reinforcement for balancing compression in the web concrete is:

\[
A_{s2} = \frac{M_c^*}{f_y \left( d - \frac{a_{\text{max}}}{2} \right) \phi_b}
\]
and the tension reinforcement for balancing the compression reinforcement is:

\[ A_{s3} = \frac{M'_t}{f_y (d - d') \phi_h} \]

Total tension reinforcement is \( A_s = A_{s1} + A_{s2} + A_{s3} \), and the total compression reinforcement is \( A'_s \). \( A_s \) is to be placed at the bottom, and \( A'_s \) is to be placed at the top.

### 9.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the maximum of the two limits:

\[ A_s \geq \frac{\sqrt{f''_c b_w d}}{4 f_y} \]  (NZS 9.3.8.2.1)

\[ A_s \geq 1.4 \frac{b_w d}{f_y} \]  (NZS 9.3.8.2.1)

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

\[ A_s \leq \begin{cases} 
0.04 bd & \text{Rectangular beam} \\
0.04 b_w d & \text{Flanged beam}
\end{cases} \]

\[ A'_s \leq \begin{cases} 
0.04 bd & \text{Rectangular beam} \\
0.04 b_w d & \text{Flanged beam}
\end{cases} \]

### 9.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, \( V' \).
Determine the shear force, $V_c$, that can be resisted by the concrete.

Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

### 9.5.2.1 Determine Shear Force and Moment

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam section are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

### 9.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_c$, is calculated as:

$$V_c = v_c A_{cv}$$  \hspace{1cm} (NZS 9.3.9.3.4)

The allowable shear stress capacity is given by:

$$v_c = k_d k_a k_n v_b$$  \hspace{1cm} (NZS 9.3.9.3.4)

The basic shear strength for a rectangular section is computed as,

$$v_b = \left[ 0.07 + 10 \frac{A_s}{b_w d} \right] \lambda \sqrt{f'_c}, \text{ where}$$  \hspace{1cm} (NZS 9.3.9.3.4)

$$f'_c \leq 50 \text{ MPa, and}$$  \hspace{1cm} (NZS 9.3.9.3.4)

$$0.08 \lambda \sqrt{f'_c} \leq v_b \leq 0.2 \lambda \sqrt{f'_c}$$  \hspace{1cm} (NZS 9.3.9.3.4)

where

$$\lambda = \begin{cases} 
1.0, & \text{normal concrete} \\
0.85, & \text{sand light-weight concrete} \\
0.75, & \text{all light-weight concrete} 
\end{cases}$$  \hspace{1cm} (NZS 9.3.9.3.5)
The factor $k_a$ allows for the influence of maximum aggregate size on shear strength. For concrete with a maximum aggregate size of 20 mm or more, $k_a$ shall be taken as 1.0. For concrete where the maximum aggregate size is 10 mm or less, the value of $k_a$ shall be taken as 0.85. Interpolation is used between these limits. The program default for $k_a$ is 1.0.

$$k_a = \begin{cases} 
0.85, & a_g \leq 10\text{mm} \\
0.85 + 0.15 \left( \frac{a_g - 10}{20} \right), & a_g < 20\text{mm} \\
1.00, & a_g \geq 20\text{mm}
\end{cases} \quad \text{(NZS 9.3.9.3.4)}$$

The factor $k_d$ allows for the influence of member depth on strength and it shall be calculated from the following conditions:

- For members with shear reinforcement equal to or greater than the nominal shear reinforcement given in NZS 9.3.9.4.15, $k_d = 1.0$
- For members with an effective depth equal to or smaller than 400 mm, $k_d = 1.0$ (NZS 9.3.9.3.4)
- For members with an effective depth greater than 400,
  \[ k_d = \left( \frac{400}{d} \right)^{0.25} \text{ where } d \text{ is in mm} \text{ (NZS 9.3.9.3.4)} \]

The factor $k_n$ allows for the influence of axial loading (NZS 10.3.10.3.1).

$$k_n = \begin{cases} 
1, & N^* = 0 \\
1 + 3 \left( \frac{N^*}{A_g f_c} \right), & N^* > 0 \quad \text{(NZS 10.3.10.3.1)} \\
1 + 12 \left( \frac{N^*}{A_g f_c} \right), & N^* < 0
\end{cases}$$

### 9.5.2.3 Determine Required Shear Reinforcement

The average shear stress is computed for rectangular and flanged sections as:
The average shear stress is limited to a maximum of,

\[ v_{\text{max}} = \min \{0.2 f'_c, 8 \text{ MPa} \} \]  \hspace{1cm} (NZS 7.5.2, 9.3.9.3.3)

The shear reinforcement is computed as follows:

- If \( v^* \leq \phi_s \left( \frac{v_c}{2} \right) \) or \( h \leq \max(300 \text{ mm}, 0.5b_w) \),
  \[ A_v = 0 \]  \hspace{1cm} (NZS 9.3.9.4.13)

- If \( \phi_s \left( \frac{v_c}{2} \right) < v^* \leq \phi_s v_c \),
  \[ A_v = \frac{16 \sqrt{f'_c b_w}}{f_{ys}} \]  \hspace{1cm} (NZS 7.5.10, 9.3.9.4.15)

- If \( \phi_s v_c < v^* \leq \phi_s v_{\text{max}} \),
  \[ A_v = \frac{\left( v^* - \phi_s v_c \right)}{\phi_s f_{ys} d} \]  \hspace{1cm} (NZS 9.3.9.4.2)

- If \( v^* > v_{\text{max}} \), a failure condition is declared.  \hspace{1cm} (NZS 7.5.2, 9.3.9.3.3)

If the beam depth \( h \) is less than the minimum of 300 mm and 0.5\( b_w \), no shear reinforcement is required (NZS 9.3.9.4.13).

The maximum of all of the calculated \( A_v/s \) values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.
9.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, $T^*$. 
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

Note that the torsion design can be turned off by choosing not to consider torsion in the Design Preferences.

9.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design $T^*$ is permitted to be reduced in accordance with the code (NZS 7.6.1.3). However, the program does not automatically redistribute the internal forces and reduce $T^*$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

9.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as $A_{co}$, $A_o$, $p_c$, $p_o$, $t_c$, and $t_o$ are calculated. These properties are described in the following (NZS 7.1).

\[
\begin{align*}
A_{co} &= \text{Area enclosed by outside perimeter of concrete cross-section} \\
A_o &= \text{Gross area enclosed by shear flow path} \\
p_c &= \text{Outside perimeter of concrete cross-section}
\end{align*}
\]
In calculating the section properties involving reinforcement, such as \(A_o\), \(p_o\), and \(t_o\), it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to a 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during \(T_{cr}\) calculation. With this assumption, the special properties for a rectangular beam section are given as:

\[
\begin{align*}
A_{co} &= bh \\
A_o &= (b - 2c)(h - 2c) \\
p_c &= 2b + 2h \\
p_o &= 2(b - 2c) + 2(h - 2c) \\
t_c &= 0.75 \frac{A_o}{p_o} \\
t_o &= 0.75 \frac{A_{co}}{p_c}
\end{align*}
\]

(NZS 7.1)

where, the section dimensions \(b\), \(h\), and \(c\) are shown in Figure 9-3. Similarly, the special section properties for a flanged beam section are given as:

\[
\begin{align*}
A_{co} &= b_sh + (b_f - b_w)h_f \\
A_o &= (b_w - 2c)(h - 2c) \\
p_c &= 2b_f + 2h \\
p_o &= 2(h - 2c) + 2(b_w - 2c) \\
t_c &= 0.75 \frac{A_o}{p_o} \\
t_o &= 0.75 \frac{A_{co}}{p_c}
\end{align*}
\]

(NZS 7.1)
where the section dimensions $b_f$, $b_w$, $h$, $h_f$, and $c$ for a flanged beam are shown in Figure 9-3. Note that the flange width on either side of the beam web is limited to the smaller of $3h_f$ (NZS 7.6.1.7).

9.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, $T_{cr}$, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \phi 0.1A_{co} t_c \sqrt{f'_c}$$  \hspace{1cm} (NZS 7.6.1.2)

where $A_{co}$ and $t_c$ are as described in the previous section, and $f'_c$ is the specified concrete compressive strength. The stress caused by torsion should also be limited in order to ignore torsion, defined as:

$$\frac{T^*}{\phi 2A_{co} t_{o}} \leq 0.08 \sqrt{f'_c}$$  \hspace{1cm} (NZS 7.6.1.3)
9.5.3.4 Determine Torsion Reinforcement

If the factored torsion, $T^*$, is less than the threshold limit, $T_{cr}$, and meets the torsion stress limit, torsion can be safely ignored (NZS 7.6.1). In that case, the program reports that no torsion reinforcement is required. However, if $T^*$ exceeds the threshold limit, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (NZS 7.6.4.1).

- If $T^* > T_{cr}$ and/or the torsion stress limit is not met, the required closed stirrup area per unit spacing, $A_t/s$, is calculated as:

$$A_t = \frac{v_{tm} T^*}{f_{yt}}$$

(NZS 7.6.4.2)

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{v_{tm} t_o P_o}{f_y}$$

(NZS 7.6.4.3)

where the torsional shear stress $v_{tm}$ is defined as:

$$v_{tm} = \frac{T^*}{\phi 2 A_o t_o}$$

(NZS 7.6.1.6)

The minimum closed stirrups and longitudinal reinforcement shall be such that the following is satisfied, where $A_t/s$ can be from any closed stirrups for shear and $A_l$ can include flexure reinforcement, provided it is fully developed.

$$\sqrt{\frac{A_t A_l}{s p_o}} = \frac{1.5 A_o t_o}{f_y A_o}$$

(NZS 7.6.2)

An upper limit of the combination of $V^*$ and $T^*$ that can be carried by the section is also checked using the equation:

$$v_n + v_{tm} < \min(0.2 f'_c, 8 \text{ MPa})$$

(NZS 7.6.1.8, 7.5.2)

For rectangular sections, $b_o$ is replaced with $b$. If the combination of $V^*$ and $T^*$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.
The maximum of all of the calculated $A_l$ and $A_i/s$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 9.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method (NZS 3101-06) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

#### 9.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and
bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

9.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

9.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

9.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limit (NZS 12.5.6.2, 8.8.1):

\[
A_i \geq \begin{cases} 
0.7 \frac{bh}{f_y} & f_y < 500 \text{ MPa} \\
0.0014bh & f_y \geq 500 \text{ MPa}
\end{cases}
\]

(NZS 12.5.6.2, 8.8.1)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

The slab reinforcement requirements reported by the program do not consider crack control. Any minimum requirements to satisfy crack limitations must be investigated independently of the program by the user.
9.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following.

9.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (NZS 12.7.1(b)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (NZS 12.7.1(b)). Figure 9-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 9-4 Punching Shear Perimeters](image)

9.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M^*$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M^*$, where
where $b_1$ is the width of the critical section measured in the direction of the span and $b_2$ is the width of the critical section measured in the direction perpendicular to the span.

### 9.6.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$
\gamma_j = \frac{1}{1 + (2/3)b_1/b_2} \quad \text{(NZS 12.7.7.2)}
$$

$$
\gamma_v = 1 - \frac{1}{1 + (2/3)b_1/b_2} \quad \text{(NZS 12.7.7.1)}
$$

$$
\beta_c \alpha_s \left[ \frac{1}{6} \left( 1 + \frac{2}{\beta_c} \right) \sqrt{f'_c} \right] \\
\beta_c \alpha_s \left[ \frac{1}{6} \left( 1 + \frac{\alpha_s d}{b_0} \right) \sqrt{f'_c} \right] \quad \text{(NZS 12.7.3.2)}
$$

where, $\beta_c$ is the ratio of the maximum to the minimum dimension of the critical section (NZS 12.1, 12.7.3.2(a)), $b_0$ is the perimeter of the critical section, and $\alpha_s$ is a scale factor based on the location of the critical section.

$$
\alpha_s = \begin{cases} 
20 & \text{for interior columns,} \\
15 & \text{for edge columns,} \\
10 & \text{for corner columns.} 
\end{cases} \quad \text{(NZS 12.7.3.2(b))}
$$

A limit is imposed on the value of $\sqrt{f'_c}$ as follows:

$$
\lambda \sqrt{f'_c} \leq \sqrt{100} \quad \text{(NZS 9.3.9.3.5(6))}
$$
9.6.2.4 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

\[
v' = \frac{V^*}{b_d} + \gamma_{22}^* \left( \frac{M_2^* - V^*(y_3 - y_1)}{I_{22}I_{33} - I_{23}^2} \right) - \gamma_{33}^* \left( \frac{M_3^* - V^*(x_3 - x_1)}{I_{22}I_{33} - I_{23}^2} \right)
\]

Eq. 1

\[
I_{22} = \sum_{sides=1}^{n} T_{22}, \text{ where "sides" refers to the sides of the critical section for punching shear}
\]

Eq. 2

\[
I_{33} = \sum_{sides=1}^{n} T_{33}, \text{ where "sides" refers to the sides of the critical section for punching shear}
\]

Eq. 3

\[
I_{23} = \sum_{sides=1}^{n} T_{23}, \text{ where "sides" refers to the sides of the critical section for punching shear}
\]

Eq. 4

The equations for \( T_{22}, T_{33}, \text{ and } T_{23} \) are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 9-5.

\[
T_{22} = Ld(y_2 - y_3)^2, \text{ for side of critical section parallel to 2-axis}
\]

Eq. 5a

\[
T_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2, \text{ for the side of the critical section parallel to the 3-axis}
\]

Eq. 5b
Figure 9-5 Shear Stress Calculations at Critical Sections

\[ T_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - y_3)^2, \]  
for the side of the critical section parallel to the 2-axis  

Eq. 6

\[ T_{33} = Ld(x_2 - x_3)^2, \]  
for the side of the critical section parallel to the 3-axis  

Eq. 6b

\[ T_{23} = Ld(x_2 - x_3)(y_2 - y_3), \]  
for the side of the critical section parallel to the 2-axis or 3-axis  

Eq. 7

NOTE: \( T_{23} \) is explicitly set to zero for corner condition.
where,

\( b_0 \) = Perimeter of critical section for punching shear

\( d \) = Effective depth at critical section for punching shear based on average of \( d \) for 2 direction and \( d \) for 3 direction

\( I_{22} \) = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 2-axis

\( I_{33} \) = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 3-axis

\( I_{23} \) = Product of inertia of critical section for punching shear with respect to the 2 and 3 planes

\( L \) = Length of the side of the critical section for punching shear currently being considered

\( M_{2}^{*} \) = Moment about line parallel to 2-axis at center of column (positive per right-hand rule)

\( M_{3}^{*} \) = Moment about line parallel to 3-axis at center of column (positive per right-hand rule)

\( V^{*} \) = Punching shear stress

\( V^{*} \) = Shear at center of column (positive upward)

\( x_1, y_1 \) = Coordinates of column centroid

\( x_2, y_2 \) = Coordinates of center of one side of critical section for punching shear

\( x_3, y_3 \) = Coordinates of centroid of critical section for punching shear

\( x_4, y_4 \) = Coordinates of location where you are calculating stress

\( \gamma_{2} \) = Percent of \( M_{2} \) resisted by shear

\( \gamma_{3} \) = Percent of \( M_{3} \) resisted by shear
9.6.2.5 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

9.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as described in the subsections that follow.

9.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is determined as:

\[ v_c = \frac{1}{6} \sqrt{f'_{c}} \]  

(NZS 12.7.3.5)

9.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ v_{\text{max}} = 0.5 \sqrt{f'_{c}} \]  

(NZS 12.7.3.4)

Given \( v' \), \( v_c \), and \( v_{\text{max}} \), the required shear reinforcement is calculated as follows, where, \( \phi \) is the strength reduction factor.
Chapter 9 - Design for NZS 3101-06

\[ A_v = \frac{v_n - v_c}{\phi f_{ys} d} \]  
(NZS 12.7.4.2(a))

Minimum punching shear reinforcement should be provided such that:

\[ V_s \geq \frac{1}{16} f_c' b_o d \]  
(NZS 12.7.4.3)

- If \( v_n > \phi v_{\text{max}} \), a failure condition is declared.  
(NZS 12.7.3.4)

- If \( v_n \) exceeds the maximum permitted value of \( \phi v_{\text{max}} \), the concrete section should be increased in size.

9.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 9-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

![Figure 9-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone](image)

The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \). The spacing between adjacent shear reinforcement in the
The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $2d$ and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

$$s_o \leq 0.5d \quad \text{(NZS 12.7.4.4)}$$

$$s \leq 0.5d \quad \text{(NZS 12.7.4.4)}$$

$$g \leq 2d \quad \text{(NZS 12.7.4.4)}$$
This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Singapore standard, Structural Use of Concrete code CP 65-99 [CP 99], is selected. The program also includes the recommendations of BC 2:2008 Design Guide of High Strength Concrete to Singapore Standard CP65 [BC 2008]. Various notations used in this chapter are listed in Table 10-1. For referencing to the pertinent sections of the Singapore code in this chapter, a prefix “CP” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

10.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_g$</td>
<td>Gross area of cross-section, mm$^2$</td>
</tr>
</tbody>
</table>
### Table 10-1 List of Symbols Used in the CP 65-99 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_l$</td>
<td>Area of longitudinal reinforcement for torsion, mm$^2$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}$</td>
<td>Total cross-sectional area of links at the neutral axis, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv,t}$</td>
<td>Total cross-sectional area of closed links for torsion, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}/S_c$</td>
<td>Area of shear reinforcement per unit length of the member, mm$^2$/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width or effective width of the section in the compression zone, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Width or effective width of flange, mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Average web width of a flanged beam, mm</td>
</tr>
<tr>
<td>$C$</td>
<td>Torsional constant, mm$^4$</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Depth to center of compression reinforcement, mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, assumed as 200,000 MPa</td>
</tr>
<tr>
<td>$f$</td>
<td>Punching shear factor considering column location</td>
</tr>
<tr>
<td>$f_{cu}$</td>
<td>Characteristic cube strength, MPa</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, MPa</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Characteristic strength of reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{yy}$</td>
<td>Characteristic strength of shear reinforcement, MPa (&lt; 460 MPa)</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section in the plane of bending, mm</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Flange thickness, mm</td>
</tr>
<tr>
<td>$h_{min}$</td>
<td>Smaller dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$h_{max}$</td>
<td>Larger dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$K$</td>
<td>Normalized design moment, $M_u/bd^2f_{cu}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$K'$</td>
<td>Maximum $\frac{M_u}{bd^2 f_{cu}}$ for a singly reinforced concrete section</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Shear strength enhancement factor for support compression</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Concrete shear strength factor, $\left[\frac{f_{cu}}{30}\right]^{\frac{1}{3}}$</td>
</tr>
<tr>
<td>$M$</td>
<td>Design moment at a section, N-mm</td>
</tr>
<tr>
<td>$M_{\text{single}}$</td>
<td>Limiting moment capacity as singly reinforced beam, N-mm</td>
</tr>
<tr>
<td>$s_v$</td>
<td>Spacing of the links along the length of the beam, mm</td>
</tr>
<tr>
<td>$T$</td>
<td>Design torsion at ultimate design load, N-mm</td>
</tr>
<tr>
<td>$u$</td>
<td>Perimeter of the punching critical section, mm</td>
</tr>
<tr>
<td>$V$</td>
<td>Design shear force at ultimate design load, N</td>
</tr>
<tr>
<td>$v$</td>
<td>Design shear stress at a beam cross-section or at a punching critical section, MPa</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Design concrete shear stress capacity, MPa</td>
</tr>
<tr>
<td>$v_{\text{max}}$</td>
<td>Maximum permitted design factored shear stress, MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Torsional shear stress, MPa</td>
</tr>
<tr>
<td>$x$</td>
<td>Neutral axis depth, mm</td>
</tr>
<tr>
<td>$x_{\text{bal}}$</td>
<td>Depth of neutral axis in a balanced section, mm</td>
</tr>
<tr>
<td>$z$</td>
<td>Lever arm, mm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Torsional stiffness constant</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>Moment redistribution factor in a member</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Partial safety factor for load</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>Partial safety factor for material strength</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Maximum concrete strain</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in tension reinforcement</td>
</tr>
<tr>
<td>$\varepsilon'_s$</td>
<td>Strain in compression reinforcement</td>
</tr>
</tbody>
</table>
10.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, \( \gamma \) (CP 2.4.1.3). If a structure is subjected to dead (D), live (L), pattern live (PL), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered (CP 2.4.3).

\[
\begin{align*}
1.4D & \\
1.4D + 1.6L & \quad \text{(CP 2.4.3)} \\
1.4D + 1.6(0.75PL) & \quad \text{(CP 2.4.3)} \\
1.0D \pm 1.4W & \\
1.4D \pm 1.4W & \quad \text{(CP 2.4.3)} \\
1.2D + 1.2L \pm 1.2W & 
\end{align*}
\]

These are also the default design load combinations in SAFE whenever the CP 65-99 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

10.3 Limits on Material Strength

The concrete compressive strength, \( f_{cu} \), should not be less than 30 MPa (CP 3.1.7.2).

The program does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsible to use the proper strength values while defining the materials.
10.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, $\gamma_m$. The values of $\gamma_m$ used in the program are listed in the table that follows and are taken from CP Table 2.2 (CP 2.4.4.1):

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforcement</td>
<td>1.15</td>
</tr>
<tr>
<td>Concrete in flexure and axial load</td>
<td>1.50</td>
</tr>
<tr>
<td>Concrete shear strength without shear reinforcement</td>
<td>1.25</td>
</tr>
</tbody>
</table>

These factors are incorporated into the design equations and tables in the code, but can be overwritten.

10.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement
10.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

10.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

10.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 10-1 (CP 3.4.4.4), where $\varepsilon_c$ is defined as:

$$
\varepsilon_c = \begin{cases} 
0.0035 & \text{if } f_{cu} \leq 60\text{MPa} \\
0.0035 - \frac{(f_{cu} - 60)}{50000} & \text{if } f_{cu} > 60\text{MPa}
\end{cases}
$$

(CP 2.5.3, BC 2.2)
Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$; CP 3.4.4.4). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \leq \begin{cases} 
0.5 & \text{for } f_{cu} \leq 60 \text{ N/mm}^2 \\
0.4 & \text{for } 60 < f_{cu} \leq 75 \text{ N/mm}^2 \\
0.33 & \text{for } 75 < f_{cu} \leq 105 \text{ N/mm}^2 
\end{cases} \quad \text{(CP 3.4.4.4, BC 2.2)}$$

...to safeguard against non-ductile failures (CP 3.4.4.4). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The depth of the compression block is given by:

$$a = \begin{cases} 
0.9x & \text{for } f_{cu} \leq 60 \text{ N/mm}^2 \\
0.8x & \text{for } 60 < f_{cu} \leq 75 \text{ N/mm}^2 \\
0.72x & \text{for } 75 < f_{cu} \leq 105 \text{ N/mm}^2 
\end{cases} \quad \text{(CP 3.4.4.4, BC 2.2)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed $(0.1f_{cu}A_g)$ (CP 3.4.4.1), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $0.1f_{cu}A_g$ and axial tensions are always included in flexural and shear design.

**10.5.1.2.1 Design of Rectangular Beams**

For rectangular beams, the limiting moment capacity as a singly reinforced beam, $M_{\text{single}}$, is first calculated for a section. The reinforcement is determined based on whether $M$ is greater than, less than, or equal to $M_{\text{single}}$. See Figure 10-1.

Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

$$M_{\text{single}} = K'f_{cu} bd^2, \quad \text{(CP 3.4.4.4)}$$
If $M \leq M_{\text{single}}$, the area of tension reinforcement, $A_s$, is given by:

$$A_s = \frac{M}{0.87 f_y z}$$

where

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right) \leq 0.95d$$

and

$$K = \frac{M}{f_{cu} bd^2}$$

Figure 10-1 Rectangular Beam Design
This reinforcement is to be placed at the bottom if $M$ is positive, or at the top if $M$ is negative.

- If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$A'_{s} = \frac{M - M_{\text{single}}}{(f'_{s} - \frac{0.67f_{\text{cu}}}{\gamma_{m}})(d - d')}$$

(CP 3.4.4.4)

where $d'$ is the depth of the compression reinforcement from the concrete compression face, and

$$f'_{s} = 0.87f_{y} \quad \text{if} \quad \frac{d'}{d} \leq \frac{1}{2} \left[ 1 - \frac{f_{y}}{800} \right]$$

(CP 3.4.4.1, 2.5.3, Fig 2.2)

$$f'_{s} = E_{s}e_{c} \left[ 1 - \frac{2d''}{d} \right] \text{if} \quad \frac{d'}{d} > \frac{1}{2} \left[ 1 - \frac{f_{y}}{800} \right]$$

(CP 3.4.4.4, 2.5.3, Fig 2.2)

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$A_{s} = \frac{M_{\text{single}}}{0.87f_{y}z} + \frac{M - M_{\text{single}}}{0.87f_{y}(d - d')}$$

(CP 3.4.4.4)

$$z = 0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \leq 0.95d$$

(CP 3.4.4.4)

10.5.1.2.2 Design of Flanged Beams

10.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.
10.5.1.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located in the flange. On the basis of this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange width, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 10-2.

![Diagram of T-Beam Section](image)

**Figure 10-2 Design of a T-Beam Section**

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

$$K = \frac{M}{f_{cu} b_f d^2}$$

(CP 3.4.4.4)

Then the moment arm is computed as:

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\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right) \leq 0.95d \]  
\hspace{1cm} (CP 3.4.4.4)

the depth of neutral axis is computed as:

\[ x = \begin{cases} 
\frac{d-z}{0.45}, & \text{for } f_{cu} \leq 60 \text{N/mm}^2 \\
\frac{d-z}{0.40}, & \text{for } 60 < f_{cu} \leq 75 \text{N/mm}^2 \\
\frac{d-z}{0.36}, & \text{for } 75 < f_{cu} \leq 105 \text{N/mm}^2 
\end{cases} \]  
\hspace{1cm} (CP 3.4.4.4, BC 2.2, Fig 2.3)

and the depth of the compression block is given by:

\[ a = \begin{cases} 
0.9x, & \text{for } f_{cu} \leq 60 \text{N/mm}^2 \\
0.8x, & \text{for } 60 < f_{cu} \leq 75 \text{N/mm}^2 \\
0.72x, & \text{for } 75 < f_{cu} \leq 105 \text{N/mm}^2 
\end{cases} \]  
\hspace{1cm} (CP 3.4.4.4, BC 2.2, Fig 2.3)

- If \( a \leq h_f \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( K > K' \).

- If \( a > h_f \),

If \( M \leq \beta_f f_{cu} bd^2 \) and \( h_f \leq 0.45d \) then,

\[ A_s = \frac{M + 0.1 f_{cu} bd (0.45d - h_f)}{0.87 f_y (d - 0.5h_f)}, \text{ where} \]  
\hspace{1cm} (BS 3.4.4.5)

\[ \beta_f = 0.45 \frac{h_f}{d} \left( 1 - \frac{b_w}{b} \right) \left( 1 - \frac{h_f}{2d} \right) + 0.15 \frac{b_w}{b} \]  
\hspace{1cm} (BS 3.4.4.5)

Otherwise the calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \), as shown in Figure 10-2.
In that case, the ultimate resistance moment of the flange is given by:

$$M_f = 0.45 f_{cu} (b_f - b_w) h_f \left( d - 0.5 h_f \right)$$  \hspace{1cm} (CP 3.4.4.5)

The moment taken by the web is computed as:

$$M_w = M - M_f$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2}$$  \hspace{1cm} (CP 3.4.4.4)

- If $K_w \leq 0.156$ (CP 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_x = \frac{M_f}{0.87 f_s z (d - 0.5 h_f)} + \frac{M_w}{0.87 f_s z}$$, where

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{K_w}{0.9}} \right) \leq 0.95 d$$

- If $K_w > K'$ (CP 3.4.4.4), compression reinforcement is required and is calculated as follows:

The ultimate moment of resistance of the web only is given by:

$$M_{uw} = K' f_{cu} b_u d^2$$  \hspace{1cm} (CP 3.4.4.4)

The compression reinforcement is required to resist a moment of magnitude $M_w - M_{uw}$. The compression reinforcement is computed as:

$$A_{c'}' = \frac{M_w - M_{uw}}{\left( f_s' - \frac{0.67 f_{cu}}{\gamma_m} \right) (d - d')} $$
where, $d'$ is the depth of the compression reinforcement from the concrete compression face, and

$$f'_{ys} = 0.87 f_y \quad \text{if} \quad d'/d \leq \frac{1}{2} \left[ 1 - \frac{f_y}{800} \right]$$  (CP 3.4.4.4, 2.5.3, Fig 2.2)

$$f'_{ys} = E_s e_s \left[ 1 - \frac{2d''}{d} \right] \quad \text{if} \quad d'/d > \frac{1}{2} \left[ 1 - \frac{f_y}{800} \right]$$  (CP 3.4.4.4, 2.5.3, Fig 2.2)

The area of tension reinforcement is obtained from equilibrium as:

$$A_s = \frac{1}{0.87 f_y} \left[ \frac{M_f}{d - 0.5 h_f} + \frac{M_{uw}}{z} + \frac{M_{w} - M_{uw}}{d - d''} \right]$$

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \right) \leq 0.95d$$

### 10.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from CP Table 3.27 (CP 3.12.5.3) with interpolation for reinforcement of intermediate strength.

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage $f_y = 250$ MPa</th>
<th>$f_y = 460$ MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>—</td>
<td>$100 \frac{A_s}{bh}$</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>T- or L-Beam with web in tension</td>
<td>$b_w/b_f &lt; 0.4$</td>
<td>$100 \frac{A_s}{b_w h}$</td>
<td>0.32</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>$b_w/b_f \geq 0.4$</td>
<td>$100 \frac{A_s}{b_w h}$</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>T-Beam with web in compression</td>
<td>—</td>
<td>$100 \frac{A_s}{b_w h}$</td>
<td>0.48</td>
<td>0.26</td>
</tr>
</tbody>
</table>
The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from CP Table 3.27 (CP 3.12.5.3).

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>—</td>
<td>$100 \frac{A_s}{bh}$</td>
<td>0.20</td>
</tr>
<tr>
<td>T- or L-Beam</td>
<td>Web in tension</td>
<td>$100 \frac{A_s}{b_fh}$</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>Web in compression</td>
<td>$100 \frac{A_s}{b_wh}$</td>
<td>0.20</td>
</tr>
</tbody>
</table>

For $f_{cu} > 40$ MPa, the minimum percentage shown in CP Table 3.27 shall be multiplied by a factor of $\left(\frac{f_{cu}}{40}\right)^{2/3}$ (CP 3.12.5.3, BC 2.2).

An upper limit of 0.04 times the gross cross-sectional area on both the tension reinforcement and the compression reinforcement is imposed upon request (CP 3.12.6.1).

### 10.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (CP 3.4.5):

- Determine the shear stress, $\nu$.
- Determine the shear stress, $\nu_c$, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.
The following three sections describe in detail the algorithms associated with these steps.

### 10.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors. The shear stress is then calculated as:

\[
v = \frac{V}{b_w d}
\]

The maximum allowable shear stress, \(v_{max}\) is defined as:

\[
v_{max} = \min (0.8 \sqrt{f_{cu}}, 7 \text{ MPa}).
\]

For light-weight concrete, \(v_{max}\) is defined as:

\[
v_{max} = \min(0.63 \sqrt{f_{cu}}, 4 \text{ MPa})
\]

### 10.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, \(v_c\), is calculated as:

\[
v' = \frac{Vh}{A_c M} \leq v_c \sqrt{1 + \frac{N}{A_c v_c}} \quad (CP 3.4.5.12)
\]

\[
v_c = \frac{0.84 k_1 k_2}{\gamma_m} \left( \frac{100 A_s}{bd} \right)^{\frac{Y}{2}} \left( \frac{400}{d} \right)^{\frac{Y}{4}}
\]

\(k_1\) is the enhancement factor for support compression, and is conservatively taken as 1

\(k_2 = \left( \frac{f_{cu}}{30} \right)^{\frac{Y}{2}}, 1 \leq k_2 \leq \left( \frac{80}{30} \right)^{\frac{Y}{2}} \quad (CP 3.4.5.4)\)
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\( \gamma_m = 1.25 \) \hspace{1cm} (CP 2.4.4.1)

However, the following limitations also apply:

\[ 0.15 \leq \frac{100 A_s}{bd} \leq 3 \] \hspace{1cm} (CP 3.4.5.4, Table 3.9)

\[ \left( \frac{400}{d} \right)^{\frac{1}{4}} \geq 0.67 \text{ (unreinforced)} \text{ or } \geq 1 \text{ (reinforced)} \] \hspace{1cm} (CP 3.4.5.4, Table 3.9)

\( f_{cu} \leq 80 \text{ MPa} \) (for calculation purpose only) \hspace{1cm} (CP 3.4.5.4, Table 3.9)

\[ \frac{Vh}{M} \leq 1 \] \hspace{1cm} (CP 3.4.5.12)

A\(_s\) is the area of tension reinforcement

10.5.2.3 Determine Required Shear Reinforcement

Given \( v, v', \text{ and } v_{\text{max}} \), the required shear reinforcement is calculated as follows (CP Table 3.8, CP 3.4.5.3):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, \( v_r \), as:

\[ v_r = \begin{cases} 
0.4 & \text{if } f_{cu} \leq 40\text{N/mm}^2 \\
0.4 \left( \frac{f_{cu}}{40} \right)^{\frac{1}{2}} & \text{if } 40 < f_{cu} \leq 80\text{N/mm}^2 
\end{cases} \] \hspace{1cm} (CP 3.4.5.3, Table 3.8)

\( f_{cu} \leq 80\text{N/mm}^2 \) (for calculation purpose only) \hspace{1cm} (CP 3.4.5.3, Table 3.8)

- If \( v \leq v' + v_r \),

\[ \frac{A_s}{s_v} = \frac{v_b}{0.87 f_{yv}} \] \hspace{1cm} (CP 3.4.5.3, Table 3.8)

- If \( v > v' + v_r \),

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If \( v > v_{\text{max}} \), a failure condition is declared. (CP 3.4.5.2)

In the preceding expressions, a limit is imposed on the \( f_{yv} \) as

\[
A_{sv} = \frac{(v - v_c)b}{0.87f_{yv}}
\]

(\text{CP 3.4.5.3, Table 3.8})

The maximum of all of the calculated \( A_{sv}/s_v \) values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

10.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, \( v_t \).
- Determine special section properties.
- Determine critical torsion stress.
- Determine the torsion reinforcement required.

Note that references in this section refer to CP 65:Part 2.

10.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding
torsions for different load cases with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following sections (CP Part 2 3.4.5.15).

The torsional shear stress, \( \tau_t \), for a rectangular section is computed as:

\[
\tau_t = \frac{2T}{h_{\text{min}}^2 \left(h_{\text{max}} - h_{\text{min}}/3\right)}
\]  

(CP Part 2 2.4.4.1)

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

\[
T_{\text{seg}} = T \left( \sum \left( \frac{h_{\text{min}}^3 h_{\text{max}}}{h_{\text{min}}^3 h_{\text{max}}} \right) \right)
\]  

(CP Part 2 2.4.4.2)

- \( h_{\text{max}} \) = Larger dimension of a rectangular section
- \( h_{\text{min}} \) = Smaller dimension of a rectangular section

If the computed torsional shear stress, \( \tau_t \), does not satisfy the following limit for sections with a larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

\[
\tau_t \leq \min \left(0.8 \sqrt{f_{cu}}, 7 \text{N/mm}^2\right) \times \frac{y_t}{550}
\]  

(CP Part 2 2.4.5)

10.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, \( \tau_{t,\text{min}} \), for which the torsion in the section can be ignored is calculated as:

\[
\tau_{t,\text{min}} = \min \left(0.067 \sqrt{f_{cu}}, 0.6 \text{N/mm}^2\right)
\]  

(CP Part 2 2.4.6)
where $f_{cu}$ is the specified concrete compressive strength.

For light-weight concrete, $v_{t,\text{min}}$ is defined as:

$$v_{t,\text{min}} = \min \left( 0.067 \sqrt{f_{cu}}, 0.6 \text{N/mm}^2 \right) \times 0.8 \quad \text{(CP Part 2 5.5)}$$

### 10.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, $v_t$, is less than the threshold limit, $v_{t,\text{min}}$, torsion can be safely ignored (CP Part 2 2.4.6). In that case, the program reports that no torsion reinforcement is required. However, if $v_t$ exceeds the threshold limit, $v_{t,\text{min}}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP Part 2 2.4.6).

- If $v_t > v_{t,\text{min}}$, the required closed stirrup area per unit spacing, $A_{sv,t}/s_v$, is calculated as:

$$A_{sv,t} = \frac{T}{0.8x_1y_1 \left( 0.87f_{yy} \right)} \quad \text{(CP Part 2 2.4.7)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = A_{sv,t} f_{yy} \left( x_1 + y_1 \right) s_v f_y \quad \text{(CP Part 2 2.4.7)}$$

In the preceding expressions, $x_1$ is the smaller center-to-center dimension of the closed link and $y_1$ is the larger center-to-center dimension of the closed link.

An upper limit of the combination of $v$ and $v_t$ that can be carried by the section also is checked using the equation:

$$v + v_t \leq \min \left( 0.8 \sqrt{f_{cu}}, 7 \text{N/mm}^2 \right) \quad \text{(CP Part 2 2.4.5)}$$

If the combination of $v$ and $v_t$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated $A_l$ and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.
The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

\[
\begin{align*}
&c \quad b - 2c \quad c \\
&h - 2c \\
&c
\end{align*}
\]

Closed Stirrup in Rectangular Beam

\[
\begin{align*}
&c \quad b_f \\
&d_s \\
&h - 2c \\
&c
\end{align*}
\]

Closed Stirrup in T-Beam Section

Figure 10-3 Closed stirrup and section dimensions for torsion design

10.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method (CP 65-99) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.
10.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps described in the subsections that follow are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

10.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

10.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the
design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

10.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limit (CP 3.12.5.3, CP Table 3.25) with interpolation for reinforcement of intermediate strength:

\[
A_r \geq \begin{cases} 
0.0024bh & \text{if } f_y = 250 \text{ MPa} \\
0.0013bh & \text{if } f_y = 460 \text{ MPa}
\end{cases} \quad \text{(CP 3.12.5.3)}
\]

For \( f_{cu} > 40 \text{ N/mm}^2 \), the preceding minimum reinforcement shall be multiplied by \( \left( f_{cu}/40\right)^{2/3} \).

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 3.12.6.1).

10.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following subsections.

10.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5\( d \) from the face of the support (CP 3.7.7.4, 3.7.7.6). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CP 3.7.7.1). Figure 10-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

10.6.2.2 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as (CP 3.7.7.4, 3.7.7.6):
\[ v_c = \frac{0.84k_1k_2}{\gamma_m} \left( \frac{100A_s}{bd} \right)^{\frac{1}{3}} \left( \frac{400}{d} \right)^{\frac{1}{4}} \]  

\((\text{CP 3.4.5.4, Table 3.9})\)

\(k_1\) is the enhancement factor for support compression, and is conservatively taken as 1  

\((\text{CP 3.4.5.8})\)

\[ k_2 = \left( \frac{f_{cu}}{30} \right)^{\frac{1}{3}} 1 \leq k_2 \leq \left( \frac{80}{30} \right)^{\frac{1}{3}} \]  

\((\text{CP 3.4.5.4, Table 3.9})\)

\[ \gamma_m = 1.25 \]  

\((\text{CP 3.4.5.2})\)

However, the following limitations also apply:

\[ 0.15 \leq \frac{100A_s}{bd} \leq 3 \]  

\((\text{CP 3.4.5.4, Table 3.9})\)
\[
\left(\frac{400}{d}\right)^{\frac{1}{4}} \geq 0.67 \text{ (unreinforced)} \text{ or } \geq 1 \text{ (reinforced)} \text{ (CP 3.4.5.4, Table 3.9)}
\]

For light-weight concrete, \(v_{\text{max}}\) is defined as:

\[
v \leq \min(0.63 \sqrt{f_{cu}}, \quad 4 \text{ MPa}) \quad \text{(CP Part 2 5.4)}
\]

\[
v \leq \min (0.8 \sqrt{f_{cu}}, \quad 7 \text{ MPa}) \quad \text{(CP 3.4.5.2, Table 3.9)}
\]

\[f_{cu} \leq 80 \text{ MPa (for calculation purpose only)} \quad \text{(CP 3.4.5.4, Table 3.9)}
\]

\(A_s = \) area of tension reinforcement, which is taken as the average tension reinforcement of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement is taken as zero in the current implementation.

### 10.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the nominal design shear stress, \(v\), is calculated as:

\[
V_{\text{eff},x} = V\left(f + \frac{1.5M_x}{V_y}\right) \quad \text{(CP 3.7.6.2, 3.7.6.3)}
\]

\[
V_{\text{eff},y} = V\left(f + \frac{1.5M_y}{V_x}\right) \quad \text{(CP 3.7.6.2, 3.7.6.3)}
\]

- \(V_{\text{max}} = \max\left(\frac{V_{\text{eff},x}}{u d}, \frac{V_{\text{eff},y}}{u d}\right) \quad \text{(CP 3.7.7.3)}\)

where,

- \(u\) is the perimeter of the critical section,

10 - 24 Slab Design
x and y are the lengths of the sides of the critical section parallel to the axis of bending

$M_x$ and $M_y$ are the design moments transmitted from the slab to the column at the connection

$V$ is the total punching shear force

$f$ is a factor to consider the eccentricity of punching shear force and is taken as:

\[
\begin{align*}
    f &= \begin{cases} 
    1.00 & \text{for interior columns,} \\
    1.25 & \text{for edge columns, and} \\
    1.25 & \text{for corner columns.}
    \end{cases} 
\end{align*}
\]  
(CP 3.7.6.2, 3.7.6.3)

10.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

10.6.3 Design Punching Shear Reinforcement

The use of shear links as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 3.7.7.5). If the slab thickness does not meet this requirement, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is completed as described in the following subsections.
Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

\[ v_{\text{max}} = 2v_c \]  

Given \( v \), \( v_c \), and \( v_{\text{max}} \), the required shear reinforcement is calculated as follows (CP 3.7.7.5):

- If \( v \leq 1.6v_c \),

\[
\frac{A_v}{s} = \frac{(v-v_c)ud}{0.87 f_{yy}} \geq \frac{0.4ud}{0.87 f_{yy}},
\]

(CP 3.7.7.5)

- If \( 1.6v_c \leq v < 2.0v_c \),

\[
\frac{A_v}{s} = \frac{5(0.7v-v_c)ud}{0.87 f_{yy}} \geq \frac{0.4ud}{0.87 f_{yy}},
\]

(CP 3.7.7.5)

- If \( v > v_{\text{max}} \), a failure condition is declared.

If \( v \) exceeds the maximum permitted value of \( v_{\text{max}} \), the concrete section should be increased in size.

Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 10-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.
The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \). The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5\( d \) measured in a direction parallel to the column face (CP 3.7.7.6).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

### 10.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 3.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, \( s_o \), between the column face and the first peripheral line of shear studs should not be smaller than 0.5\( d \). The spacing between adjacent shear studs, \( g \), at the first peripheral line of studs shall not ex-
ceed 1.5$d$. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

\[ s_o \leq 0.5d \]  
\[ s \leq 0.75d \]  
\[ g \leq 1.5d \]  

(CP 3.7.7.6)
Chapter 11
Design for AS 3600-09

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Australian code AS 3600-2009 [AS 2009] is selected. Various notations used in this chapter are listed in Table 11-1. For referencing to the pertinent sections of the AS code in this chapter, a prefix “AS” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

11.1 Notations

| Table 11-1 List of Symbols Used in the AS 3600-2009 Code |
|-----------------|-------------------------------------------------|
| $A_g$           | Gross area of concrete, mm$^2$                  |
| $A_{lt}$        | Area of longitudinal reinforcement for torsion, mm$^2$ |
| $A_{sc}$        | Area of compression reinforcement, mm$^2$      |
Table 11-1 List of Symbols Used in the AS 3600-2009 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{st}$</td>
<td>Area of tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{st \text{(required)}}$</td>
<td>Area of required tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}$</td>
<td>Area of shear reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv \text{,min}}$</td>
<td>Minimum area of shear reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}/s$</td>
<td>Area of shear reinforcement per unit length, mm$^2$/mm</td>
</tr>
<tr>
<td>$A_{sv}/s$</td>
<td>Area of shear reinforcement per unit length consisting of closed ties, mm$^2$/mm</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Area of a polygon with vertices at the center of longitudinal bars at the corners of a section, mm$^2$</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$a_b$</td>
<td>Depth of compression block at balanced condition, mm</td>
</tr>
<tr>
<td>$a_{\text{max}}$</td>
<td>Maximum allowed depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of member, mm</td>
</tr>
<tr>
<td>$b_{cf}$</td>
<td>Effective width of flange (flanged section), mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Concrete cover to compression reinforcement, mm</td>
</tr>
<tr>
<td>$d_o$</td>
<td>Distance from the extreme compression fiber to the centroid of the outermost tension reinforcement, mm</td>
</tr>
<tr>
<td>$d_{om}$</td>
<td>Mean value of $d_o$, averaged around the critical shear perimeter, mm</td>
</tr>
<tr>
<td>$D$</td>
<td>Overall depth of a section, mm</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Thickness of slab (flanged section), mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, MPa</td>
</tr>
<tr>
<td>$f'_{c}$</td>
<td>Specified compressive strength of concrete, MPa</td>
</tr>
<tr>
<td>$f'_{cf}$</td>
<td>Characteristic flexural tensile strength of concrete, MPa</td>
</tr>
</tbody>
</table>
### Table 11-1 List of Symbols Used in the AS 3600-2009 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{cv}$</td>
<td>Concrete shear strength, MPa</td>
</tr>
<tr>
<td>$f_{sy}$</td>
<td>Specified yield strength of flexural reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{sy,f}$</td>
<td>Specified yield strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, MPa</td>
</tr>
<tr>
<td>$J_t$</td>
<td>Torsional modulus, mm$^3$</td>
</tr>
<tr>
<td>$k_u$</td>
<td>Ratio of the depth to the neutral axis from the compression face, to the effective depth, $d$</td>
</tr>
<tr>
<td>$M_{ud}$</td>
<td>Reduced ultimate strength in bending without axial force, N-mm</td>
</tr>
<tr>
<td>$M^*$</td>
<td>Factored moment at section, N-mm</td>
</tr>
<tr>
<td>$N^*$</td>
<td>Factored axial load at section, N</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of shear reinforcement along the beam, mm</td>
</tr>
<tr>
<td>$T_{uc}$</td>
<td>Torsional strength of section without torsional reinforcement, N-mm</td>
</tr>
<tr>
<td>$T_{u,max}$</td>
<td>Maximum permitted total factored torsion at a section, N-mm</td>
</tr>
<tr>
<td>$T_{us}$</td>
<td>Torsion strength of section with torsion reinforcement, N-mm</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Factored torsional moment at a section, N-mm</td>
</tr>
<tr>
<td>$u_t$</td>
<td>Perimeter of the polygon defined by $A_t$, mm</td>
</tr>
<tr>
<td>$V^*$</td>
<td>Factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_{u,max}$</td>
<td>Maximum permitted total factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_{u,min}$</td>
<td>Shear strength provided by minimum shear reinforcement, N</td>
</tr>
<tr>
<td>$V_{ac}$</td>
<td>Shear force resisted by concrete, N</td>
</tr>
<tr>
<td>$V_{us}$</td>
<td>Shear force resisted by reinforcement, N</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Factor for obtaining depth of compression block in concrete</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching critical section</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in concrete</td>
</tr>
<tr>
<td>$\varepsilon_{c, max}$</td>
<td>Maximum usable compression strain allowed in extreme concrete fiber, (0.003 mm/mm)</td>
</tr>
</tbody>
</table>
Table 11-1 List of Symbols Used in the AS 3600-2009 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>Strain in reinforcement</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Strength reduction factor</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>Angle of compression strut for torsion, degrees</td>
</tr>
<tr>
<td>$\theta_v$</td>
<td>Angle of compression strut for shear, degrees</td>
</tr>
</tbody>
</table>

11.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For AS 3600-09, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be defined (AS 2.4.2):

- $1.35D$ (AS/NZS 1170.0-02, 4.2.2(a))
- $1.2D + 1.5L$ (AS/NZS 1170.0-02, 4.2.2(b))
- $1.2D + 1.5(0.75 PL)$ (AS/NZS 1170.0-02, 4.2.2(b))
- $1.2D + 0.4L + 1.0S$ (AS/NZS 1170.0-02, 4.2.2(g))
- $0.9D \pm 1.0W$ (AS/NZS 1170.0-02, 4.2.2(e))
- $1.2D \pm 1.0W$ (AS/NZS 1170.0-02, 4.2.2(d))
- $1.2D + 0.4L \pm 1.0W$ (AS/NZS 1170.0-02, 4.2.2(d))
- $1.0D \pm 1.0E$ (AS/NZS 1170.0-02, 4.2.2(f))
- $1.0D + 0.4L \pm 1.0E$ (AS/NZS 1170.0-02, 4.2.2(f))

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in SAFE whenever the AS 3600-2009 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.
11.3 Limits on Material Strength

The upper and lower limits of $f'_c$ are 100 MPa and 20 MPa, respectively, for all framing types (AS 3.1.1.1(b)).

\[ f'_c \leq 100 \text{ MPa} \quad \text{(AS 3.1.1.1)} \]
\[ f'_c \geq 20 \text{ MPa} \quad \text{(AS 3.1.1.1)} \]

The upper limit of $f_{sy}$ is 500 MPa for all frames (AS 3.2.1, Table 3.2.1).

The code allows use of $f'_c$ and $f_{sy}$ beyond the given limits, provided special care is taken regarding the detailing and ductility (AS 3.1.1, 3.2.1, 17.2.1.1).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

11.4 Strength Reduction Factors

The strength reduction factor, $\phi$, is defined as given in AS 2.2.2(ii), Table 2.2.2:

For members with Class N reinforcement only

\[ \phi = 0.80 \text{ for flexure (tension controlled)} \quad \text{(Table 2.2.2(b))} \]
\[ \phi = 0.60 \text{ for flexure (compression controlled)} \quad \text{(Table 2.2.2(b))} \]

For members with Class L reinforcement

\[ \phi = 0.64 \text{ for flexure (tension controlled)} \quad \text{(Table 2.2.2(b))} \]
\[ \phi = 0.60 \text{ for flexure (compression controlled)} \quad \text{(Table 2.2.2(b))} \]
\[ \phi = 0.70 \text{ for shear and torsion} \quad \text{(Table 2.2.2(b))} \]

These values can be overwritten; however, caution is advised.
11.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, axial, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement for bending and axial loading
- Design shear reinforcement
- Design torsion reinforcement

11.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments and factored axial forces
- Determine required flexural reinforcement for bending and axial forces

11.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments and axial forces for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases the
beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

11.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, \( f_y \), is limited to 500MPa (AS 3.2.1), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 11-1 (AS 8.1.2).

The following assumptions apply to the stress block used to compute the flexural bending capacity of rectangular sections (AS 8.1.2).

- The maximum strain in the extreme compression fiber is taken as 0.003 (AS 8.1.3(a)).
- A uniform compressive stress of \( \alpha_2 f'_c \) acts on an area (AS 8.1.3(b)) bounded by:
  - The edges of the cross-sections.
  - A line parallel to the neutral axis at the strength limit under the loading concerned, and located at a distance \( \gamma_k d \) from the extreme compression fiber.

The maximum allowable depth of the rectangular compression block, \( a_{\text{max}} \), is given by

\[
a_{\text{max}} = \gamma k_u d \quad \text{where,} \tag{AS 8.1.3(b)}
\]

\[
\alpha_2 = 1.0 - 0.003 f'_c \quad \text{where,} \ 0.67 \leq \alpha_2 \leq 0.85 \tag{AS 8.1.3(1)}
\]

\[
\gamma = 1.05 - 0.007 f'_c \quad \text{where,} \ 0.67 \leq \gamma \leq 0.85 \tag{AS 8.1.3(2)}
\]
The design procedure used by SAFE for both rectangular and flanged sections (L- and T-beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed \( A_{sfy} > 0.15N \) (AS 10.7.1a); hence, all beams are designed for major direction flexure, shear, and torsion only.

### 11.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, \( M' \) (i.e., designing top or bottom reinforcement), the depth of the compression block is given by \( a \) (see Figure 11-1), where,

\[
a = d - \sqrt{d^2 - \frac{2|M'|}{\alpha_2 f'_e \varphi b}}
\]

\( \text{(AS 8.1.3)} \)

![Figure 11-1 Rectangular Beam Design](image)

---

11 - 8 Beam Design
where, the value of $\phi$ is taken as that for a tension controlled section $(k_u \leq 0.36)$, which by default is 0.80 (AS 2.2.2) in the preceding and following equations. The selection of Reinforcement Class can be made using the Design Preferences.

- If $a \leq a_{\text{max}}$, the area of tension reinforcement is then given by:

\[
A_{tt} = \frac{M^*}{\phi f_{sy} \left( d - \frac{a}{2} \right)}
\]

This reinforcement is to be placed at the bottom if $M^*$ is positive, or at the top if $M^*$ is negative.

- If $a > a_{\text{max}}$, i.e., $k_u > 0.36$, compression reinforcement is required (AS 8.1.5) and is calculated as follows:

The compressive force developed in the concrete alone is given by:

\[
C = \alpha_2 f' c b a_{\text{max}} \quad \text{(AS 8.1.3)}
\]

and the moment resisted by concrete compression and tension reinforcement is:

\[
M_{uc} = C \left( d - \frac{a_{\text{max}}}{2} \right) \phi
\]

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

\[
M_{us} = M^* - M_{uc}
\]

The required compression reinforcement is given by:

\[
A_{cc} = \frac{M_{us}}{(f'_s - \alpha_2 f' c)(d - d')\phi}, \text{ where}
\]

\[
f'_s = 0.003 E_s \left[ \frac{c - d'}{c} \right] \leq f_{sy} \quad \text{(AS 8.1.2.1, 3.2.2)}
\]
The required tension reinforcement for balancing the compression in the concrete is:

\[ A_{s1} = \frac{M_{uc}}{f_{sy} \left[ d - \frac{a_{max}}{2} \right] \phi} \]

and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A_{s2} = \frac{M_{us}}{f_{sy} (d - d') \phi} \]

Therefore, the total tension reinforcement is \( A_{st} = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A_{sc} \). \( A_{st} \) is to be placed at the bottom and \( A_{sc} \) is to be placed at the top if \( M^* \) is positive, and vice versa if \( M^* \) is negative.

### 11.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 11-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 11-1 is assumed on the compression side (AS 8.1.5).
11.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, \( M^* \) (i.e., designing top reinforcement), the calculation of the reinforcement is exactly the same as above, i.e., no flanged beam data is used.

11.5.1.2.2.2 Flanged Beam Under Positive Moment

If \( M^* > 0 \), the depth of the compression block is given by:

\[
a = d - \sqrt{d^2 - \frac{2M^*}{\alpha_2 f'_c \phi b_f}}
\]

where, the value of \( \phi \) is taken as that for \( k_u \leq 0.36 \), which is 0.80 by default (AS 2.2.2) in the preceding and the following equations.

The maximum allowable depth of the rectangular compression block, \( a_{\text{max}} \), is given by:

\[
a_{\text{max}} = \gamma k_u d \quad \text{where, } k_u = 0.36
\]

- If \( a \leq D_s \), the subsequent calculations for \( A_{st} \) are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( a > a_{\text{max}} \).

- If \( a > D_s \), the calculation for \( A_{st} \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \), as shown in Figure 11-2. \( C_f \) is given by:

\[
C_f = \alpha_2 f'_c \left( b_{ef} - b_w \right) \times \min \left( D_s, a_{\text{max}} \right)
\]

Therefore, \( A_{st} = \frac{C_f}{f_{sy}} \) and the portion of \( M^* \) that is resistsed by the flange is given by:

\[
M_{uf} = \phi C_f \left( d - \frac{\min \left( D_s, a_{\text{max}} \right)}{2} \right)
\]
Therefore, the balance of the moment $M'$ to be carried by the web is:

$$M_{uw} = M' - M_{uf}$$

The web is a rectangular section of dimensions $b_w$ and $d$, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{\alpha_2 f'_c \phi b_w}}$$

- If $a_1 \leq a_{max}$, the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi f_y \left( d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_{st} = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_{max}$, compression reinforcement is required and is calculated as follows:

The compression force in the web concrete alone is given by:

$$C_w = \alpha_2 f'_c b_w a_{max} \quad \text{(AS 8.1.3)}$$

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{uc} = C_w \left( d - \frac{a_{max}}{2} \right) \phi$$

and the moment resisted by compression and tension reinforcement is:

$$M_{ws} = M_{uw} - M_{uc}$$

Therefore, the compression reinforcement is computed as:
\[ A_{sc} = \frac{M_{sc}}{(f_z' - \alpha_z f_c') (d - d') \phi}, \text{ where} \]

\[ f_z' = 0.003 E_s \left[ \frac{c_{\text{max}} - d'}{c_{\text{max}}} \right] \leq f_{sy} \]  

(AS 8.1.2.1, 3.2.2)

The tension reinforcement for balancing compression in the web concrete is:

\[ A_{s2} = \frac{M_{sc}}{f_{sy} \left[ d - \frac{a_{\text{max}}}{2} \right] \phi} \]

and the tension reinforcement for balancing the compression reinforcement is:

\[ A_{s3} = \frac{M_{sc}}{f_{sy} (d - d') \phi} \]

The total tensile reinforcement is \( A_{st} = A_{s1} + A_{s2} + A_{s3} \), and the total compression reinforcement is \( A_{sc} \). \( A_{st} \) is to be placed at the bottom and \( A_{sc} \) is to be placed at the top.

### 11.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following limit (AS 8.1.6.1):

\[ A_{st, \text{min}} = \alpha_t \left( \frac{D}{d} \right)^2 \frac{f_z'}{f_{sy}} b d, \text{ where} \]

\[ \alpha_t = 20, \quad \text{for Rectangular Section} \]  

(AS 8.1.6.1(2))

for L- and T-Sections with the web in tension:

\[ \alpha_t = 0.20 + \left( \frac{b_f}{b_w} - 1 \right) \left( 0.4 \frac{D}{D} - 0.18 \right) \geq 0.20 \left( \frac{b_f}{b_w} \right)^{1/4}, \]  

(AS 8.1.6.1(2))

for L- and T-Sections with the flange in tension.
An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_u \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_{w,d} & \text{Flanged beam} \end{cases}$$

$$A_w \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_{w,d} & \text{Flanged beam} \end{cases}$$

### 11.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, $V^*$.  
- Determine the shear force, $V_{uc}$, that can be resisted by the concrete.  
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

#### 11.5.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.
### 11.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_{uc}$, is calculated as:

$$ V_{uc} = \beta_1 \beta_2 \beta_3 b_d d_o f_{cv}' \left[ \frac{A_{sl}}{b_d d_o} \right]^{1/3} $$  \hspace{1cm} (AS 8.2.7.1)

where,

- $f_{cv}' = (f_{cv}')^{1/3} \leq 4 \text{MPa}$  \hspace{1cm} (AS 8.2.7.1)
- $\beta_1 = 1.1 \left( 1.6 - \frac{d_o}{1000} \right) \geq 1.1$  \hspace{1cm} (AS 8.2.7.1)
- $\beta_2 = 1$, or
  - $\beta_3 = 1$  \hspace{1cm} (AS 8.2.7.1)
  - $\beta_3 = 1$  \hspace{1cm} (AS 8.2.7.1)

$\beta_3$ = 1

#### 11.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to:

$$ V_{u,\text{min}} = V_{uc} + 0.6 b_v d_o $$  \hspace{1cm} (AS 8.2.9)

$$ V_{u,\text{max}} = 0.2 f_{cv}' b d_o $$  \hspace{1cm} (AS 8.2.6)

Given $V^*$, $V_{uc}$, and $V_{u,\text{max}}$, the required shear reinforcement is calculated as follows, where, $\phi$, the strength reduction factor, is 0.75 by default (AS 2.2.2).

- If $V^* \leq \phi V_{uc} / 2$, 

---

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\[ \frac{A_{sv}}{s} = 0, \text{ if } D \leq 750 \text{ mm}; \text{ otherwise } A_{sv,\min} \text{ shall be provided. (AS 8.2.5).} \]

- If \( \left( \frac{\phi V_{uc}}{2} \right) < V^* \leq \phi V_{u,\min}, \)

\[ \frac{A_{sv}}{s} = 0, \text{ if } D < b_w/2 \text{ or } 250 \text{ mm, whichever is greater (AS 8.2.5(c)(i)); otherwise } A_{sv,\min} \text{ shall be provided.} \]

- If \( \phi V_{u,\min} < V^* \leq \phi V_{u,\max}, \)

\[ A_{sv} = \frac{V^* - \phi V_{uc}}{\phi f_{sy,f} d_o \cot \theta_v}, \quad \text{(AS 8.2.10)} \]

and greater than \( A_{sv,\min} \), defined as:

\[ \frac{A_{sv,\min}}{s} = \left( 0.35 \frac{b_w}{f_{sy,f}} \right), \quad \text{(AS 8.2.8)} \]

\( \theta_v = \) the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when \( V^* = \phi V_{u,\min} \) to 45 degrees when \( V^* = \phi V_{u,\max}. \)

- If \( V^* > \phi V_{max}, \) a failure condition is declared. \quad \text{(AS 8.2.6)}

- If \( V^* \) exceeds its maximum permitted value \( \phi V_{max}, \) the concrete section size should be increased (AS 8.2.6).

Note that if torsion design is considered and torsion reinforcement is required, the calculated shear reinforcement is ignored. Closed stirrups are designed for combined shear and torsion according to AS 8.3.4(b).

The maximum of all of the calculated \( A_{sv}/s \) values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup require-
ments to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

11.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, $T^*$.  
- Determine special section properties.  
- Determine critical torsion capacity.  
- Determine the torsion reinforcement required.

11.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design $T^*$ is permitted to be reduced in accordance with the code (AS 8.3.2). However, the program does not automatically redistribute the internal forces and reduce $T^*$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

11.5.3.2 Determine Special Section Properties

For torsion design, special section properties such as $A_t$, $J_t$, and $u_t$ are calculated. These properties are described in the following (AS 8.3).

$$A_t = \text{Area of a polygon with vertices at the center of longitudinal bars at the corners of the cross-section}$$
\[ u_t = \text{Perimeter of the polygon defined by } A_t \]
\[ J_t = \text{Torsional modulus} \]

In calculating the section properties involving reinforcement, such as \( A_{sw}/s \) and \( A_t \), it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm-diameter stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during \( T_{uc} \) calculation. With this assumption, the special properties for a rectangular beam section are given as:

\[ A_t = (b-2c)(h-2c), \quad (AS \ 8.3.5) \]
\[ u_t = 2(b-2c) + 2(h-2c), \quad (AS \ 8.3.6) \]
\[ J_t = 0.33x^2y, \quad (AS \ 8.3.3) \]

where, the section dimensions \( b, h \) and \( c \) are shown in Figure 11-3. Similarly, the special section properties for a flanged beam section are given as:

\[ A_t = (b_w-2c)(h-2c), \quad (AS \ 8.3.5) \]
\[ u_t = 2(h-2c) + 2(b_w-2c), \quad (AS \ 8.3.6) \]
\[ J_t = 0.33\Sigma x^2y, \quad (AS \ 8.3.3) \]

where the section dimensions \( b_w, h, \) and \( c \) for a flanged beam are shown in Figure 11-3. The values \( x \) and \( y \) refer to the smaller and larger dimensions of a component rectangle, respectively.
11.5.3.3 Determine Torsion Reinforcement

The torsional strength of the section without torsion reinforcement, $T_{uc}$, is calculated as:

$$T_{uc} = 0.3 J_t \sqrt{J_t}$$  \hspace{1cm} (AS 8.3.5)

where $J_t$ is the torsion modulus of the concrete cross-section as described in detail in the previous section.

Torsion reinforcement also can be ignored if any one of the following is satisfied:

$$T^* \leq 0.25 \phi T_{uc}$$  \hspace{1cm} (AS 8.3.4(a)(i))

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 0.5$$  \hspace{1cm} (AS 8.3.4(a)(ii))

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1 \text{ and } D \leq \max(250\text{mm}, b/2)$$  \hspace{1cm} (AS 8.3.4(a)(iii))
SAFE Reinforced Concrete Design

If the factored torsion $T^*$ alone or in combination with $V^*$ does not satisfy any of the three conditions in the preceding description, torsion reinforcement is needed. It is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (AS 8.3).

- If $T^* > T_{cr}$, the required closed stirrup area per unit spacing, $A_{sw}/s$, is calculated as:

$$A_{sw} = \frac{T^* \tan \theta_t}{\phi 2 f_{sy,f} A_f}$$

(AS 8.3.5(b))

where, the minimum value of $A_{sw}/s$ is taken as follows:

$$A_{sw,\text{min}} = \frac{0.35 b_w}{f_{sy,f}}$$

(AS 8.2.8)

The value $\theta_t$ is the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $T^* = \phi T_{uc}$ to 45 degrees when $T^* = \phi T_{u,\text{max}}$.

The following equation shall also be satisfied for combined shear and torsion by adding additional shear stirrups.

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} \leq 1.0$$

(AS 8.3.4(b))

where,

$$T_{us} = f_{sy,f} \left( \frac{A_{sw}}{s} \right) 2 A_t \cot \theta_t$$

(AS 8.3.5(b))

$$V_{us} = (A_{sy,f} d_o / s) \cot \theta_v$$

(AS 8.2.10(a))

The required longitudinal rebar area is calculated as:

$$A_l = \frac{0.5 f_{sy,f} \left( \frac{A_{sw}}{s} \right) \mu_t \cot^2 \theta_t}{f_{sy}}$$

(AS 8.3.6(a))
An upper limit of the combination of $V^*$ and $T^*$ that can be carried by the section is also checked using the equation:

$$\frac{T^*}{\phi T^*_{u, max}} + \frac{V^*}{\phi V^*_{u, max}} \leq 1.0$$  \hspace{1cm} (AS 8.3.3)

where,

$$V^*_{u, max} = 0.2 f'_c b_w d_o$$  \hspace{1cm} (AS 8.2.6)

$$T^*_{u, max} = 0.2 f'_c J_t$$  \hspace{1cm} (AS 8.3.3)

For rectangular sections, $b_w$ is replaced with $b$. If the combination of $V^*$ and $T^*$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T^* > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups satisfy the following limit.

$$\left( \frac{A_{vw}}{s} + 2 \frac{A_{sw}}{s} \right) \geq \frac{0.35b}{f_{vy,f}}$$  \hspace{1cm} (AS 8.3.7, 8.2.8)

If this equation is not satisfied with the originally calculated $A_{sv}/s$ and $A_{sw}/s$, $A_{sv}/s$ is increased to satisfy this condition. In that case, $A_{sv}/s$ does not need to satisfy AS Section 8.2.8 independently.

The maximum of all the calculated $A_t$ and $A_{sv}/s$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 11.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of
the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (AS 3600-2001) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

11.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments and axial forces for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the following subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

11.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.
11.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

11.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (AS 9.1.1):

\[
A_t = 0.24 \left( \frac{h}{d} \right)^2 \frac{f_{ct,f}}{f_{sy,f}} b h \text{ for flat slabs} \quad \text{(AS 9.1.1(a))}
\]

\[
A_t = 0.19 \left( \frac{h}{d} \right)^2 \frac{f_{ct,f}}{f_{sy,f}} b h \quad \text{for slabs supported by beams/walls and slab footings.} \quad \text{(AS 9.1.1(b))}
\]

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

11.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following subsections.
11.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d_{om}/2$ from the face of the support (AS 9.2.1.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (AS 9.2.1.3). Figure 11-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 11-4 Punching Shear Perimeters](Image)

11.6.2.2 Determine Concrete Capacity

The shear capacity, $f_{cv}$, is calculated based on the minimum of the two expressions from AS 9.2.3, as shown, with the $d_{om}$ and $u$ terms removed to convert force to stress.
where, $\beta_h$ is the ratio of the longest dimension to the shortest dimension of the critical section.

### 11.6.2.3 Determine Maximum Shear Stress

The maximum design shear stress is computed along the major and minor axis of column separately using the following equation:

\[
\nu_{\text{max}} = \frac{V^*}{ud_{\text{om}}} \left[ 1.0 + \frac{uM_v}{8V^* ad_{\text{om}}} \right] 
\]

(AS 9.2.4(a))

### 11.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

### 11.6.3 Design Punching Shear Reinforcement

The design guidelines for shear links or shear studs are not available in AS 3600-2009. SAFE uses the NZS 3101-06 guidelines to design shear studs or shear links.

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earli-
er sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

11.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

11.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ V_{\text{max}} = 3 \ V_{u,\text{min}} = 3*V_u \]  

(AS 9.2.4(a), (d))

where \( V_u \) is computed from AS 9.2.3 or 9.2.4. Given \( V^* \), \( V_u \), and \( V_{u,\text{max}} \), the required shear reinforcement is calculated as follows, where, \( \phi \) is the strength reduction factor.

\[
\frac{A_s}{s} = \left( \frac{V^* - \phi V_u}{f_y d_{om}} \right) 
\]  

(AS 8.2.10)

Minimum punching shear reinforcement should be provided such that:

\[ V_s \geq \frac{1}{16} \sqrt{f'_{cu} d_{om}} \]  

(NZS 12.7.4.3)

- If \( V^* > \phi V_{\text{max}} \), a failure condition is declared.  
  (NZS 12.7.3.4)

- If \( V^* \) exceeds the maximum permitted value of \( \phi V_{\text{max}} \), the concrete section should be increased in size.

11.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 11-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.
The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \). The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed \( 2d \) measured in a direction parallel to the column face (NZS 12.7.4.4).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

![Diagram showing typical stud arrangement and critical sections](image)

**Figure 11-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone**

### 11.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus one-half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, \( s_{as} \), between the column face and the first peripheral line of shear studs should not be smaller than \( 0.5d \). The spacing between adjacent shear studs, \( g \), at the first peripheral line of studs shall not exceed \( 2d \) and in the case of studs in a radial pattern, the angle between adjacent...
stud rails shall not exceed 60 degrees. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

\[
\begin{align*}
  s_o & \leq 0.5d & \text{(NZS 12.7.4.4)} \\
  s & \leq 0.5d & \text{(NZS 12.7.4.4)} \\
  g & \leq 2d & \text{(NZS 12.7.4.4)}
\end{align*}
\]
Chapter 12
Design for ACI 318-11

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code ACI 318-11 [ACI 2011] is selected. Various notations used in this chapter are listed in Table 12-1. For referencing to the pertinent sections or equations of the ACI code in this chapter, a prefix “ACI” followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

12.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{cp}$</td>
<td>Area enclosed by the outside perimeter of the section, sq-in</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Gross area of concrete, sq-in</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Area of longitudinal reinforcement for torsion, sq-in</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Area enclosed by the shear flow path, sq-in</td>
</tr>
<tr>
<td>$A_{oh}$</td>
<td>Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, sq-in</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_t/s$</td>
<td>Area of closed shear reinforcement per unit length of member for torsion, sq-in/in</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Area of shear reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_v/s$</td>
<td>Area of shear reinforcement per unit length, sq-in/in</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, in</td>
</tr>
<tr>
<td>$a_{max}$</td>
<td>Maximum allowed depth of compression block, in</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of section, in</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Effective width of flange (flanged section), in</td>
</tr>
<tr>
<td>$b_o$</td>
<td>Perimeter of the punching shear critical section, in</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), in</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Width of the punching shear critical section in the direction of bending, in</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Width of the punching shear critical section perpendicular to the direction of bending, in</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, in</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, in</td>
</tr>
<tr>
<td>$d'$</td>
<td>Distance from compression face to compression reinforcement, in</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, psi</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, psi</td>
</tr>
<tr>
<td>$f'c$</td>
<td>Specified compressive strength of concrete, psi</td>
</tr>
</tbody>
</table>
### Table 12-1 List of Symbols Used in the ACI 318-11 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, psi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Specified yield strength of flexural reinforcement, psi</td>
</tr>
<tr>
<td>$f_{yt}$</td>
<td>Specified yield strength of shear reinforcement, psi</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section, in</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Height of the flange, in</td>
</tr>
<tr>
<td>$M_u$</td>
<td>Factored moment at a section, lb-in</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Factored axial load at a section occurring simultaneously with $V_u$ or $T_u$, lb</td>
</tr>
<tr>
<td>$P_u$</td>
<td>Factored axial load at a section, lb</td>
</tr>
<tr>
<td>$p_{cp}$</td>
<td>Outside perimeter of concrete cross-section, in</td>
</tr>
<tr>
<td>$p_h$</td>
<td>Perimeter of centerline of outermost closed transverse torsional reinforcement, in</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of shear reinforcement along the beam, in</td>
</tr>
<tr>
<td>$T_{cr}$</td>
<td>Critical torsion capacity, lb-in</td>
</tr>
<tr>
<td>$T_u$</td>
<td>Factored torsional moment at a section, lb-in</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear force resisted by concrete, lb</td>
</tr>
<tr>
<td>$V_{max}$</td>
<td>Maximum permitted total factored shear force at a section, lb</td>
</tr>
<tr>
<td>$V_e$</td>
<td>Shear force resisted by transverse reinforcement, lb</td>
</tr>
<tr>
<td>$V_u$</td>
<td>Factored shear force at a section, lb</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>Punching shear scale factor based on column location</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching shear critical section</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Factor for obtaining depth of the concrete compression block</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in the concrete</td>
</tr>
<tr>
<td>$\varepsilon_{c_{max}}$</td>
<td>Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)</td>
</tr>
</tbody>
</table>
### Table 12-1 List of Symbols Used in the ACI 318-11 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in the reinforcement</td>
</tr>
<tr>
<td>$\varepsilon_{s,\text{min}}$</td>
<td>Minimum tensile strain allowed in the reinforcement at nominal strength for tension controlled behavior (0.005 in/in)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Strength reduction factor</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Fraction of unbalanced moment transferred by flexure</td>
</tr>
<tr>
<td>$\gamma_v$</td>
<td>Fraction of unbalanced moment transferred by eccentricity of shear</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shear strength reduction factor for light-weight concrete</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of compression diagonals, degrees</td>
</tr>
</tbody>
</table>

### 12.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-11, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (ACI 9.2.1):

1. $1.4D$ \hspace{1cm} (ACI 9-1)
2. $1.2D + 1.6L$ \hspace{1cm} (ACI 9-2)
3. $1.2D + 1.6 (0.75 \text{ PL})$ \hspace{1cm} (ACI 13.7.6.3, 9-2)
4. $0.9D \pm 1.0W$ \hspace{1cm} (ACI 9-6)
5. $1.2D + 1.0L \pm 1.0W$ \hspace{1cm} (ACI 9-4)
6. $0.9D \pm 1.0E$ \hspace{1cm} (ACI 9-7)
7. $1.2D + 1.0L \pm 1.0E$ \hspace{1cm} (ACI 9-5)
8. $1.2D + 1.6L + 0.5S$ \hspace{1cm} (ACI 9-2)
9. $1.2D + 1.0L + 1.6S$ \hspace{1cm} (ACI 9-3)
10. $1.2D + 1.6S \pm 0.5W$ \hspace{1cm} (ACI 9-3)
11. $1.2D + 1.0L + 0.5S \pm 1.0W$ \hspace{1cm} (ACI 9-4)
12. $1.2D + 1.0L + 0.2S \pm 1.0E$ \hspace{1cm} (ACI 9-5)
These are the default design load combinations in SAFE whenever the ACI 318-11 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

12.3 Limits on Material Strength

The concrete compressive strength, $f'_c$, should not be less than 2,500 psi (ACI 5.1.1). The upper limit of the reinforcement yield strength, $f_y$, is taken as 80 ksi (ACI 9.4) and the upper limit of the reinforcement shear strength, $f_{yt}$, is taken as 60 ksi (ACI 11.4.2).

If the input $f'_c$ is less than 2,500 psi, SAFE continues to design the members based on the input $f'_c$ and does not warn the user about the violation of the code. The user is responsible for ensuring that the minimum strength is satisfied.

12.4 Strength Reduction Factors

The strength reduction factors, $\phi$, are applied to the specified strength to obtain the design strength provided by a member. The $\phi$ factors for flexure, shear, and torsion are as follows:

\[
\phi = 0.90 \text{ for flexure (tension controlled)} \quad \text{(ACI 9.3.2.1)}
\]

\[
\phi = 0.75 \text{ for shear and torsion} \quad \text{(ACI 9.3.2.3)}
\]

These values can be overwritten; however, caution is advised.

12.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.
Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

### 12.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

#### 12.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

#### 12.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added
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when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, $f_y$, is limited to 80 ksi (ACI 9.4), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 12-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 10.3.4) when the concrete in compression reaches its assumed strain limit of 0.003. When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed $(0.1f'_cA_g)$ (ACI 10.3.5), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $(0.1f'_cA_g)$ and axial tensions are always included in flexural and shear design.

12.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, $M_u$ (i.e., designing top or bottom reinforcement), the depth of the compression block is given by $a$ (see Figure 12-1), where,

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85f'_c\phi b}}$$  \hspace{1cm} (ACI 10.2)

and the value of $\phi$ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, $c_{\text{max}}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{\text{min}}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):
\[ c_{\text{max}} = \frac{\varepsilon_{\text{cmax}}}{\varepsilon_{\text{cmax}} + \varepsilon_{\text{min}}} \cdot d \]  

\[(\text{ACI 10.2.2)} \]

\[ 0.003 = c_{0.85} f_{c}^\prime \]

\[ 1_{ac} \]

\[ \beta = \frac{c s T_s T_s}{s} \]

\[ c_s c \]

\[ C_s \]

\[ a = \beta c \]

Figure 12-1 Rectangular Beam Design

where,

\[ \varepsilon_{\text{cmax}} = 0.003 \]  

\[(\text{ACI 10.2.3)} \]

\[ \varepsilon_{\text{min}} = 0.005 \]  

\[(\text{ACI 10.3.4)} \]

The maximum allowable depth of the rectangular compression block, \( a_{\text{max}} \), is given by:

\[ a_{\text{max}} = \beta c_{\text{max}} \]  

\[(\text{ACI 10.2.7.1)} \]

where \( \beta \) is calculated as:

\[ \beta = 0.85 - 0.05 \left( \frac{f_{c}^\prime - 4000}{1000} \right), \quad 0.65 \leq \beta \leq 0.85 \]  

\[(\text{ACI 10.2.7.3)} \]
• If \( a \leq a_{\text{max}} \) (ACI 10.3.4), the area of tension reinforcement is then given by:

\[
A_y = \frac{M_u}{\phi f_y \left( d - \frac{a}{2} \right)}
\]

This reinforcement is to be placed at the bottom if \( M_u \) is positive, or at the top if \( M_u \) is negative.

• If \( a > a_{\text{max}} \), compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

  – The compressive force developed in the concrete alone is given by:

\[
C = 0.85 f'_c \, b a_{\text{max}}
\]

  (ACI 10.2.7.1)

  and the moment resisted by concrete compression and tension reinforcement is:

\[
M_{uc} = \phi C \left( d - \frac{a_{\text{max}}}{2} \right)
\]

  (ACI 9.3.2.1)

  – Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

\[
M_{us} = M_u - M_{uc}
\]

  – The required compression reinforcement is given by:

\[
A'_{sc} = \frac{M_{us}}{\phi \left( f'_{ys} - 0.85 f'_c \right) \left( d - d' \right)}, \text{ where}
\]

\[
f'_{ys} = E_y \varepsilon_{c, \text{max}} \left( \frac{c_{\text{max}} - d'}{c_{\text{max}}} \right) \leq f_y
\]

  (ACI 10.2.2, 10.2.3, 10.2.4)

  – The required tension reinforcement for balancing the compression in the concrete is:
\[ A_{s1} = \frac{M_{wc}}{\phi f_y \left[ d - \frac{a_{max}}{2} \right]} \]

and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A_{s2} = \frac{M_{uw}}{\phi f_y (d - d')} \]

Therefore, the total tension reinforcement is \( A_s = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A' \). \( A_s \) is to be placed at the bottom and \( A' \) is to be placed at the top if \( M_0 \) is positive, and vice versa if \( M_0 \) is negative.

### 12.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 12-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 12-1 is assumed on the compression side.

![Figure 12-2 T-Beam Design](image)

**Figure 12-2 T-Beam Design**
12.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M_u$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

12.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85 f'c \phi b_f}}$$  \hfill (ACI 10.2)

where, the value of $\phi$ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, $c_{max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{s_{\text{min}}}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\text{max}} = \frac{\varepsilon_{c_{\text{max}}}}{\varepsilon_{c_{\text{max}}} + \varepsilon_{s_{\text{min}}}}d$$  \hfill (ACI 10.2.2)

where,

$$\varepsilon_{c_{\text{max}}} = 0.003$$  \hfill (ACI 10.2.3)

$$\varepsilon_{s_{\text{min}}} = 0.005$$  \hfill (ACI 10.3.4)

The maximum allowable depth of the rectangular compression block, $a_{max}$, is given by:

$$a_{\text{max}} = \beta_1 c_{\text{max}}$$  \hfill (ACI 10.2.7.1)

where $\beta_1$ is calculated as:

$$\beta_1 = 0.85 - 0.05 \left(\frac{f'c - 4000}{1000}\right), \quad 0.65 \leq \beta_1 \leq 0.85$$  \hfill (ACI 10.2.7.3)
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- If \( a \leq h_f \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as \( b_f \). Compression reinforcement is required if \( a > a_{\text{max}} \).

- If \( a > h_f \), the calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \), as shown in Figure 12-2. \( C_f \) is given by:

\[
C_f = 0.85 f' c \left( b_f - b_w \right) \min(h_f, a_{\text{max}})
\]  

(ACI 10.2.7.1)

Therefore, \( A_{s1} = \frac{C_f}{f_y} \) and the portion of \( M_u \) that is resisted by the flange is given by:

\[
M_{uf} = \phi C_f \left( d - \frac{\min(h_f, a_{\text{max}})}{2} \right)
\]  

(ACI 9.3.2.1)

Again, the value for \( \phi \) is 0.90 by default. Therefore, the balance of the moment, \( M_u \), to be carried by the web is:

\[
M_{uw} = M_u - M_{uf}
\]

The web is a rectangular section with dimensions \( b_w \) and \( d \), for which the design depth of the compression block is recalculated as:

\[
a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f' c \phi b_w}}
\]  

(ACI 10.2)

- If \( a_1 \leq a_{\text{max}} \) (ACI 10.3.4), the area of tension reinforcement is then given by:

\[
A_{s2} = \frac{M_{uw}}{\phi f_y \left( d - \frac{a_1}{2} \right)}, \text{ and}
\]

\[
A_s = A_{s1} + A_{s2}
\]

This reinforcement is to be placed at the bottom of the flanged beam.
If \( a_t > a_{\text{max}} \), compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

- The compressive force in the web concrete alone is given by:

\[
C_w = 0.85 f'_c b_w a_{\text{max}} \tag{ACI 10.2.7.1}
\]

Therefore the moment resisted by the concrete web and tension reinforcement is:

\[
M_{uc} = C_w \left( d - \frac{a_{\text{max}}}{2} \right) \phi
\]

and the moment resisted by compression and tension reinforcement is:

\[
M_{us} = M_{uw} - M_{uc}
\]

Therefore, the compression reinforcement is computed as:

\[
A'_{s1} = \frac{M_{us}}{(f'_s - 0.85 f'_c)(d - d') \phi}, \quad \text{where}
\]

\[
f'_s = E_s \varepsilon_{c_{\text{max}}} \left[ \frac{c_{\text{max}} - d'}{c_{\text{max}}} \right] \leq f_y \tag{ACI 10.2.2, 10.2.3, 10.2.4}
\]

The tension reinforcement for balancing compression in the web concrete is:

\[
A'_{s2} = \frac{M_{uc}}{f_y \left[ d - \frac{a_{\text{max}}}{2} \right] \phi}
\]

and the tension reinforcement for balancing the compression reinforcement is:

\[
A'_{s3} = \frac{M_{us}}{f_y (d - d') \phi}
\]
The total tension reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is $A'$. $A_s$ is to be placed at the bottom and $A'$ is to be placed at the top.

### 12.5.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the two following limits:

$$ A_{s,\text{min}} = \max\left(3\sqrt{f'_{c}} b_d d, \frac{200}{f_y} b_w d\right) \quad \text{(ACI 10.5.1)} $$

$$ A_s \geq \frac{4}{3} A_{s(\text{required})} \quad \text{(ACI 10.5.3)} $$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$ A_s \leq \begin{cases} 0.4b_d & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases} $$

$$ A' \leq \begin{cases} 0.4b_d & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases} $$

### 12.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, $V_u$.
- Determine the shear force, $V_c$, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.
12.5.2.1 Determine Factored Shear Force
In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

12.5.2.2 Determine Concrete Shear Capacity
The shear force carried by the concrete, $V_c$, is calculated as:

$$V_c = 2\lambda \sqrt{f'_c b_w d}$$  \hspace{1cm} \text{(ACI 11.2.1.2)}

A limit is imposed on the value of $\sqrt{f'_c}$ as $f'_c \leq 100$ \hspace{1cm} \text{(ACI 11.1.2)}

The value of $\lambda$ should be specified in the material property definition.

12.5.2.3 Determine Required Shear Reinforcement
The shear force is limited to a maximum of:

$$V_{\text{max}} = V_c + \left(8\sqrt{f'_c}\right) b_w d$$  \hspace{1cm} \text{(ACI 11.4.7.9)}

Given $V_u$, $V_c$, and $V_{\text{max}}$, the required shear reinforcement is calculated as follows where, $\phi$, the strength reduction factor, is 0.75 (ACI 9.3.2.3). The flexural reinforcement strength, $f_{yf}$, is limited to 60 ksi (ACI 11.5.2) even if the material property is defined with a higher value.

- If $V_u \leq 0.5\phi V_c$,

$$\frac{A_v}{s} = 0$$  \hspace{1cm} \text{(ACI 11.4.6.1)}

- If $0.5\phi V_c < V_u \leq \phi V_{\text{max}}$,

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{yf} d}$$  \hspace{1cm} \text{(ACI 11.4.7.1, 11.4.7.2)}

- If $V_u > \phi V_{\text{max}}$, a failure condition is declared.  \hspace{1cm} \text{(ACI 11.4.7.9)}
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If $V_u$ exceeds the maximum permitted value of $\phi V_{\text{max}}$, the concrete section should be increased in size (ACI 11.4.7.9).

Note that if torsion design is considered and torsion reinforcement is required, the equation given in ACI 11.4.6.3 does not need to be satisfied independently. See the subsequent section Design of Beam Torsion Reinforcement for details.

If the beam depth $h$ is

$$h \leq 10'' \text{ for rectangular,}$$

$$h \leq \min \left\{ 24'', \max \left( 2.5h, \frac{b}{2} \right) \right\} \text{ for T-beam,}$$

the minimum shear reinforcement given by ACI 11.4.6.3 is not enforced (ACI 11.4.6.1).

$$\frac{A_v}{s} \geq \max \left( \frac{0.75 \sqrt{f'_{\text{c}}}}{f_{\text{yr}}}, b_w, \frac{50b_w}{f_{\text{yr}}} \right) \quad \text{(ACI 11.4.6.3)}$$

The maximum of all of the calculated $A_v/s$ values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

12.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, $T_u$.
- Determine special section properties.

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- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

12.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 11.6.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design $T_u$ is permitted to be reduced in accordance with the code (ACI 11.5.2.2). However, the program does not automatically redistribute the internal forces and reduce $T_u$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

12.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as $A_{cp}$, $A_{oh}$, $A_o$, $p_{cp}$, and $p_h$, are calculated. These properties are described in the following (ACI 2.1).

\[
\begin{align*}
A_{cp} &= \text{Area enclosed by outside perimeter of concrete cross-section} \\
A_{oh} &= \text{Area enclosed by centerline of the outermost closed transverse torsional reinforcement} \\
A_o &= \text{Gross area enclosed by shear flow path} \\
p_{cp} &= \text{Outside perimeter of concrete cross-section} \\
p_h &= \text{Perimeter of centerline of outermost closed transverse torsional reinforcement}
\end{align*}
\]

In calculating the section properties involving reinforcement, such as $A_{oh}$, $A_o$, and $p_h$, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to a 1.5 inch clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion rein-
forcement calculation. However, the flange is considered during $T_{cr}$ calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{cp} = bh$$  \hspace{1cm} (ACI 11.5.1, 2.1)

$$A_{oh} = (b-2c)(h-2c)$$  \hspace{1cm} (ACI 11.5.3.1, 2.1, R11.5.3.6(b))

$$A_o = 0.85 A_{oh}$$  \hspace{1cm} (ACI 11.5.3.6, 2.1)

$$p_{cp} = 2b + 2h$$  \hspace{1cm} (ACI 11.5.1, 2.1)

$$p_h = 2(b-2c) + 2(h-2c)$$  \hspace{1cm} (ACI 11.5.3.1, 2.1)

where, the section dimensions $b$, $h$, and $c$ are shown in Figure 12-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_{cp} = b_w h + (b_f - b_w) h_f$$  \hspace{1cm} (ACI 11.5.1, 2.1)

$$A_{oh} = (b_w - 2c)(h - 2c)$$  \hspace{1cm} (ACI 11.5.3.1, 2.1, R11.5.3.6(b))

$$A_o = 0.85 A_{oh}$$  \hspace{1cm} (ACI 11.5.3.6, 2.1)

$$p_{cp} = 2b_f + 2h$$  \hspace{1cm} (ACI 11.5.1, 2.1)

$$p_h = 2(h-2c) + 2(b_w-2c)$$  \hspace{1cm} (ACI 11.5.3.1, 2.1)

where the section dimensions $b_f$, $b_w$, $h$, $h_f$, and $c$ for a flanged beam are shown in Figure 12-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h-h_f)$ (ACI 13.2.4).

### 12.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, $T_{cr}$, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \phi 4\lambda \sqrt{f'_c} \left( \frac{A_{cp}}{p_{cp}} \right)^2 \left[ 1 + \frac{P_u}{4A_{cp} f'_c} \right]$$  \hspace{1cm} (ACI 11.5.2.2(c))

where $A_{cp}$ and $p_{cp}$ are the area and perimeter of the concrete cross-section as described in the previous section, $P_u$ is the factored axial force (compression
positive), \( \phi \) is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 9.3.2.3), and \( f'_{c} \) is the specified concrete compressive strength.

### 12.5.3.4 Determine Torsion Reinforcement

If the factored torsion \( T_{u} \) is less than the threshold limit, \( T_{cr} \), torsion can be safely ignored (ACI 11.5.1). In that case, the program reports that no torsion reinforcement is required. However, if \( T_{u} \) exceeds the threshold limit, \( T_{cr} \), it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI R11.5.3.6). Note that the longitudinal reinforcement strength, \( f_{s} \), is limited to 80 ksi (ACI 9.4) and the transverse reinforcement strength, \( f_{yt} \), is limited to 60 ksi, even if the material property is defined with a higher value.

If \( T_{u} > T_{cr} \), the required closed stirrup area per unit spacing, \( A_{t}/s \), is calculated as:

\[
\frac{A_{t}}{s} = \frac{T_{u} \tan \theta}{\phi 2 A_{o} f_{yt}}
\]

and the required longitudinal reinforcement is calculated as:

\[
A_{l} = \frac{T_{u} P_{h}}{\phi 2 A_{o} f_{y} \tan \theta}
\]

where, the minimum value of \( A_{l}/s \) is taken as:

\[
\frac{A_{l}}{s} = \frac{25}{f_{yt}} b_{w}
\]

and the minimum value of \( A_{l} \) is taken as:

\[
A_{l} \geq 5 \sqrt{f'_{c} A_{yp}} - \left( \frac{A_{l}}{s} \right) P_{h} \left( \frac{f_{yt}}{f_{y}} \right)
\]

In the preceding expressions, \( \theta \) is taken as 45 degrees. The code allows any value between 30 and 60 degrees (ACI 11.5.3.6).
Figure 12-3 Closed stirrup and section dimensions for torsion design

An upper limit of the combination of $V_u$ and $T_u$ that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_y}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 8 \sqrt{f_c'}\right)$$

(ACI 11.5.3.1, 11.4.7.9)

For rectangular sections, $b_w$ is replaced with $b$. If the combination of $V_u$ and $T_u$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s} + 2 \frac{A_s}{s}\right) \geq \max \left\{0.75 \sqrt{\frac{f_c'}{f_{ys}} b_w}, \frac{50}{f_y} b_w\right\}$$

(ACI 11.5.5.2)
If this equation is not satisfied with the originally calculated $A_v/s$ and $A_t/s$, $A_v/s$ is increased to satisfy this condition. In that case, $A_v/s$ does not need to satisfy the ACI Section 11.4.6.3 independently.

The maximum of all of the calculated $A_t$ and $A_t/s$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 12.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (ACI 318-11) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

#### 12.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

12.6.1.1 Determine Factored Moments for the Strip
For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

12.6.1.2 Design Flexural Reinforcement for the Strip
The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

12.6.1.3 Minimum and Maximum Slab Reinforcement
The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2, 13.3.1):

\[ A_{s,\text{min}} = 0.0020 \, bh \text{ for } f_y = 40 \text{ ksi or 50 ksi} \quad (\text{ACI 7.12.2.1(a)}) \]

\[ A_{s,\text{min}} = 0.0018 \, bh \text{ for } f_y = 60 \text{ ksi} \quad (\text{ACI 7.12.2.1(b)}) \]

\[ A_{s,\text{min}} = \frac{0.0018 \times 60000}{f_y} \, bh \text{ for } f_y > 60 \text{ ksi} \quad (\text{ACI 7.12.2.1(c)}) \]
In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

### 12.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following sections.

#### 12.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (ACI 11.11.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.11.1.3). Figure 12-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 12-4 Punching Shear Perimeters](image-url)
12.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$.

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}}$$  \hspace{1cm} \text{(ACI 13.5.3.2)}

$$\gamma_v = 1 - \gamma_f$$  \hspace{1cm} \text{(ACI 13.5.3.1)}

For flat plates, $\gamma_v$ is determined from the following equations taken from ACI 421.2R-07 *Seismic Design of Punching Shear Reinforcement in Flat Plates* [ACI 2007].

For interior columns,

$$\gamma_{vx} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x/l_y}}$$  \hspace{1cm} \text{(ACI 421.2 C-11)}

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_y/l_x}}$$  \hspace{1cm} \text{(ACI 421.2 C-12)}

For edge columns,

$$\gamma_{vx} = \text{same as for interior columns}$$  \hspace{1cm} \text{(ACI 421.2 C-13)}

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x/l_y} - 0.2}$$  \hspace{1cm} \text{(ACI 421.2 C-14)}

$\gamma_{vy} = 0$ when $l_x/l_y \leq 0.2$

For corner columns,

$$\gamma_{vx} = 0.4$$  \hspace{1cm} \text{(ACI 421.2 C-15)}

$$\gamma_{vy} = \text{same as for edge columns}$$  \hspace{1cm} \text{(ACI 421.2 C-16)}

**NOTE:** Program uses ACI 421.2-12 and ACI 421.2-15 equations in lieu of ACI 421.2 C-14 and ACI 421.2 C-16 which are currently NOT enforced.
where $b_1$ is the width of the critical section measured in the direction of the span and $b_2$ is the width of the critical section measured in the direction perpendicular to the span. The values $l_x$ and $l_y$ are the projections of the shear-critical section onto its principal axes, $x$ and $y$, respectively.

### 12.6.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

$$
v_c = \min \left( \begin{array}{l}
\left( 2 + \frac{4}{\beta_c} \right) \lambda \sqrt{f'_c} \\
\left( 2 + \frac{\alpha_c d}{b_o} \right) \lambda \sqrt{f'_c} \\
4\lambda \sqrt{f'_c}
\end{array} \right)
$$

(ACI 11.11.2.1)

where, $\beta_c$ is the ratio of the maximum to the minimum dimensions of the critical section, $b_o$ is the perimeter of the critical section, and $\alpha_c$ is a scale factor based on the location of the critical section.

$$
\alpha_c = \begin{cases} 
40 & \text{for interior columns,} \\
30 & \text{for edge columns,} \\
20 & \text{for corner columns.}
\end{cases}
$$

(ACI 11.11.2.1)

A limit is imposed on the value of $\sqrt{f'_c}$ as:

$$
\sqrt{f'_c} \leq 100
$$

(ACI 11.1.2)

### 12.6.2.4 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.
\[ V_U = \frac{V_{U'}}{bod} + \gamma_v [M_U - V_U (y_2 - y_1)] \left[ I_{22} (y_4 - y_3) - I_{23} (x_4 - x_3) \right] - \frac{I_{22} I_{33} - I_{23}^2}{I_{22} I_{33} - I_{23}^2} \gamma_v [M_U - V_U (x_3 - x_1)] \left[ I_{22} (x_4 - x_3) - I_{23} (y_4 - y_3) \right] \]

\[ I_{22} = \sum_{sides=1}^{n} T_{22}, \text{ where "sides" refers to the sides of the critical section for punching shear} \]

\[ I_{33} = \sum_{sides=1}^{n} T_{33}, \text{ where "sides" refers to the sides of the critical section for punching shear} \]

\[ I_{23} = \sum_{sides=1}^{n} T_{23}, \text{ where "sides" refers to the sides of the critical section for punching shear} \]

The equations for \( T_{22}, T_{33}, \) and \( T_{23} \) are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figure 12-5.

\[ T_{22} = Ld (y_2 - y_3)^2, \text{ for the side of the critical section parallel to the 2-axis} \]

\[ T_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld (y_2 - y_3)^2, \text{ for the side of the critical section parallel to the 3-axis} \]

\[ T_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld (x_2 - x_3)^2, \text{ for the side of the critical section parallel to the 2-axis} \]

\[ T_{33} = Ld (x_2 - x_3)^2, \text{ for the side of the critical section parallel} \]
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Figure 12-5 Shear Stress Calculations at Critical Sections

\[ T_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \]

Eq. 7

NOTE: \( T_{23} \) is explicitly set to zero for corner condition.

where,

\[ b_0 = \text{Perimeter of the critical section for punching shear} \]

\[ d = \text{Effective depth at the critical section for punching shear based on the average of } d \text{ for 2 direction and } d \text{ for 3 direction} \]
12.6.2.5 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported.
as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

12.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 11.11.3). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

12.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

\[ v_c \leq 2\lambda \sqrt{f'_c} \]  
for shear links  

\[ v_c \leq 3\lambda \sqrt{f'_c} \]  
for shear studs  

(ACI 11.11.3.1)

(ACI 11.11.5.1)

12.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ V_{\text{max}} = 6 \sqrt{f'_c} b_d \]  
for shear links  

\[ V_{\text{max}} = 8 \sqrt{f'_c} b_d \]  
for shear studs  

(ACI 11.11.3.2)

(ACI 11.11.5.1)

Given \( V_u \), \( V_c \), and \( V_{\text{max}} \), the required shear reinforcement is calculated as follows, where, \( \phi \) the strength reduction factor, is 0.75 (ACI 9.3.2.3).
\[ A_{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} \]  

\[ A_{s} \geq \frac{2f'_{c}}{f_y} b_o \] for shear studs  

(ACI 11.4.7.1, 11.4.7.2)

(ACI 11.11.5.1)

- If \( V_u > \phi V_{\text{max}} \), a failure condition is declared.  

(ACI 11.11.3.2)

- If \( V_u \) exceeds the maximum permitted value of \( \phi V_{\text{max}} \), the concrete section should be increased in size.

12.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 12-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \) (ACI R11.11.3.3, 11.11.5.2). The spacing between adjacent
shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

### 12.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $2d$, and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

$$s_o \leq 0.5d$$

$$s \leq \begin{cases} 
0.75d & \text{for } v_u \leq 6\phi \sqrt{f_c'} \\
0.50d & \text{for } v_u > 6\phi \sqrt{f_c'}
\end{cases}$$

$$g \leq 2d$$

The limits of $s_o$ and the spacing, $s$, between for the links are specified as:

$$s_o \leq 0.5d$$

$$s \leq 0.50d$$
This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code TS 500-2000 [TS 500] is selected. Various notations used in this chapter are listed in Table 13-1. For referencing to the pertinent sections or equations of the TS code in this chapter, a prefix “TS” followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

### 13.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{cp}$</td>
<td>Area enclosed by the outside perimeter of the section</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Gross area of concrete</td>
<td>mm$^2$</td>
</tr>
</tbody>
</table>
### Table 13-1 List of Symbols Used in the TS 500-2000 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>Area of longitudinal reinforcement for torsion, mm$^2$</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Area enclosed by the shear flow path, mm$^2$</td>
</tr>
<tr>
<td>$A_{oh}$</td>
<td>Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{ot}/s$</td>
<td>Area of transverse torsion reinforcement (closed stirrups) per unit length of the member, mm$^2$/mm</td>
</tr>
<tr>
<td>$A_{os}/s$</td>
<td>Area of transverse shear reinforcement per unit length of the member, mm$^2$/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$A_{sw}$</td>
<td>Area of shear reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sw}/s$</td>
<td>Area of shear reinforcement per unit length of the member, mm$^2$/mm</td>
</tr>
<tr>
<td>$a_{max}$</td>
<td>Maximum allowed depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of section, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Effective width of flange (flanged section), mm</td>
</tr>
<tr>
<td>$b_o$</td>
<td>Perimeter of the punching shear critical section, mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), mm</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Width of the punching shear critical section in the direction of bending, mm</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Width of the punching shear critical section perpendicular to the direction of bending, mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Distance from compression face to compression reinforcement, in</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, N/mm$^2$</td>
</tr>
</tbody>
</table>
Table 13-1 List of Symbols Used in the TS 500-2000 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, N/mm²</td>
</tr>
<tr>
<td>$f_{cd}$</td>
<td>Designed compressive strength of concrete, N/mm²</td>
</tr>
<tr>
<td>$f_{ck}$</td>
<td>Characteristic compressive strength of concrete, N/mm²</td>
</tr>
<tr>
<td>$f_{ctk}$</td>
<td>Characteristic tensile strength of concrete, N/mm²</td>
</tr>
<tr>
<td>$f_{yd}$</td>
<td>Designed yield stress of flexural reinforcement, N/mm².</td>
</tr>
<tr>
<td>$f_{yk}$</td>
<td>Characteristic yield stress of flexural reinforcement, N/mm².</td>
</tr>
<tr>
<td>$f_{ywd}$</td>
<td>Designed yield stress of transverse reinforcement, N/mm².</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section, mm</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Height of the flange, mm</td>
</tr>
<tr>
<td>$M_d$</td>
<td>Design moment at a section, N/mm</td>
</tr>
<tr>
<td>$N_d$</td>
<td>Design axial load at a section, N</td>
</tr>
<tr>
<td>$p_{cp}$</td>
<td>Outside perimeter of concrete cross-section, mm</td>
</tr>
<tr>
<td>$p_h$</td>
<td>Perimeter of centerline of outermost closed transverse torsional reinforcement, mm</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of shear reinforcement along the beam, mm</td>
</tr>
<tr>
<td>$T_{cr}$</td>
<td>Critical torsion capacity, N/mm</td>
</tr>
<tr>
<td>$T_d$</td>
<td>Design torsional moment at a section, N/mm</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear force resisted by concrete, N</td>
</tr>
<tr>
<td>$V_{max}$</td>
<td>Maximum permitted total factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_t$</td>
<td>Shear force resisted by transverse reinforcement, N</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Design shear force at a section, N</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Punching shear scale factor based on column location</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching shear critical section</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Factor for obtaining depth of the concrete compression block</td>
</tr>
</tbody>
</table>
### Table 13-1 List of Symbols Used in the TS 500-2000 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in the concrete</td>
</tr>
<tr>
<td>$\varepsilon_{c_{\text{max}}}$</td>
<td>Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 mm / mm)</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in the reinforcement</td>
</tr>
<tr>
<td>$\varepsilon_{cu}$</td>
<td>Maximum usable compression strain allowed in extreme concrete fiber (0.003 mm/mm)</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in reinforcing steel</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>Material factor</td>
</tr>
<tr>
<td>$\gamma_{mc}$</td>
<td>Material factor for concrete</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shear strength reduction factor for lightweight concrete</td>
</tr>
</tbody>
</table>

### 13.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For TS 500-2000, if a structure is subjected to dead (G), live (Q), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (TS 6.2.6):

- $1.4G + 1.6Q$ (TS Eqn. 6.3)
- $0.9G \pm 1.3W$ (TS Eqn. 6.6)
- $1.0G + 1.3Q \pm 1.3W$ (TS Eqn. 6.5)
- $0.9G \pm 1.0E$ (TS Eqn. 6.8)
- $1.0G + 1.0Q \pm 1.0E$ (TS Eqn. 6.7)

These are the default design load combinations in SAFE whenever the TS 500-2000 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.
13.3 Limits on Material Strength

The characteristic compressive strength of concrete, $f_{ck}$, should not be less than 16 N/mm$^2$. The upper limit of the reinforcement yield stress, $f_y$, is taken as 420 N/mm$^2$ and the upper limit of the reinforcement shear strength, $f_{sh}$ is taken as 420 N/mm$^2$.

The program enforces the upper material strength limits for flexure and shear design of beams and columns or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

13.4 Design Strength

The design strength for concrete and steel is obtained by dividing the characteristic strength of the material by a partial factor of safety, $\gamma_{mc}$ and $\gamma_{ms}$. The values used in the program are as follows:

- Partial safety factor for steel, $\gamma_{ms} = 1.15$, and $\gamma_{mc} = 1.5$. (TS 6.2.5)

These factors are already incorporated in the design equations and tables in the code. Although not recommended, the program allows them to be overwritten. If they are overwritten, the program uses them consistently by modifying the code-mandated equations in every relevant place.

13.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.
SAFE Reinforced Concrete Design

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

13.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

13.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

13.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.
The design procedure is based on the simplified rectangular stress block, as shown in Figure 13-1 (TS 7.1). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by the program for both rectangular and flanged sections (T beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed \( 0.1 c_k A_g \) (TS 7.3, Eqn. 7.2); hence, all of the beams are designed ignoring axial force.

### 13.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, \( M_d \) (i.e., designing top or bottom reinforcement), the depth of the compression block is given by \( a \) (see Figure 13-1), where,

\[
a = d - \sqrt{d^2 - \frac{2|M_d|}{0.85f_{cd}b}}, \quad \text{(TS 7.1)}
\]

The maximum depth of the compression zone, \( c_b \), is calculated based on the compressive strength of the concrete and the tensile steel tension using the following equation (TS 7.1):

\[
c_b = \frac{\epsilon_{cu}E_s}{\epsilon_{cu}E_s + f_{yd}}d \quad \text{(TS 7.1)}
\]

The maximum allowable depth of the rectangular compression block, \( a_{\text{max}} \), is given by

\[
a_{\text{max}} = 0.85k_1c_b \quad \text{(TS 7.11, 7.3, Eqn. 7.4)}
\]

where \( k_1 \) is calculated as follows:

\[
k_1 = 0.85 - 0.006(f_{ck} - 25), \quad 0.70 \leq k_1 \leq 0.85. \quad \text{(TS 7.1, Table 7.1)}
\]
If $a \leq a_{\text{max}}$, the area of tension reinforcement is then given by:

$$A_s = \frac{M_d}{f_{yd} \left( d - \frac{a}{2} \right)}$$

This reinforcement is to be placed at the bottom if $M_d$ is positive, or at the top if $M_d$ is negative.

If $a > a_{\text{max}}$, compression reinforcement is required (TS 7.1) and is calculated as follows:

- The compressive force developed in the concrete alone is given by:

$$C = 0.85 f_{cd} b a_{\text{max}}, \quad (\text{TS 7.1})$$

and the moment resisted by concrete compression and tension reinforcement is:
\[ M_{dc} = C \left( d - \frac{a_{\text{max}}}{2} \right). \]

Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

\[ M_{ds} = M_d - M_{dc}. \]

The required compression reinforcement is given by:

\[ A'_s = \frac{M_{ds}}{(\sigma'_s - 0.85 f_{yd})(d - d')}, \]

where

\[ \sigma'_s = E_s e_{cu} \left( \frac{c_{\text{max}} - d'}{c_{\text{max}}} \right) \leq f_{yd}. \] (TS 7.1)

The required tension reinforcement for balancing the compression in the concrete is:

\[ A_s = \frac{M_{ds}}{f_{yd} \left[ d - \frac{a_{\text{max}}}{2} \right]}, \]

and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A'_s = \frac{M_{ds}}{f_{yd} (d - d')} \]

Therefore, the total tension reinforcement is \( A_t = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A' \). \( A_s \) is to be placed at the bottom and \( A'_s \) is to be placed at the top if \( M_d \) is positive, and vice versa if \( M_d \) is negative.

### 13.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 13-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ig-
nored. In that case, a simplified stress block similar to that shown in Figure 13-1 is assumed on the compression side.

In designing for a factored negative moment, \( M_d \) (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

### 13.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, \( M_d \) (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

### 13.5.1.2.2.2 Flanged Beam Under Positive Moment

If \( M_d > 0 \), the depth of the compression block is given by:

\[
\alpha = d - \frac{d^2}{\sqrt{\frac{2M_d}{0.85f_{cu}b_f}}}
\]

The maximum depth of the compression zone, \( c_b \), is calculated based on the compressive strength of the concrete and the tensile steel tension using the following equation (TS 7.1):

\[
c_b = \frac{\sigma_b E_s}{\sigma_{cu} E_s + f_{yd}} d
\]  

(TS 7.1)
The maximum allowable depth of the rectangular compression block, $a_{\text{max}}$, is given by

$$a_{\text{max}} = 0.85k_{1}c_{b}$$

(TS 7.11, 7.3, Eqn. 7.4)

where $k_{1}$ is calculated as follows:

$$k_{1} = 0.85 - 0.006\left(f_{ck} - 25\right), \quad 0.70 \leq k_{1} \leq 0.85.$$  

(TS 7.1, Table 7.1)

- If $a \leq d_{s}$, the subsequent calculations for $A_{s}$ are exactly the same as previously defined for the Rectangular section design. However, in that case, the width of the beam is taken as $b_{f}$, as shown in Figure 13-2. Compression reinforcement is required if $a > a_{\text{max}}$.

- If $a > d_{s}$, the calculation for $A_{s}$ has two parts. The first part is for balancing the compressive force from the flange, $C_{f}$, and the second part is for balancing the compressive force from the web, $C_{w}$, as shown in Figure 13-2. $C_{f}$ is given by:

$$C_{f} = 0.85f_{\text{cd}}\left(b_{f} - b_{w}\right)\times \min\left(d_{s}, a_{\text{max}}\right)$$

(TS 7.1)

Therefore, $A_{s1} = \frac{C_{f}}{f_{\text{yd}}}$ and the portion of $M_{d}$ that is resisted by the flange is given by:

$$M_{df} = C_{f}\left(d - \frac{\min\left(d_{s}, a_{\text{max}}\right)}{2}\right)$$

Therefore, the balance of the moment, $M_{d}$, to be carried by the web is given by:

$$M_{dw} = M_{d} - M_{df}$$

The web is a rectangular section of dimensions $b_{w}$ and $d$, for which the design depth of the compression block is recalculated as:

$$a_{t} = d - \sqrt{d^{2} - \frac{2M_{dw}}{0.85f_{\text{cd}}b_{w}}}$$

(TS 7.1)

- If $a_{t} \leq a_{\text{max}}$ (TS 7.1), the area of tensile steel reinforcement is then given by:
This steel is to be placed at the bottom of the T-beam.

- If \( a_1 > a_{\text{max}} \), compression reinforcement is required and is calculated as follows:

The compression force in the web concrete alone is given by:

\[
C = 0.85 f_{ce} b_y a_{\text{max}} \tag{TS 7.1}
\]

Therefore the moment resisted by the concrete:

\[
M_{dc} = C \left( d - \frac{a_{\text{max}}}{2} \right),
\]

The tensile steel for balancing compression in the web concrete is:

\[
A_{s2} = \frac{M_{dc}}{f_{yd} \left[ d - \frac{a_{\text{max}}}{2} \right]},
\]

The moment resisted by compression steel and tensile steel is:

\[
M_{ds} = M_{dw} - M_{dc}
\]

Therefore, the compression steel is computed as:

\[
A' = \frac{M_{ds}}{\left( \sigma' - 0.85 f_{ce} \right) \left( d - d' \right)}, \text{ where}
\]

\[
\sigma' = E_s \epsilon_{cu} \left[ \frac{c_{\text{max}} - d'}{c_{\text{max}}} \right] \leq f_{yd}, \text{ and} \tag{TS 7.1}
\]

the tensile steel for balancing the compression steel is:
\[
A_{s3} = \frac{M_{ds}}{f_{yd}(d - d')}
\]

The total tensile reinforcement is \(A_s = A_{s1} + A_{s2} + A_{s3}\), and the total compression reinforcement is \(A'_c\). \(A_s\) is to be placed at the bottom and \(A'_c\) is to be placed at the top.

### 13.5.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required in a beam section is given by the following limit:

\[
A_s \geq 0.8 \frac{f_{cat} b_w d}{f_{yd}} \quad \text{(TS 7.3, Eqn. 7.3)}
\]

The maximum flexural tensile steel required in a beam section is given by the following limit:

\[
A_s - A'_c \leq 0.85 \rho_b bd
\]

An upper limit of 0.02 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed as follows:

\[
A_s \leq \begin{cases} 
0.02bd & \text{Rectangular Beam} \\
0.02b_w d & \text{T-Beam}
\end{cases}
\]

\[
A'_c \leq \begin{cases} 
0.02bd & \text{Rectangular Beam} \\
0.02b_w d & \text{T-Beam}
\end{cases}
\]

### 13.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, \(V_d\).
- Determine the shear force, $V_c$, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

**13.5.2.1 Determine Factored Shear Force**

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

**13.5.2.2 Determine Concrete Shear Capacity**

Given the design force set $N_d$ and $V_d$, the shear force carried by the concrete, $V_c$, is calculated as follows:

- If the beam is subjected to axial loading, $N_d$ is positive in this equation regardless of whether it is a compressive or tensile force,

$$V_{cr} = 0.65 f_{cd} b_n d \left( 1 + \frac{\gamma N_d}{A_g} \right),$$  \hspace{1cm} (TS 8.1.3, Eqn. 8.1)

where,

- $0.07$ for axial compression
- $\gamma = -0.3$ for axial tension
- $0$ when tensile stress $< 0.5$ MPa

$$V_c = 0.8 V_{cr},$$  \hspace{1cm} (TS 8.1.4, Eqn. 8.4)

**13.5.2.3 Determine Required Shear Reinforcement**

Given $V_d$ and $V_c$, the required shear reinforcement in the form of stirrups or ties within a spacing, $s$, is given for rectangular and circular columns by the following:

- The shear force is limited to a maximum of
The required shear reinforcement per unit spacing, \( A_v/s \), is calculated as follows:

- If \( V_d \leq V_{cr} \),
  \[
  \frac{A_{sw}}{s} = 0.3 \frac{f_{cd} b_w}{f_{ywd}},
  \]
  (TS 8.1.5, Eqn. 8.6)

- Else if \( V_{cr} < V_d \leq V_{max} \),
  \[
  \frac{A_{sw}}{s} = \frac{(V_d - V_c)}{f_{ywd} d},
  \]
  (TS 8.1.4, Eqn. 8.5)

- Else if \( V_d > V_{max} \),
  a failure condition is declared. (TS 8.1.5b)

If \( V_d \) exceeds its maximum permitted value \( V_{max} \), the concrete section size should be increased (TS 8.1.5b).

Note that if torsion design is performed and torsion rebar is needed, the equation given in TS 8.1.5 does not need to be satisfied independently. See the next section Design of Beam Torsion Reinforcement for details.

The maximum of all of the calculated \( A_v/s \) values, obtained from each design load combination, is reported along with the controlling shear force and associated design load combination name.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.
13.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, \( T_d \).
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

13.5.3.1 Determine Factored Torsion

In the design of torsion reinforcement of any beam, the factored torsions for each design load combination at a particular design station are obtained by factoring the corresponding torsion for different analysis cases with the corresponding design load combination factors (TS 8.2).

In a statically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the design \( T_d \) is permitted to be reduced in accordance with code (TS 8.2.3). However, the program does not try to redistribute the internal forces and to reduce \( T_d \). If redistribution is desired, the user should release the torsional DOF in the structural model.

13.5.3.2 Determine Special Section Properties

For torsion design, special section properties such as \( A_e \), \( S \) and \( u_e \) are calculated. These properties are described as follows (TS 8.2.4).

\[
A_e = \text{Area enclosed by centerline of the outermost closed transverse torsional reinforcement}
\]

\[
S = \text{Shape factor for torsion}
\]

\[
u_e = \text{Perimeter of area} \ A_e
\]
In calculating the section properties involving reinforcement, such as $A_{oc/s}$, $A_{ov/s}$, and $u_e$, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 30 mm. This is equivalent to 25-mm clear cover and a 10-mm-diameter stirrup placement. For torsion design of T beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during $T_{cr}$ calculation. With this assumption, the special properties for a Rectangular beam section are given as follows:

$$A_e = (b - 2c)(h - 2c), \quad (TS\ 8.2.4)$$

$$u_t = 2(b - 2c) + 2(h - 2c), \quad (TS\ 8.2.4)$$

$$S = \frac{x^2y}{3} \quad (TS\ 8.2.4)$$

where, the section dimensions $b$, $h$ and $c$ are shown in Figure 3-9. Similarly, the special section properties for a T beam section are given as follows:
\[ A_e = (b_w - 2c)(h - 2c), \]  
\[ u_t = 2(h - 2c) + 2(b_w - 2c), \]  
\[ S = \Sigma x_{v}/3 \]  

where the section dimensions \( b_w, h \) and \( c \) for a T-beam are shown in Figure 13-3.

### 13.5.3.3 Determine Critical Torsion Capacity

Design for torsion may be ignored if either of the following is satisfied:

(i) The critical torsion limits, \( T_{cr} \), for which the torsion in the section can be ignored, is calculated as follows:

\[ T_d \leq T_{cr} = 0.65 f_{ctd} S \]  

(TS 8.2.3, Eqn 8.12)

In that case, the program reports shear reinforcement based on TS 8.1.5, Eqn. 8.6, i.e.,

\[ \frac{A_{sw}}{s} \geq 0.3 \frac{f_{ctd}}{f_{ywd}} b_w \]  

(TS 8.1.5, Eqn. 8.6)

(ii) When design shear force and torsional moment satisfy the following equation, there is no need to compute torsional stirrups. However, the minimum stirrups and longitudinal reinforcement shown below must be provided:

\[ \left( \frac{V_d}{V_{cr}} \right)^2 + \left( \frac{T_d}{T_{cr}} \right)^2 \leq 1 \]  

(TS 8.2.2, Eqn 8.10)

where \( T_{cr} \) is computed as follows:

\[ T_{cr} = 1.35 f_{ctd} S \]  

(TS 8.2.2, Eqn 8.11)

The required minimum closed stirrup area per unit spacing, \( A_o/s \), is calculated as:

\[ \frac{A_o}{s} = 0.15 \frac{f_{ctd}}{f_{ywd}} \left( 1 + \frac{1.3 T_d}{V_d b_w} \right) b_w \]  

(TS 8.2.4, Eqn. 8.17)
In Eqn. 8.17, \( \frac{T_d}{V_d b_w} \leq 1.0 \) and for the case of statically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, minimum reinforcement will obtained by taking \( T_d \) equal to \( T_{cr} \).

and the required minimum longitudinal rebar area, \( A_{sl} \), is calculated as:

\[
A_{sl} = \frac{T_d u_e}{2 A_e f_{yd}} \quad \text{(TS 8.2.5, Eqn. 8.18)}
\]

### 13.5.3.4 Determine Torsion Reinforcement

If the factored torsion \( T_d \) is less than the threshold limit, \( T_{cr} \), torsion can be safely ignored (TS 8.2.3), when the torsion is not required for equilibrium. In that case, the program reports that no torsion is required. However, if \( T_d \) exceeds the threshold limit, \( T_{cr} \), it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (TS 8.2.4 and 8.2.5).

If \( T_d > T_{cr} \), the required longitudinal rebar area, \( A_{sl} \), is calculated as:

\[
A_{sl} = \frac{T_d u_e}{2 A_e f_{yd}} \quad \text{(TS 8.2.4, Eqn. 8.16)}
\]

and the required closed stirrup area per unit spacing, \( A_{ov}/s \), is calculated as:

\[
\frac{A_{ov}}{s} = \frac{A_{ov}}{s} + \frac{A_{ov}}{s} \quad \text{(TS 8.2.4, Eqn. 8.13)}
\]

\[
\frac{A_{ov}}{s} = \frac{(V_a - V_c)}{d f_{yd}} \quad \text{(TS 8.2.4, Eqn. 8.14)}
\]

\[
\frac{A_{ot}}{s} = \frac{T_d}{2 A_e f_{yd}} \quad \text{(TS 8.2.4, Eqn. 8.15)}
\]

where, the minimum value of \( A_{ov}/s \) is taken as:
\[
\frac{A_w}{s} = 0.15 \frac{f_{cd}}{f_{ys,ld}} \left( 1 + \frac{1.3T_d}{V_d b_w} \right) b_w \quad (TS\ 8.2.4,\ Eqn.\ 8.17)
\]

where, \( \frac{1.3T_d}{V_d b_w} \leq 1.0 \)

An upper limit of the combination of \( V_d \) and \( T_d \) that can be carried by the section also is checked using the following equation.

\[
\frac{T_d}{S} + \frac{V_d}{b_w d} \leq 0.22 f_{cd} \quad (TS\ 8.2.5b,\ Eqn.\ 8.19)
\]

The maximum of all the calculated \( A_d \) and \( A_o/s \) values obtained from each design load combination is reported along with the controlling combination names.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 13.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (TS 500-2000) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

#### 13.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal dis-
placement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

13.6.1.1 Determine Factored Moments for the Strip
For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

13.6.1.2 Design Flexural Reinforcement for the Strip
The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.
13.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (TS 11.4.5):

\[ A_{s,min} = 0.0020 \, bh \text{ for steel grade S220} \]  
\[ A_{s,min} = 0.00175 \, bh \text{ for steel grade S420 and S500} \]  

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

13.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following sections.

13.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of \( d/2 \) from the face of the support (TS 8.3.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (TS 8.3.1). Figure 13-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.
13.6.2.2 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the following limit:

\[ \nu_{pc} = f_{cd} = 0.35 \sqrt{f_{ck}} \gamma_c \]  

(TS 8.3.1)

13.6.2.3 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, \( \nu_{pd} \), is calculated as:

\[ \nu_{pd} = \frac{V_{pd}}{u_p d} \left[ 1 + \eta \left( \frac{0.4M_{pd,1}u_p d}{V_{pd}W_{m,2}} + \frac{0.4M_{pd,3}u_p d}{V_{pd}W_{m,3}} \right) \right], \text{ where} \]  

(TS 8.3.1)

\[ \eta \]  

factor to be used in punching shear check
$$\eta = \frac{1}{1 + \sqrt{b_2 / b_1}} \text{ where } b_2 \geq 0.7b_1$$

When the aspect ratio of loaded area is greater than 3, the critical perimeter is limited assuming $h = 3b$

$c_p$ is the effective perimeter of the critical section

d is the mean effective depth of the slab

$M_{pd}$ is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3

$V_{pd}$ is the total punching shear force

$W_m$ section modulus of area within critical punching perimeter ($c_p$) along bending axis 2 and 3.

### 13.6.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

### 13.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the slab thickness is greater than or equal to 250 mm, a (TS 8.3.2). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

13 - 24 Slab Design
13.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

\[ v_{pr} = f_{crtd} = 0.35 \sqrt{f_{ck}} / \gamma_c \]  

(TS 8.3.1)

13.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ v_{pr,max} = 1.5 \gamma f_{crtd} = 0.525 \gamma \sqrt{f_{ck}} / \gamma_c \] for shear links/shear studs  

(TS 8.3.1)

Given \( V_{pd} \), \( V_{pr} \), and \( V_{pr,max} \), the required shear reinforcement is calculated as follows,

\[ A_s = \frac{V_{pd} - V_{pr}}{f_{yd} d} \]  

(TS8.1.4)

- If \( V_{pd} > V_{pr,max} \), a failure condition is declared.  
  
  (TS 8.3.1)

- If \( V_{pd} \) exceeds the maximum permitted value of \( V_{pr,max} \), the concrete section should be increased in size.

13.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 13-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

NOTE: Shear Stud and shear links requirements are computed based on ACI 318-08 code as Turkish TS 500-2000 refers to special literature on this topic.
Figure 13-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \) (ACI R11.3.3, 11.11.5.2). The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed \( 2d \) measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

13.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, \( s_{xy} \), between the column face and the first peripheral line of shear studs should not be smaller than \( 0.5d \). The spacing between adjacent shear studs, \( g \), at the first peripheral line of studs shall not exceed \( 2d \), and in the case of studs in a radial pattern, the angle between adja-
cent stud rails shall not exceed 60 degrees. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

\[ s_o \leq 0.5d \]  
\[ s \leq \begin{cases} 
0.75d & \text{for } v_a \leq 6\phi \lambda \sqrt{f'_c} \\
0.50d & \text{for } v_a > 6\phi \lambda \sqrt{f'_c} 
\end{cases} \]  
\[ g \leq 2d \]

The limits of $s_o$ and the spacing, $s$, between for the links are specified as:

\[ s_o \leq 0.5d \]  
\[ s \leq 0.50d \]
Chapter 14
Design for Italian NTC 2008

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Italian code NTC2008 [D.M. 14/01/2008] is selected. For the load combinations, reference is also made to NTC2008. Various notations used in this chapter are listed in Table 14-1.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

### 14.1 Notations

<table>
<thead>
<tr>
<th>Table 14-1 List of Symbols Used in the NTC2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
</tr>
<tr>
<td>$A_t$</td>
</tr>
<tr>
<td>$A'_s$</td>
</tr>
</tbody>
</table>
Table 14-1 List of Symbols Used in the NTC2008

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{sl}$</td>
<td>Area of longitudinal reinforcement for torsion, mm$^2$</td>
</tr>
<tr>
<td>$A_{sw}$</td>
<td>Total cross-sectional area of links at the neutral axis, mm$^2$</td>
</tr>
<tr>
<td>$A_{sw}/s$</td>
<td>Area of shear reinforcement per unit length, mm$^2$/mm</td>
</tr>
<tr>
<td>$A_{t}/s$</td>
<td>Area of transverse reinforcement per unit length for torsion, mm$^2$/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width or effective width of the section in the compression zone, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Width or effective width of flange, mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Average web width of a flanged beam, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Effective depth of compression reinforcement, mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement</td>
</tr>
<tr>
<td>$f_{cd}$</td>
<td>Design concrete strength = $\alpha_{cc} f_{ck} / \gamma_c$, MPa</td>
</tr>
<tr>
<td>$f_{ck}$</td>
<td>Characteristic compressive concrete cylinder strength at 28 days, MPa</td>
</tr>
<tr>
<td>$f_{ctm}$</td>
<td>Mean value of concrete axial tensile strength, MPa</td>
</tr>
<tr>
<td>$f_{cw}$</td>
<td>Design concrete compressive strength for shear design = $\alpha_{cc} f_{cwk} / \gamma_c$, MPa</td>
</tr>
<tr>
<td>$f_{cwk}$</td>
<td>Characteristic compressive cylinder strength for shear design, MPa</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Compressive stress in compression reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{yd}$</td>
<td>Design yield strength of reinforcement = $f_{yk} / \gamma_s$, MPa</td>
</tr>
<tr>
<td>$f_{yk}$</td>
<td>Characteristic strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{ywd}$</td>
<td>Design strength of shear reinforcement = $f_{ywk} / \gamma_s$, MPa</td>
</tr>
<tr>
<td>$f_{ywk}$</td>
<td>Characteristic strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of section, mm</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( h_f )</td>
<td>Flange thickness, mm</td>
</tr>
<tr>
<td>( M_{Ed} )</td>
<td>Design moment at a section, N-mm</td>
</tr>
<tr>
<td>( m )</td>
<td>Normalized design moment, ( M/bd^2 \eta_{fc} )</td>
</tr>
<tr>
<td>( m_{\text{lim}} )</td>
<td>Limiting normalized moment capacity as a singly reinforced beam</td>
</tr>
<tr>
<td>( s_v )</td>
<td>Spacing of the shear reinforcement, mm</td>
</tr>
<tr>
<td>( T_{Ed} )</td>
<td>Torsion at ultimate design load, N-mm</td>
</tr>
<tr>
<td>( T_{Rdc} )</td>
<td>Torsional cracking moment, N-mm</td>
</tr>
<tr>
<td>( T_{Rd,\text{max}} )</td>
<td>Design torsional resistance moment, N-mm</td>
</tr>
<tr>
<td>( u )</td>
<td>Perimeter of the punch critical section, mm</td>
</tr>
<tr>
<td>( V_{Rdc} )</td>
<td>Design shear resistance from concrete alone, N</td>
</tr>
<tr>
<td>( V_{Rd,\text{max}} )</td>
<td>Design limiting shear resistance of a cross-section, N</td>
</tr>
<tr>
<td>( V_{Ed} )</td>
<td>Shear force at ultimate design load, N</td>
</tr>
<tr>
<td>( x )</td>
<td>Depth of neutral axis, mm</td>
</tr>
<tr>
<td>( x_{\text{lim}} )</td>
<td>Limiting depth of neutral axis, mm</td>
</tr>
<tr>
<td>( z )</td>
<td>Lever arm, mm</td>
</tr>
<tr>
<td>( \alpha_{cc} )</td>
<td>Coefficient accounting for long-term effects on the concrete compressive strength</td>
</tr>
<tr>
<td>( \alpha_{cw} )</td>
<td>Coefficient accounting for the state of stress in the compression chord</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Redistribution factor</td>
</tr>
<tr>
<td>( \varepsilon_c )</td>
<td>Concrete strain</td>
</tr>
<tr>
<td>( \varepsilon_s )</td>
<td>Strain in tension reinforcement</td>
</tr>
<tr>
<td>( \varepsilon'_s )</td>
<td>Strain in compression steel</td>
</tr>
<tr>
<td>( \gamma_c )</td>
<td>Partial safety factor for concrete strength</td>
</tr>
<tr>
<td>( \gamma_s )</td>
<td>Partial safety factor for reinforcement strength</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Factor defining the effective depth of the compression zone</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Effectiveness factor for shear resistance without concrete crush-</td>
</tr>
</tbody>
</table>
Table 14-1 List of Symbols Used in the NTC2008

- \( \eta \) Concrete strength reduction factor for sustained loading and stress block
- \( \rho_t \) Tension reinforcement ratio
- \( \sigma_{cp} \) Axial stress in the concrete, MPa
- \( \theta \) Angle of the concrete compression strut
- \( \omega \) Normalized tension reinforcement ratio
- \( \omega' \) Normalized compression reinforcement ratio
- \( \omega_{lim} \) Normalized limiting tension reinforcement ratio

14.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be checked. NTC2008 allows load combinations to be defined based on NTC2008 Equation 2.5.1.

\[
\sum_{j=1}^{\gamma_{G1,j}} G_{1,k,j} + \sum_{l=1}^{\gamma_{G2,k,j}} G_{2,k,j} + P + \gamma_{Q1,k,j} + \sum_{i=1}^{\gamma_{Q,i,j}} \psi_{Q,i,j} Q_{k,j} \quad \text{(Eq. 2.5.1)}
\]

Load combinations considering seismic loading are automatically generated based on NTC2008 Equation 2.5.5.

\[
\sum_{j=1}^{\gamma_{G1,k,j}} G_{1,k,j} + \sum_{l=1}^{\gamma_{G2,k,j}} G_{2,k,j} + P + E + \sum_{r=1}^{\gamma_{Q,r,j}} \psi_{Q,r,j} Q_{k,j} \quad \text{(Eq. 2.5.5)}
\]

For both sets of load combinations, the variable values are defined in the list that follows.

- \( \gamma_{G1,\text{sup}} = 1.30 \) (NTC2008 Table 2.6.1)
- \( \gamma_{G1,\text{inf}} = 1.00 \) (NTC2008 Table 2.6.1)
- \( \gamma_{G2,\text{sup}} = 1.50 \) (NTC2008 Table 2.6.1)
- \( \gamma_{G2,\text{inf}} = 0.00 \) (NTC2008 Table 2.6.1)
\[ \gamma_{Q,1,\text{sup}} = 1.5 \quad \text{(NTC2008 Table 2.6.I)} \]
\[ \gamma_{Q,1,\text{inf}} = 0.0 \quad \text{(NTC2008 Table 2.6.I)} \]
\[ \gamma_{Q,I,\text{sup}} = 1.5 \quad \text{(NTC2008 Table 2.6.I)} \]
\[ \gamma_{Q,I,\text{inf}} = 0.0 \quad \text{(NTC2008 Table 2.6.I)} \]

\[ \psi_{0,i} = 0.7 \text{ (live load, assumed not to be storage) (Table 2.5.I)} \]
\[ \psi_{0,i} = 0.6 \text{ (wind load) (Table 2.5.I)} \]
\[ \psi_{0,i} = 0.5 \text{ (snow load, assumed } H \leq 1,000 \text{ m) (Table 2.5.I)} \]
\[ \psi_{2,i} = 0.3 \text{ (live, assumed office/residential space) (Table 2.5.I)} \]
\[ \psi_{2,i} = 0.0 \text{ (snow, assumed } H \leq 1,000 \text{ m) (Table 2.5.I)} \]

If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

### 14.3 Limits on Material Strength

The concrete compressive strength, \( f_{ck} \), should not be greater than 90 MPa (NTC2008 Tab. 4.1.I). The reinforcement material should be B450C or B450A (NTC2008 §11.3.2).

<table>
<thead>
<tr>
<th>NTC Table 11.3.Ia:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{y,\text{nom}} ) &amp; 450 N/mm²</td>
</tr>
<tr>
<td>( f_{t,\text{nom}} ) &amp; 540 N/mm²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NTC Table 11.3.Ib: Material TYPE B450C Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>Characteristic yield stress, ( f_{yk} )</td>
</tr>
<tr>
<td>Characteristic rupture stress, ( f_{tk} )</td>
</tr>
</tbody>
</table>
(f_y/ f_y,nom)_k ≥ 1.15
< 1.35
10.0

Elongation at rupture
(f_y/ f_y,nom)_k
(A_g)_k
< 1.25
≥ 7.5 %
10.0

NTC Table 11.3.1c: Material TYPE B450A Properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Prerequisite</th>
<th>Fracture %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic yield stress, f_yk</td>
<td>≥ f_y,nom</td>
<td>5.0</td>
</tr>
<tr>
<td>Characteristic rupture stress, f_uk</td>
<td>≥ f_y,nom</td>
<td>5.0</td>
</tr>
<tr>
<td>(f_y/ f_y,nom)_k</td>
<td>≥ 1.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt; 1.25</td>
<td>10.0</td>
</tr>
<tr>
<td>Elongation at rupture</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f_y/ f_y,nom)_k</td>
<td>&lt; 1.25</td>
<td>10.0</td>
</tr>
<tr>
<td>(A_g)_k</td>
<td>≥ 2.5 %</td>
<td>10.0</td>
</tr>
</tbody>
</table>

14.4 Partial Safety Factors

The design strengths for concrete and steel are obtained by dividing the characteristic strengths of the materials by the partial safety factors, γ_s and γ_c as shown here.

\[ f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \]  
(NTC Eq. 4.1.4)

\[ f_{yd} = \frac{f_{yk}}{\gamma_s} \]  
(NTC Eq. 4.1.6)

\[ f_{ywd} = \frac{f_{ywk}}{\gamma_s} \]  
(NTC Eq. 4.1.6)

\( \alpha_{cc} \) is the coefficient taking account of long term effects on the compressive strength. \( \alpha_{cc} \) is taken as 0.85 (NTC2008 4.1.2.1.1.1) by default and can be overwritten by the user.

14 - 6 Partial Safety Factors
The partial safety factors for the materials and the design strengths of concrete and reinforcement are given in the text that follows (NTC2008 4.1.2.1.1.1-3):

Partial safety factor for reinforcement, $\gamma_s = 1.15$

Partial safety factor for concrete, $\gamma_c = 1.5$

These values can be overwritten; however, caution is advised.

14.5 Beam Design

In the design of concrete beams, the program calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, torsions, design load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at a user-defined number of output stations along the beam span.

All beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

14.5.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at output stations along the beam span. The following steps are involved in designing the flexural reinforcement for the major moment for a particular beam, at a particular section:

Determine the maximum factored moments

- Determine the required reinforcing steel
14.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each design load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding design load combination factors.

The beam section is then designed for the factored moments obtained from each of the design load combinations. Positive moments produce bottom steel. In such cases, the beam may be designed as a rectangular or a T-beam section. Negative moments produce top steel. In such cases, the beam is always designed as a rectangular section.

14.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user can avoid the need for compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on a simplified rectangular stress block, as shown in Figure 14-7 (NTC Fig. 4.1.3). When the applied moment exceeds the moment capacity, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by the program for both rectangular and flanged sections (T-beams) is summarized in the following subsections. It is assumed that the design ultimate axial force is negligible, hence all beams are designed ignoring axial force.
In designing for a factored negative or positive moment, $M_{Ed}$ (i.e., designing top or bottom steel), the effective strength and depth of the compression block are given by $\alpha_{cc} f_{cd}$ and $\beta x$ (see Figure 14-7) respectively, where:

$\beta = 0.8 \quad$ (NTC § 4.1.2.1.2.2)

$\alpha_{cc} = 0.85 \quad$ (NTC § 4.1.2.1.2.2)

$f_{cd} = \alpha_{cc} \frac{f_{ck}}{\gamma_c}$ if $f_{ck} \leq 50N/mm^2$

$\alpha_{cc} = 0.85$

$\gamma_c = 1.5$

if $f_{ck} > 50N/mm^2$ NTC 2008 refer to Eurocode 2:

$f_{cd} = \alpha_{cc} \eta \frac{f_{ck}}{\gamma_c}$

$\eta = 1.0 - (f_{ck} - 50)/200$ for $50 < f_{ck} \leq 90$ MPa \hspace{1cm} (EC2 Eq. 3.22)
For the design of the beams a ductility criterion, suggested in Eurocode 2 § 5.5, is followed.

The limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, \((x/d)_{\text{lim}}\), is expressed as a function of the ratio of the redistributed moment to the moment before redistribution, \(\delta\), as follows:

\[
\frac{x}{d}_{\text{lim}} = \left(\delta - k_1\right)/k_2 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \quad (\text{EC2 Eq. 5.10a})
\]

\[
\frac{x}{d}_{\text{lim}} = \left(\delta - k_3\right)/k_4 \quad \text{for } f_{ck} > 50 \text{ MPa} \quad (\text{EC2 Eq. 5.10b})
\]

No redistribution is assumed, such that \(\delta\) is assumed to be 1. The four factors, \(k_1, k_2, k_3, \text{ and } k_4\), are defined as:

\[
k_1 = 0.44 \quad (\text{EC2 5.5(4)})
\]

\[
k_2 = 1.25 \left(0.6 + 0.0014/\varepsilon_{cu}\right) \quad (\text{EC2 5.5(4)})
\]

\[
k_3 = 0.54 \quad (\text{EC2 5.5(4)})
\]

\[
k_4 = 1.25 \left(0.6 + 0.0014/\varepsilon_{cu}\right) \quad (\text{EC2 5.5(4)})
\]

where the ultimate strain, \(\varepsilon_{cu2}\), is determined from EC2 Table 3.1 as:

\[
\varepsilon_{cu2} = 0.0035 \text{ for } f_{ck} < 50 \text{ MPa} \quad (\text{NTC § 4.1.2.1.2.2})
\]

\[
\varepsilon_{cu2} = 2.6 + 35 \left[\left(90 - f_{ck}\right)/100\right]^4 \text{ for } f_{ck} \geq 50 \text{ MPa} \quad (\text{NTC § 4.1.2.1.2.2})
\]

### 14.5.1.2.1 Rectangular Beam Flexural Reinforcement

For rectangular beams, the normalized moment, \(m\), and the normalized section capacity as a singly reinforced beam, \(m_{\text{lim}}\), are determined as:

\[
m = \frac{M}{bd^2 f_{cd}}
\]

\[
m_{\text{lim}} = \beta \left(\frac{x}{d}\right)_{\text{lim}} \left[1 - \frac{\beta}{2} \left(\frac{x}{d}\right)_{\text{lim}}\right]
\]
The reinforcing steel area is determined based on whether $m$ is greater than, less than, or equal to $m_{\text{lim}}$.

- If $m \leq m_{\text{lim}}$, a singly reinforced beam will be adequate. Calculate the normalized steel ratio, $\omega$, and the required area of tension reinforcement, $A_s$, as:

$$\omega = 1 - \sqrt{1 - 2m}$$

$$A_s = \omega \frac{f_{cd} b d}{f_{yd}}$$

This area of reinforcing steel is to be placed at the bottom if $M_{Ed}$ is positive, or at the top if $M_{Ed}$ is negative.

- If $m > m_{\text{lim}}$, compression reinforcement is required. Calculate the normalized steel ratios, $\omega'$, $\omega_{\text{lim}}$, and $\omega$, as:

$$\omega_{\text{lim}} = \beta \left( \frac{x}{d} \right)_{\text{lim}} = 1 - \sqrt{1 - 2m_{\text{lim}}}$$

$$\omega' = \frac{m - m_{\text{lim}}}{1 - d'/d}$$

$$\omega = \omega_{\text{lim}} + \omega'$$

where $d'$ is the depth to the compression steel, measured from the concrete compression face.

Calculate the required area of compression and tension reinforcement, $A_s'$ and $A_s$, as:

$$A_s' = \omega' \frac{f_{cd} b d}{f'_{s}}$$

$$A_s = \omega \frac{f_{cd} b d}{f_{yd}}$$

where $f'_{s}$, the stress in the compression steel, is calculated as:
\[ f' = E_s \varepsilon_c \left( 1 - \frac{d'}{x_{lim}} \right) \leq f_{yd} \]

\( A_s \) is to be placed at the bottom and \( A'_s \) is to be placed at the top if \( M_{Ed} \) is positive, and \( A'_s \) is to be placed at the bottom and \( A_s \) is to be placed at the top if \( M_{Ed} \) is negative.

### 14.5.1.2.2 T-Beam Flexural Reinforcement

In designing a T-beam, a simplified stress block, as shown in Figure 14-8, is assumed if the flange is in compression, i.e., if the moment is positive. If the moment is negative, the flange is in tension, and therefore ignored. In that case, a simplified stress block, similar to that shown in Figure 14-8, is assumed on the compression side.

#### 14.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, \( M_{Ed} \) (i.e., designing top steel), the calculation of the reinforcing steel area is exactly the same as described for a rectangular beam, i.e., no specific T-beam data is used.

#### 14.5.1.2.2.2 Flanged Beam Under Positive Moment

In designing for a factored positive moment, \( M_{Ed} \), the program analyzes the section by considering the depth of the stress block. If the depth of the stress block is less than or equal to the flange thickness, the section is designed as a rectangular beam with a width \( b_f \). If the stress block extends into the web, additional calculation is required.

For T-beams, the normalized moment, \( m \), and the normalized section capacity as a singly reinforced beam, \( m_{lim} \), are calculated as:

\[ m = \frac{M}{b_f d^2 f_{yd}} \]

\[ m_{lim} = \beta \left( \frac{x}{d} \right)_{lim} \left[ 1 - \frac{\beta}{2} \left( \frac{x}{d} \right)_{lim} \right] \]
Calculate the normalized steel ratios $\omega_{\text{lim}}$ and $\omega$, as:

$$\omega_{\text{lim}} = \beta \left( \frac{x}{d} \right)_{\text{lim}}$$

$$\omega = 1 - \sqrt{1 - 2m}$$

Calculate the maximum depth of the concrete compression block, $x_{\text{max}}$, and the effective depth of the compression block, $x$, as:

$$x_{\text{max}} = \omega_{\text{lim}} d$$

$$x = \omega d$$

The reinforcing steel area is determined based on whether $m$ is greater than, less than, or equal to $m_{\text{lim}}$.

- If $x \leq h_f$, the subsequent calculations for $A_s$ are exactly the same as previously defined for rectangular beam design. However, in this case, the width of the beam is taken as $b_f$, as shown in Figure 14-8. Compression reinforcement is required if $m > m_{\text{lim}}$.

- If $x > h_f$, the calculation for $A_s$ has two parts. The first part is for balancing the compressive force from the flange, and the second part is for balancing the compressive force from the web, as shown in Figure 14-8.

- The required reinforcing steel area, $A_{s2}$, and corresponding resistive moment, $M_2$, for equilibrating compression in the flange outstands are calculated as:

$$A_{s2} = \frac{(b_f - h_s)h_f f_{yd}}{f_{yd}}$$

$$M_2 = A_{s2} f_{yd} \left( d - \frac{h_f}{2} \right)$$
Now calculate the required reinforcing steel area $A_{s1}$ for the rectangular section of width $b_w$ to resist the remaining moment $M_1 = M_{Ed} - M_2$. The normalized moment, $m_1$, is calculated as:

$$m_1 = \frac{M_1}{b_w d^2 f_{cd}}$$

The reinforcing steel area is determined based on whether $m_1$ is greater than, less than, or equal to $m_{lim}$.

- If $m_1 \leq m_{lim}$, a singly reinforced beam will be adequate. Calculate the normalized steel ratio, $\omega_1$, and the required area of tension reinforcement, $A_{s1}$, as:

$$\omega_1 = 1 - \sqrt{1-2m}$$

$$A_{s1} = \omega_1 \left[ \frac{f_{yd}bd}{f_{yd}} \right]$$

- If $m_1 > m_{lim}$, compression reinforcement is required. Calculate the normalized steel ratios, $\omega'$, $\omega_{lim}$, and $\omega$, as:
\[ \omega_{\text{lim}} = \beta \left( \frac{x}{d} \right)_{\text{lim}} \]

\[ \omega' = \frac{m - m_{\text{lim}}}{1 - d'/d} \]

\[ \omega_1 = \omega_{\text{lim}} + \omega' \]

where \( d' \) is the depth to the compression steel, measured from the concrete compression face.

Calculate the required area of compression and tension reinforcement, \( A_{s'} \) and \( A_s \), as:

\[ A_{s'} = \omega' \left[ \frac{f_{cd}bd}{f'_{s}} \right] \]

\[ A_{s1} = \omega_1 \left[ \frac{f_{cd}bd}{f'_{yd}} \right] \]

where \( f'_{s} \), the stress in the compression steel, is calculated as:

\[ f'_{s} = E_s \varepsilon_c \left[ 1 - \frac{d'}{x_{\text{lim}}} \right] \leq f_{yd} \]

The total tensile reinforcement is \( A_s = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A'_{s} \). \( A_s \) is to be placed at the bottom and \( A'_{s} \) is to be placed at the top of the section.

### 14.5.1.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel reinforcement, \( A_{s,\text{min}} \), required in a beam section is given as the maximum of the following two values:

\[ A_{s,\text{min}} = 0.26 \left( \frac{f_{cd}}{f_{yk}} \right) b_t d \quad \text{(NTC Eq. 4.1.43)} \]

\[ A_{s,\text{min}} = 0.0013 b_t d \quad \text{(NTC Eq. 4.1.43)} \]
where $b_t$ is the mean width of the tension zone, equal to the web width for T-beams, and $f_{c_{tm}}$ is the mean value of axial tensile strength of the concrete, calculated as:

$$f_{c_{tm}} = 0.30 f_{ck}^{(2/3)} \quad \text{for } f_{ck} \leq 50 \text{ MPa} \quad \text{(NTC Eq. 11.2.3a)}$$

$$f_{c_{tm}} = 2.12 \ln \left(1 + \frac{f_{cm}}{10}\right) \quad \text{for } f_{ck} > 50 \text{ MPa} \quad \text{(NTC Eq. 11.2.3b)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \quad \text{(NTC Eq. 11.2.2)}$$

The maximum flexural steel reinforcement, $A_{s_{max}}$, permitted as either tension or compression reinforcement is defined as:

$$A_{s_{max}} = 0.04 A_c \quad \text{(NTC § 4.1.6.1.1)}$$

where $A_c$ is the gross cross-sectional area.

### 14.5.2 Design Beam Shear Reinforcement

The required beam shear reinforcement is calculated for each design load combination at each output station along the beam length. The following assumptions are made for the design of beam shear reinforcement:

- The beam section is assumed to be prismatic. The shear capacity is based on the beam width at the output station and therefore a variation in the width of the beam is neglected in the calculation of the concrete shear capacity at each particular output station.

- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement. Inclined shear steel is not handled.

The following steps are involved in designing the shear reinforcing for a particular beam, for a particular design load combination resulting from shear forces in a particular direction:

- Determine the design forces acting on the section, $N_{Ed}$ and $V_{Ed}$. Note that $N_{Ed}$ is needed for the calculation of $V_{Red}$.

- Determine the maximum design shear force that can be carried without crushing of the notional concrete compressive struts, $V_{Red}$.
Determine the required shear reinforcement as area per unit length, \( A_{sw}/s \).

The following three sections describe in detail the algorithms associated with this process.

### 14.5.2.1 Determine Design Shear Force

In the design of the beam shear reinforcement, the shear forces and moments for a particular design load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding design load combination factors.

### 14.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, \( V_{rd,c} \), is calculated as:

\[
V_{rd,c} = \left[ C_{rd,c} k \left( 100 \rho_t f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] b_w d
\]

where \( f_{ck} \) is in MPa.

\[
k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm}
\]

\[
\rho_t = \text{tension reinforcement ratio} = \frac{A_{s1}}{b_w d} \leq 0.02
\]

\[
A_{s1} = \text{area of tension reinforcement}
\]

\[
\sigma_{cp} = N_{Ed}/A_c < 0.2 f_{cd} \text{ MPa}
\]

The value of \( C_{rd,c} \), \( v_{min} \) and \( k_1 \) for use in a country may be found in its National Annex. The program default values for \( C_{rd,c} \), \( v_{min} \) and \( k_1 \) are given as follows (EC2 6.2.2(1)):
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\[ CR_{d,c} = 0.18/\gamma_c \]  
(C2 6.2.2(1))

\[ v_{\text{min}} = 0.035 \ k^{3/2} \ f_{ck}^{1/2} \]  
(C2 6.2.2(1))

\[ k_1 = 0.15. \]  
(C2 6.2.2(1))

For light-weight concrete:

\[ CR_{d,c} = 0.18/\gamma_c \]  
(C2 11.6.1(1))

\[ v_{\text{min}} = 0.03 \ k^{3/2} \ f_{ck}^{1/2} \]  
(C2 11.6.1(1))

\[ k_1 = 0.15. \]  
(C2 11.6.1(1))

14.5.2.3 Determine Maximum Design Shear Force

To prevent crushing of the concrete compression struts, the design shear force \( V_{Ed} \) is limited by the maximum sustainable design shear force, \( V_{Rcd} \). If the design shear force exceeds this limit, a failure condition occurs. The maximum sustainable shear force is defined as:

\[
V_{Rcd} = 0.9 \cdot d \cdot b_w \cdot \alpha_c \cdot f'_{cd} \cdot \frac{\cot \alpha + \cot \theta}{1 + \cot^2 \theta} \\
\text{(NTC Eq. 4.1.19)}
\]

\[ \alpha_c = 1 \] for members not subjected to axial compression

\[
= 1 + \frac{\sigma_{cp}}{f_{cd}} \text{ for } 0 \leq \sigma_{cp} \leq 0.25 f_{cd}
\]

\[
= 1.25 \text{ for } 0.25 f_{cd} \leq \sigma_{cp} \leq 0.5 f_{cd}
\]

\[
= 2.5 \left( 1 + \frac{\sigma_{cp}}{f_{cd}} \right) \text{ for } 0.5 f_{cd} \leq \sigma_{cp} \leq f_{cd}
\]

\[ f'_{cd} = 0.5 f_{cd} \]

\[ \alpha \] angle between the shear reinforcement and the column axis. In the case of vertical stirrups \( \alpha = 90 \) degrees

\[ \theta \] angle between the concrete compression struts and the column axis. NTC 2008 allows \( \theta \) to be taken between 21.8 and 45 degrees.

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If torsion is significant i.e., $T_{Ed} > T_{cr}$ where $T_{cr}$ is defined as:

$$T_{cr} = T_{Rd,c} \left(1 - \frac{V_{Ed}}{V_{Rd,c}}\right)$$  \hspace{1cm} \text{(EC2 Eq. 6.31)}

and if the load combination include seismic, the value of $\theta$ is taken as $45^\circ$. However, for other cases $\theta$ is optimized using the following relationship:

$$(\cot \theta + \tan \theta) = 0.9 \alpha_{cw} v_{f,cd} / V_{Ed}$$

where

$$21.8^\circ \leq \theta \leq 45^\circ$$

14.5.2.4 Determine Required Shear Reinforcement

If $V_{Ed}$ is less than $V_{Rcd}$, the required shear reinforcement in the form of stirrups or ties per unit spacing, $A_{sw} / s$, is calculated as:

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.9d \cdot f_{yed}} \cdot \frac{1}{(\cot \alpha + \cot \theta) \sin \alpha}$$  \hspace{1cm} \text{(NTC Eq. 4.1.18)}

with $\alpha = 90^\circ$ degrees and $\theta$ given in the previous section.

The maximum of all of the calculated $A_{sw} / s$ values, obtained from each design load combination, is reported for the major and minor directions of the beam, along with the controlling combination name.

The calculated shear reinforcement must be greater than the minimum reinforcement:

$$A_{sw, \text{min}} = 1.5 \cdot b$$

with $b$ in millimeters and $A_{sw, \text{min}}$ in mm²/mm.

The beam shear reinforcement requirements reported by the program are based purely on shear strength consideration. Any minimum stirrup requirements to satisfy spacing considerations or transverse reinforcement volumetric considerations must be investigated independently by the user.
14.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at a user-defined number of output stations along the beam span. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to beam torsion:

- Determine the factored torsion, $T_{Ed}$.
- Determine torsion section properties.
- Determine the reinforcement steel required.

14.5.3.1 Determine Factored Torsion

In the design of torsion reinforcement of any beam, the factored torsions for each design load combination at a particular design station are obtained by factoring the corresponding torsion for different load cases with the corresponding design load combination factors.

14.5.3.2 Determine Torsion Section Properties

For torsion design, special torsion section properties, including $A$, $t$, $u$, and $u_m$ are calculated. These properties are described as follows (NTC § 4.1.2.1.4).

\[ A = \text{area enclosed by centerlines of the connecting walls, where the centerline is located a distance of } t/2 \text{ from the outer surface} \]

\[ t = A_c / u = \text{effective wall thickness} \]

\[ A_c = \text{area of the section} \]

\[ u = \text{outer perimeter of the cross-section} \]

\[ u_m = \text{perimeter of the area } A \]

For torsion design of T-beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during calculation of the torsion section properties.

With this assumption, the special properties for a Rectangular beam section are given as follows:
\[ A_c = bh \]
\[ A = (b - t)(h - t) \]
\[ u = 2b + 2h \]
\[ u_m = 2(b - t) + 2(h - t) \]

where, the section dimensions \( b, h, \) and \( t \) are shown in Figure 14-9. Similarly, the special section properties for a T-beam section are given as follows:

\[ A_c = b_u h + (b_f - b_w) d_s \]
\[ A = (b_f - t)(h - t) \]
\[ u = 2b_f + 2h \]
\[ u_m = 2(b_f - t) + 2(h - t) \]

where the section dimensions \( b_f, b_w, h, \) and \( d_s \) for a T-beam are shown in Figure 14-9.

Figure 14-9 Closed stirrup and section dimensions for torsion design
14.5.3.3 Determine Torsion Reinforcement

It is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals.

The ultimate resistance of compression diagonals is:

\[
T_{Red} = 2 \cdot A_t \cdot f'_{cd} \cdot \frac{\cot \theta}{1 + \cot^2 \theta} \quad (\text{NTC Eq. 4.1.27})
\]

\[
f'_{cd} = 0.5 f_{cd}
\]

\(\theta\) is the angle of the compression diagonals, as previously defined for beam shear. The code allows any value between 21.8° and 68.2° (NTC Eq. 4.1.30), while the program assumes the conservative value of 45°.

An upper limit of the combination of \(V_{Ed}\) and \(T_{Ed}\) that can be carried by the section without exceeding the capacity of the concrete struts is checked using the following equation.

\[
\frac{T_{Ed}}{T_{Red}} + \frac{V_{Ed}}{V_{Red}} \leq 1.0 \quad (\text{NTC Eq. 4.1.32})
\]

If the combination of \(V_{Ed}\) and \(T_{Ed}\) exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The ultimate resistance of the closed stirrups is:

\[
T_{Rsd} = 2 \cdot A_s \cdot \frac{A_{fs}}{s} \cdot f_{yd} \cdot \cot \theta \quad (\text{NTC Eq. 4.1.28})
\]

\(A_s = \) stirrups’ area

By reversing the equation 4.1.28 and imposing \(T_{Ed} = T_{Rsd}\) it is possible to calculate the necessary steel area.
Finally the resistance of longitudinal bars is:

\[
T_{Rld} = 2 \cdot A \cdot \sum A_1 \cdot \frac{f_{yd}}{u_m \cot \theta}
\]  
(NTC Eq. 4.1.28)

\[\sum A_1 \text{ area of the total longitudinal bars}\]

By reversing the equation 4.1.28 and imposing \(T_{Rld} = T_{Rsd}\) it is possible to calculate the necessary steel area.

The maximum of all the calculated \(A_l\) and \(A_s/s\) values obtained from each design load combination is reported along with the controlling combination names.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 14.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (NTC2008) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

#### 14.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal
displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

14.6.1.1 Determine Factored Moments for Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

14.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.
14.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits:

\[ A_{s, \text{min}} = 0.26 \frac{f_{cm}}{f_{yk}} bd \]  
\[ A_{s, \text{min}} = 0.0013bd \]  

where \( f_{cm} \) is the mean value of axial tensile strength of the concrete and is computed as:

\[ f_{cm} = 0.30 f_{ck}^{(2/3)} \text{ for } f_{ck} \leq 50 \text{ MPa} \]  
\[ f_{cm} = 2.12 \ln(1 + f_{cm}/10) \text{ for } f_{ck} > 50 \text{ MPa} \]  
\[ f_{cm} = f_{ck} + 8 \text{ MPa} \]

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (NTC § 4.1.6.1.1).

14.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following subsections.

NTC2008 for the punching shear check refers to Eurocode2-2004.

14.6.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (EC2 6.4.1(4)) and at a critical section at a distance of 2.0d from the face of the support (EC2 6.4.2(1)). The perimeter of the critical section should be constructed such that its length is minimized. Figure 6-4 shows the auto punching perimeters consid-
erased by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Diagram showing different column shapes: Interior Column, Edge Column, Corner Column, Circular Column, T-Shape Column, L-Shape Column.](image)

**Figure 6-4 Punching Shear Perimeters**

### 14.6.2.2 Determination of Concrete Capacity

The concrete punching shear stress capacity is taken as:

\[
V_{Rd,c} = \left[ C_{Rd,c}k \left( 100\rho_1 f_{ck} \right)^{1/3} + k_1\sigma_{cp} \right]
\]

with a minimum of:

\[
V_{Rd,c} = \left( v_{min} + k_1\sigma_{cp} \right)
\]

where \( f_{ck} \) is in MPa and

\[
k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm}
\]

\[
\rho_1 = \sqrt{\rho_{xx} \rho_{yy}} \leq 0.02
\]
where \( \rho_x \) and \( \rho_y \) are the reinforcement ratios in the \( x \) and \( y \) directions respectively, conservatively taken as zeros, and

\[
\sigma_{cp} = \frac{\sigma_{cx} + \sigma_{cy}}{2} \tag{EC2 6.4.4(1)}
\]

where \( \sigma_{cx} \) and \( \sigma_{cy} \) are the normal concrete stresses in the critical section in the \( x \) and \( y \) directions respectively, conservatively taken as zeros.

\[
C_{Rd,c} = 0.18 / \gamma_c \tag{EC2 6.4.4(1)}
\]

\[
\nu_{\min} = 0.035 k^{3/2} f_{ck}^{1/2} \tag{EC2 6.4.4(1)}
\]

\[
k_1 = 0.15 \tag{EC2 6.4.4(1)}
\]

### 14.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, \( v_{Ed} \), is calculated as:

\[
v_{Ed} = \frac{V_{Ed}}{u_d} \left[ 1 + k \frac{M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + k \frac{M_{Ed,3} u_1}{V_{Ed} W_{1,3}} \right], \text{ where} \tag{EC2 6.4.4(2)}
\]

- \( k \) is the function of the aspect ratio of the loaded area in Table 14.1 of EN 1992-1-1
- \( u_1 \) is the effective perimeter of the critical section
- \( d \) is the mean effective depth of the slab
- \( M_{Ed} \) is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3
- \( V_{Ed} \) is the total punching shear force
- \( W_1 \) accounts for the distribution of shear based on the control perimeter along bending axis 2 and 3.
14.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

14.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as described in the subsections that follow.

14.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

14.6.3.2 Determine Required Shear Reinforcement

The shear is limited to a maximum of \( V_{Rcd} \) calculated in the same manner as explained previously for beams.

Given \( v_{Ed} \), \( v_{Rdc} \), and \( v_{Rcd} \), the required shear reinforcement is calculated as follows (EC2 6.4.5).

- If \( v_{Rdc} < v_{Ed} \leq v_{Rcd} \)
  \[
  A_{sv} = \frac{v_{Ed} - 0.75v_{Rdc}}{1.5f_{ywd,ef}} (u_d) s_r
  \]  
  (EC2 6.4.5)

- If \( v_{Ed} > v_{Rcd} \), a failure condition is declared. (EC2 6.2.3(3))

- If \( v_{Ed} \) exceeds the maximum permitted value of \( V_{Rcd} \) the concrete section should be increased in size.
14.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 6-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $2d$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $1.5d$ measured in a direction parallel to the column face (EC2 9.4.3(1)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

14.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in EC2 4.4.1 plus half of the diameter of the flexural reinforcement.
Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.3d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $1.5d$ and should not exceed $2d$ at additional perimeters. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

\[
0.3d \leq s_o \leq 2d \\
\leq 0.75d \\
g \leq 1.5d \text{ (first perimeter)} \\
g \leq 2d \text{ (additional perimeters)}
\] (EC2 9.4.3(1))
Chapter 15
Design for Hong Kong CP-2013

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Hong Kong limit state code CP-2013 [CP 2013] is selected. The various notations used in this chapter are listed in Table 15-1. For referencing to the pertinent sections of the Hong Kong code in this chapter, a prefix “CP” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

15.1 Notations

| Table 15-1 List of Symbols Used in the CP-2013 Code |
|-----------------|----------------------------------|
| $A_g$           | Gross area of cross-section, mm$^2$ |
| $A_{lt}$        | Area of longitudinal reinforcement for torsion, mm$^2$ |
| $A_t$           | Area of tension reinforcement, mm$^2$ |
### Table 15-1 List of Symbols Used in the CP-2013 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv}$</td>
<td>Total cross-sectional area of links at the neutral axis, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv,t}$</td>
<td>Total cross-sectional area of closed links for torsion, mm$^2$</td>
</tr>
<tr>
<td>$A_{sv} / S_v$</td>
<td>Area of shear reinforcement per unit length, mm$^2$/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width or effective width of the section in the compression zone, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Width or effective width of flange, mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Average web width of a flanged beam, mm</td>
</tr>
<tr>
<td>$C$</td>
<td>Torsional constant, mm$^4$</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of tension reinforcement, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Depth to center of compression reinforcement, mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, N/mm$^2$</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, assumed as 200,000 N/mm$^2$</td>
</tr>
<tr>
<td>$f$</td>
<td>Punching shear factor considering column location</td>
</tr>
<tr>
<td>$f_{cu}$</td>
<td>Characteristic cube strength, N/mm$^2$</td>
</tr>
<tr>
<td>$f'_s$</td>
<td>Stress in the compression reinforcement, N/mm$^2$</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Characteristic strength of reinforcement, N/mm$^2$</td>
</tr>
<tr>
<td>$f_{yv}$</td>
<td>Characteristic strength of shear reinforcement, N/mm$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section in the plane of bending, mm</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Flange thickness, mm</td>
</tr>
<tr>
<td>$h_{min}$</td>
<td>Smaller dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$h_{max}$</td>
<td>Larger dimension of a rectangular section, mm</td>
</tr>
<tr>
<td>$K$</td>
<td>Normalized design moment, $\frac{M_u}{bd^2 f_{cu}}$</td>
</tr>
<tr>
<td>$K'$</td>
<td>Maximum $\frac{M_u}{bd^2 f_{cu}}$ for a singly reinforced concrete section</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Shear strength enhancement factor for support compression</td>
</tr>
</tbody>
</table>
### Table 15-1 List of Symbols Used in the CP-2013 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_2)</td>
<td>Concrete shear strength factor, (\left[\frac{f_{cu}}{25}\right]^{\frac{1}{2}})</td>
</tr>
<tr>
<td>(M)</td>
<td>Design moment at a section, N-mm</td>
</tr>
<tr>
<td>(M_{\text{single}})</td>
<td>Limiting moment capacity as singly reinforced beam, N-mm</td>
</tr>
<tr>
<td>(s_v)</td>
<td>Spacing of the links along the length of the beam, mm</td>
</tr>
<tr>
<td>(T)</td>
<td>Design torsion at ultimate design load, N-mm</td>
</tr>
<tr>
<td>(u)</td>
<td>Perimeter of the punch critical section, mm</td>
</tr>
<tr>
<td>(V)</td>
<td>Design shear force at ultimate design load, N</td>
</tr>
<tr>
<td>(v)</td>
<td>Design shear stress at a beam cross-section or at a punching critical section, N/mm²</td>
</tr>
<tr>
<td>(v_c)</td>
<td>Design concrete shear stress capacity, N/mm²</td>
</tr>
<tr>
<td>(v_{\text{max}})</td>
<td>Maximum permitted design factored shear stress, N/mm²</td>
</tr>
<tr>
<td>(v_t)</td>
<td>Torsional shear stress, N/mm²</td>
</tr>
<tr>
<td>(x)</td>
<td>Neutral axis depth, mm</td>
</tr>
<tr>
<td>(x_{\text{bal}})</td>
<td>Depth of neutral axis in a balanced section, mm</td>
</tr>
<tr>
<td>(z)</td>
<td>Lever arm, mm</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Torsional stiffness constant</td>
</tr>
<tr>
<td>(\beta_0)</td>
<td>Moment redistribution factor in a member</td>
</tr>
<tr>
<td>(\gamma_f)</td>
<td>Partial safety factor for load</td>
</tr>
<tr>
<td>(\gamma_m)</td>
<td>Partial safety factor for material strength</td>
</tr>
<tr>
<td>(\varepsilon_c)</td>
<td>Maximum concrete strain</td>
</tr>
<tr>
<td>(\varepsilon_s)</td>
<td>Strain in tension reinforcement</td>
</tr>
<tr>
<td>(\varepsilon_s')</td>
<td>Strain in compression reinforcement</td>
</tr>
</tbody>
</table>

### 15.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, \(\gamma_f\) (CP 2.3.1.3). For CP-2013, if a structure is subjected to dead (G),
live (Q), pattern live (PQ), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered. (CP 2.3.2.1, Table 2.1).

1.4G  
1.4G + 1.6Q (CP 2.3.2)  
1.4G + 1.6(0.75PQ) (CP 2.3.2)  
1.0G ± 1.4W  
1.4G ± 1.4W (CP 2.3.2)  
1.2G + 1.2Q ± 1.2W

These are also the default design load combinations in SAFE whenever the CP-2013 code is used. If roof live load is separately treated or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

15.3 Limits on Material Strength

The concrete compressive strength, $f_{cu}$, should not be less than 20 N/mm$^2$ (CP 3.1.3). The program does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsible to use the proper strength values while defining the materials.

15.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, $\gamma_m$. The values of $\gamma_m$ used in the program are listed in the following table, as taken from CP Table 2.2 (CP 2.4.3.2):
### 15.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the sections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

#### 15.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the length of the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement
15.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

15.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 15-1 (CP 6.1.2.4(a)), where $\varepsilon_{c,\text{max}}$ is defined as:

$$\varepsilon_{c,\text{max}} = \begin{cases} 0.0035 & \text{if } f_{cu} \leq 60 \text{N/mm}^2 \\ 0.0035 - 0.00006(f_{cu} - 60)^{1/2} & \text{if } f_{cu} > 60 \text{N/mm}^2 \end{cases}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$; CP 6.1.2.4(b)). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \leq \begin{cases} 0.5 & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\ 0.4 & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\ 0.33 & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 \end{cases}$$ (CP 6.1.2.4(b))
to safeguard against non-ductile failures (CP 6.1.2.4(b)). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The depth of the compression block is given by:

\[
\alpha = \begin{cases} 
0.9x & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\
0.8x & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\
0.72x & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 
\end{cases} 
\]

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed \(0.1f_{cu}A_g\) (CP 6.1.2.4(a)), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than \(0.1f_{cu}A_g\) and axial tensions are always included in flexural and shear design.

### 15.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam, \(M_{\text{single}}\), is obtained first for a section. The reinforcing is determined based on whether \(M\) is greater than, less than, or equal to \(M_{\text{single}}\). See Figure 15-1

Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

\[
M_{\text{single}} = K'f_{cu}bd^2, \quad \text{where} \quad (CP \ 6.1.2.4(c), \ Eqn. \ 6.8)
\]

\[
K' = \begin{cases} 
0.156 & \text{for } f_{cu} \leq 45 \text{ N/mm}^2 \\
0.120 & \text{for } 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\
0.094 & \text{for } 70 < f_{cu} \leq 100 \text{ N/mm}^2 
\end{cases} 
\]

- If \(M \leq M_{\text{single}}\), the area of tension reinforcement, \(A_t\), is obtained from:

\[
A_t = \frac{M}{0.87f_{y}z}, \quad \text{where} \quad (CP \ 6.1.2.4(c), \ Eqn. \ 6.12)
\]
This reinforcement is to be placed at the bottom if $M$ is positive, or at the top if $M$ is negative.

- If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$A'_s = \frac{M - M_{\text{single}}}{\left( \frac{f'_s - 0.67 f_{cu}}{\gamma_c} \right)(d - d')}$$  

(CP 6.1.2.4(c)), Eqn. 6.15)
where $d'$ is the depth of the compression reinforcement from the concrete compression face, and

$$ f'_s = E_s \varepsilon_s \left(1 - \frac{d'}{x}\right) \leq 0.87 f_s, \quad (CP\ 6.1.2.4(c),\ 3.2.6,\ Fig.\ 3.9) $$

$$ f = \begin{cases} \frac{d-z}{0.45}, & \text{for } f_{cu} \leq 45 \text{N/mm}^2 \\ \frac{d-z}{0.40}, & \text{for } 45 < f_{cu} \leq 70 \text{N/mm}^2 \\ \frac{d-z}{0.36}, & \text{for } 70 < f_{cu} \leq 100 \text{N/mm}^2 \end{cases} (CP\ 6.1.2.4(a),\ Fig\ 6.1,\ Eqn.\ 6.11) $$

$$ z = d \left(0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right) \leq 0.95d \quad (CP\ 6.1.2.4(c)) $$

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$ A_s = \frac{M_{\text{single}}}{0.87f_s z} + \frac{M - M_{\text{single}}}{0.87f_s (d - d')} \quad (CP\ 6.1.2.4(c)) $$

### 15.5.1.2.2 Design of Flanged Beams

#### 15.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

#### 15.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located in the flange. On the basis of this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange depth, the contribution
of the web to the flexural strength of the beam is taken into account. See Figure 15-2.

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

\[
K = \frac{M}{f_{cu} b_f d^2}.
\]

Then the moment arm is computed as:

\[
z = d \left(0.5 + \frac{0.25 - \frac{K}{0.9}}{0.5} \right) \leq 0.95d,
\]

the depth of the neutral axis is computed as:

\[
x = \begin{cases} 
\frac{d-z}{0.45}, & \text{for } f_{cu} \leq 45 \text{N/mm}^2 \\
\frac{d-z}{0.40}, & \text{for } 45 < f_{cu} \leq 70 \text{N/mm}^2 \\
\frac{d-z}{0.36}, & \text{for } 70 < f_{cu} \leq 100 \text{N/mm}^2 
\end{cases}
\]

and the depth of the compression block is given by:

\[
a = \begin{cases} 
0.9x, & \text{for } f_{cu} \leq 45 \text{N/mm}^2 \\
0.8x, & \text{for } 45 < f_{cu} \leq 70 \text{N/mm}^2 \\
0.72x, & \text{for } 70 < f_{cu} \leq 100 \text{N/mm}^2 
\end{cases}
\]

- If \(a \leq h_f\), the subsequent calculations for \(A_s\) are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as \(b_f\). Compression reinforcement is required when \(K > K'\).

- If \(a > h_f\), the calculation for \(A_s\) has two parts. The first part is for balancing the compressive force from the flange, \(C_f\), and the second part is for balancing the compressive force from the web, \(C_w\), as shown in Figure 15-2.
In that case, the ultimate resistance moment of the flange is given by:

\[ M_f = \frac{0.67}{\gamma_C} f_{cu} (b_f - b_w) h_f \left( d - 0.5 h_f \right) \]

The moment taken by the web is computed as:

\[ M_w = M - M_f \]

and the normalized moment resisted by the web is given by:

\[ K_w = \frac{M_w}{f_{cu} b_w d^2} \]

- If \( K_w \leq K' \) (CP 6.1.2.4(c)), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.
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\[ A_s = \frac{M_f}{0.87f_y(d - 0.5h_f)} + \frac{M_w}{0.87f_yz} \]

where

\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K_w}{0.9}} \right) \leq 0.95d \]

- If \( K_w > K' \), compression reinforcement is required and is calculated as follows:

The ultimate moment of resistance of the web only is given by:

\[ M_{uw} = Kf_{cu}b_wd^2 \]

The compression reinforcement is required to resist a moment of magnitude \( M_w - M_{uw} \). The compression reinforcement is computed as:

\[ A'_s = \frac{M_w - M_{uw}}{f' + \frac{0.67f_{cu}}{\gamma_c}(d - d')} \]

where, \( d' \) is the depth of the compression reinforcement from the concrete compression face, and

\[ f' = E_sE_c \left( 1 - \frac{d}{x} \right) \leq 0.87f_y \]

(CP 6.1.2.4(c), 3.2.6, Fig 3.9)

The area of tension reinforcement is obtained from equilibrium as:

\[ A_s = \frac{1}{0.87f_y} \left[ \frac{M_f}{d - 0.5h_f} + \frac{M_{uw}}{z} + \frac{M_w - M_{uw}}{d - d'} \right] \]

\[ z = d \left( 0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \right) \leq 0.95d \]

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15.5.1.2.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from CP Table 9.1 (CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage f&lt;sub&gt;y&lt;/sub&gt; = 250 MPa</th>
<th>Minimum percentage f&lt;sub&gt;y&lt;/sub&gt; = 460 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>—</td>
<td>100 ( \frac{A_s}{bh} )</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>T or L-Beam with web in tension</td>
<td>( \frac{b_w}{b_f} &lt; 0.4 )</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.32</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>( \frac{b_w}{b_f} \geq 0.4 )</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>T-Beam with web in compression</td>
<td>—</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.48</td>
<td>0.26</td>
</tr>
<tr>
<td>L-Beam with web in compression</td>
<td>—</td>
<td>100 ( \frac{A_s}{b_w h} )</td>
<td>0.36</td>
<td>0.20</td>
</tr>
</tbody>
</table>

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from CP Table 9.1 (CP 9.2.1.1).

<table>
<thead>
<tr>
<th>Section</th>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>—</td>
<td>100 ( \frac{A'_c}{bh} )</td>
<td>0.20</td>
</tr>
<tr>
<td>T or L-Beam</td>
<td>Web in tension</td>
<td>100 ( \frac{A'_c}{b_f h_f} )</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>Web in compression</td>
<td>100 ( \frac{A'_c}{b_c h} )</td>
<td>0.20</td>
</tr>
</tbody>
</table>

An upper limit of 0.04 times the gross cross-sectional area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows (CP 9.2.1.3):
15.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (CP 6.1.2.5):

- Determine the shear stress, \( v \).
- Determine the shear stress, \( v_c \), that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

15.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors. The shear stress is then calculated as:

\[
v = \frac{V}{bd}
\]  

(CP 6.1.2.5(a))

The maximum allowable shear stress, \( v_{\text{max}} \) is defined as:

\[
v_{\text{max}} = \min(0.8 \sqrt{f_{\text{cu}}}, \hspace{1em} 7 \text{ MPa})
\]  

(CP 6.1.2.5(a))

15.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, \( v_c \), is calculated as:

\[
A_i \leq \begin{cases} 
0.04bd & \text{Rectangular beam} \\
0.04b_n d & \text{Flanged beam} 
\end{cases} 
\]  

(CP 9.2.1.3)

\[
A'_i \leq \begin{cases} 
0.04bd & \text{Rectangular beam} \\
0.04b_n d & \text{Flanged beam} 
\end{cases} 
\]  

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\[ v'_c = v_c + 0.6 \frac{NVh}{A_c M} \leq v_c \sqrt{1 + \frac{N}{A_c v_c}} \]  
(CP 6.1.2.5(k), Eqn. 6.22)

\[ v_c = \frac{0.79 k_1 k_2 \left( \frac{100 A_s}{b d} \right)^{\frac{1}{2}} \left( \frac{400}{d} \right)^{\frac{1}{4}}}{\gamma_m} \]  
(CP 6.1.2.5(c), Table 6.3)

\( k_1 \) is the enhancement factor for support compression, and is conservatively taken as 1  
(CP 6.1.2.5(g))

\[ k_2 = \left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}}, \quad 1 \leq k_2 \leq \left( \frac{80}{25} \right)^{\frac{1}{3}} \]  
(CP 6.1.2.5(c), Table 6.3)

\[ \gamma_m = 1.25 \]  
(CP 6.1.2.5(c), Table 6.3)

However, the following limitations also apply:

\[ 0.15 \leq \frac{100 A_s}{b d} \leq 3, \]  
(CP 6.1.2.5(c), Table 6.3)

\[ \left( \frac{400}{d} \right)^{\frac{1}{4}} \geq 0.67, \quad \text{Members without shear reinforcement} \]
\[ \left( \frac{400}{d} \right)^{\frac{1}{4}} \geq 1.00, \quad \text{Members with shear reinforcement} \]  
(CP 6.1.2.5(c), Table 6.3)

\[ \frac{Vh}{M} \leq 1 \]  
(CP 6.1.2.5(k))

**15.5.2.3 Determine Required Shear Reinforcement**

Given \( v, v_c \), and \( v_{\text{max}} \), the required shear reinforcement is calculated as follows (CP Table 6.2, CP 6.1.2.5(b)):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, \( v_r \), as:
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\[
v_r = \begin{cases} 
0.4 & \text{if } f_{cu} \leq 40 \, \text{N/mm}^2 \\
0.4 \left( \frac{f_{cu}}{40} \right)^{\frac{2}{3}} & \text{if } 40 < f_{cu} \leq 80 \, \text{N/mm}^2 \\
0.4 \left( \frac{80}{40} \right)^{\frac{2}{3}} & \text{if } f_{cu} > 80 \, \text{N/mm}^2 
\end{cases} \quad \text{(CP 6.1.2.5(b), Table 6.2)}
\]

- If \( v \leq v'_{c} + v_{r} \), minimum reinforcement is required:
  \[
  \frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}} \quad \text{(CP 6.1.2.5(b))}
  \]

- If \( v > v'_{c} + v_{r} \),
  \[
  \frac{A_{st}}{s_v} = \frac{(v - v'_{c}) b}{0.87 f_{yv}} \quad \text{(CP 6.1.2.5(b))}
  \]

- If \( v > v_{\text{max}} \), a failure condition is declared. \quad \text{(CP 6.1.2.5(b))}

The maximum of all the calculated \( A_{sv}/s_v \) values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

### 15.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, \( v_r \).
- Determine special section properties.
15.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given, as follows (CP 6.3.1).

The torsional shear stress, $v_t$, for a rectangular section is computed as:

\[
v_t = \frac{2T}{h_{\text{min}}^2 (h_{\text{max}} - h_{\text{min}} / 3)} \quad \text{(CP 6.3.3(a))}
\]

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

\[
T_{\text{seg}} = T \left( \frac{h_{\text{min}}^3 h_{\text{max}}}{\sum (h_{\text{min}}^3 h_{\text{max}})} \right) \quad \text{(CP 6.3.3(b))}
\]

$h_{\text{max}}$ = Larger dimension of a rectangular section

$h_{\text{min}}$ = Smaller dimension of a rectangular section

If the computed torsional shear stress, $v_t$, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

\[
v_t \leq \min(0.8 \sqrt{f_{\text{cu}}}, 7 \text{ N/mm}^2) \times \frac{v_{\text{cu}}}{550} \quad \text{(CP 6.3.4, Table 6.17)}
\]
SAFE Reinforced Concrete Design

Where $y_1$ is the larger center-to-center dimension of a rectangular link.

15.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, $\nu_{t,\text{min}}$, for which the torsion in the section can be ignored is calculated as:

$$\nu_{t,\text{min}} = \min\left(0.067\sqrt{f_{cu}}, 0.6 \text{ N/mm}^2\right)$$

(CP 6.3.4, Table 6.17)

where $f_{cu}$ is the specified concrete compressive strength.

15.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, $\nu_t$ is less than the threshold limit, $\nu_{t,\text{min}}$, torsion can be safely ignored (CP 6.3.5). In that case, the program reports that no torsion reinforcement is required. However, if $\nu_t$ exceeds the threshold limit, $\nu_{t,\text{min}}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP 6.3.5).

![Figure 15-3 Closed stirrup and section dimensions for torsion design](image)

Figure 15-3 Closed stirrup and section dimensions for torsion design

15 - 18 Beam Design
If $v_t > v_{t,\text{min}}$, the required closed stirrup area per unit spacing, $A_{sv,t} / s_v$, is calculated as:

$$A_{sv,t} = \frac{T}{0.8x_1 y_1 (0.87 f_{yv})}$$  \hspace{1cm} (CP 6.3.6, Eqn. 6.67)

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{w,t} f_{vm} (x_1 + y_1)}{s_v f_y}$$  \hspace{1cm} (CP 6.3.6, Eqn. 6.68)

In the preceding expressions, $x_1$ is the smaller center-to-center dimension of the closed link, and $y_1$ is the larger center-to-center dimension of the closed link.

An upper limit of the combination of $v$ and $v_t$ that can be carried by the section also is checked using the equation:

$$v + v_t \leq \min \left( 0.8 \sqrt{f_{cm}}, 7 \text{ N/mm}^2 \right)$$  \hspace{1cm} (CP 6.3.4)

If the combination of $v$ and $v_t$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated $A_l$ and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 15.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is performed based on the ultimate strength design method (CP-2013).
for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

15.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

15.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

15.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across
the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

15.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required in each direction of a slab is given by the following limits (CP 9.3.1.1), with interpolation for reinforcement of intermediate strength:

\[
A_t \geq \begin{cases} 
0.0024bh & \text{if } f_t \leq 250 \text{ MPa} \\
0.0013bh & \text{if } f_t \geq 460 \text{ MPa}
\end{cases} 
\]  

(CP 9.3.1.1(a))

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 9.2.1.3).

15.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following subsections.

15.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5\(d\) from the face of the support (CP 6.1.5.7(d)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CP 6.1.5.7). Figure 15-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.
15.6.2.2 Determine Concrete Capacity

The concrete punching shear factored strength is taken as (CP 6.1.5.7(d), Table 6.3):

\[
v_c = 0.79 k_1 k_2 \left( \frac{100 A_e}{bd} \right)^{\frac{1}{2}} \left( \frac{400}{d} \right)^{\frac{1}{4}}
\]  

\[\text{(CP 6.1.2.5(d), Table 6.3)}\]

\[k_1\] is the enhancement factor for support compression, and is conservatively taken as 1  
\[\text{(CP 6.1.2.5(g), 6.1.5.7(d))}\]

\[k_2 = \left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}} \quad 1 \leq k_2 \leq \left( \frac{80}{25} \right)^{\frac{1}{3}} \]  
\[\text{(CP 6.1.2.5(c), Table 6.3)}\]

\[\gamma_m = 1.25\]  
\[\text{(CP 2.4.3.2, Table 2.2)}\]

However, the following limitations also apply:
\[ 0.15 \leq \frac{100A_s}{bd} \leq 3, \quad \text{(CP 6.1.2.5(c), Table 6.3)} \]

\[
\begin{cases}
\left( \frac{400}{\frac{d}{4}} \right)^\frac{1}{4} & \geq 0.67, \quad \text{Members without shear reinforcement} \\
1.00 & , \quad \text{Members with shear reinforcement} \\
\end{cases} 
\quad \text{(CP 6.1.2.5(c), Table 6.3)}
\]

\[ A_s = \text{area of tension reinforcement, which is taken as the average tension} \]
\[ \text{reinforcement of design strips in Layer A and layer B where Layer} \]
\[ \text{A and Layer design strips are in orthogonal directions. When design} \]
\[ \text{strips are not present in both orthogonal directions then tension rein} \]
\[ \text{forcement is taken as zero in the current implementation.} \]

\[ v \leq \min(0.8 \sqrt{f_{cu}}, 7 \text{ MPa}) \quad \text{(CP 6.1.5.7(b))} \]

\[ f_{cu} \leq 80 \text{ MPa (for calculation purpose only)} \quad \text{(CP Table 6.3)} \]

**15.6.2.3 Determine Maximum Shear Stress**

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, \( \nu_{\text{max}} \), is calculated as:

\[ V_{\text{eff},x} = V \left( f + \frac{1.5M_x}{V_y} \right) \quad \text{(CP 6.1.5.6(b), 6.1.5.6(c))} \]

\[ V_{\text{eff},y} = V \left( f + \frac{1.5M_y}{V_x} \right) \quad \text{(CP 6.1.5.6(b), 6.1.5.6(c))} \]

\[ \nu_{\text{max}} = \max \left\{ \frac{V_{\text{eff},x}}{ud}, \frac{V_{\text{eff},y}}{ud} \right\} \quad \text{(CP 6.1.5.7)} \]

where,
\( u \) is the perimeter of the critical section,

\( x \) and \( y \) are the lengths of the sides of the critical section parallel to the axis of bending,

\( M_x \) and \( M_y \) are the design moments transmitted from the slab to the column at the connection,

\( V \) is the total punching shear force, and

\( f \) is a factor to consider the eccentricity of punching shear force and is taken as

\[
 f = \begin{cases} 
 1.00 & \text{for interior columns} \\
 1.25 & \text{for edge columns} \\
 1.25 & \text{for corner columns} 
\end{cases} 
\] (CP 6.1.5.6(b), 6.1.5.6(c))

**15.6.2.4 Determine Capacity Ratio**

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

**15.6.3 Design Punching Shear Reinforcement**

The use of shear links as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 6.1.5.7(e)). The use of shear studs is not covered in Hong Kong CP 2013. However, program uses the identical clauses for shear studs when CP 20013 code is selected. If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear and Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.
15.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

15.6.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

\[ v_{\text{max}} = 2v_c \]  

(CP 6.1.5.7(e))

Given \( v \), \( v_c \), and \( v_{\text{max}} \), the required shear reinforcement is calculated as follows (CP 6.1.5.7(e)).

- If \( v \leq 1.6v_c \),

\[ \frac{A_v}{s} = \frac{(v - v_c)ud}{0.87f_{cy}} \geq \frac{v_c ud}{0.87f_{cy}} \]  

(CP 6.1.5.7(e))

- If \( 1.6v_c \leq v < 2.0v_c \),

\[ \frac{A_v}{s} = \frac{5(0.7v - v_c)ud}{0.87f_{cy}} \geq \frac{v_c ud}{0.87f_{cy}} \]  

(CP 6.1.5.7(e))

\[ v_r = \begin{cases} 
0.4 & \text{N/mm}^2 \\
0.4\left(\frac{f_{cu}}{40}\right)^{2/3} & \text{N/mm}^2 
\end{cases} \]  

(CP 6.1.5.7, Table 6.2)

- If \( v > 2.0v_c \), a failure condition is declared.  

(CP 6.1.5.7(e))

If \( v \) exceeds the maximum permitted value of \( v_{\text{max}} \), the concrete section should be increased in size.

15.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 15-5 shows a typical arrangement
of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed \(d/2\). The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed \(1.5d\) measured in a direction parallel to the column face (CP 6.1.5.7(f)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

![Typical Studrail and Critical Sections](image)

**Figure 15-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone**

### 15.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 4.2.4 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.
When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, $g$, at the first peripheral line of studs shall not exceed $1.5d$. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

$$s_o \leq 0.5d$$  \hspace{1cm} (CP 6.1.5.7(f))

$$s \leq 0.75d$$  \hspace{1cm} (CP 6.1.5.7(f))

$$g \leq 1.5d$$  \hspace{1cm} (CP 6.1.5.7(f))

Stirrups are only permitted when slab thickness is greater than 200 mm (CP 6.1.5.7(e)).
This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code ACI 318-14 [ACI 2014] is selected. Various notations used in this chapter are listed in Figure 16-1. For referencing to the pertinent sections or equations of the ACI code in this chapter, a prefix “ACI” followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

### 16.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{cp}$</td>
<td>Area enclosed by the outside perimeter of the section, sq-in</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Gross area of concrete, sq-in</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_l$</td>
<td>Area of longitudinal reinforcement for torsion, sq-in</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Area enclosed by the shear flow path, sq-in</td>
</tr>
<tr>
<td>$A_{oh}$</td>
<td>Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of tension reinforcement, sq-in</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_{l/s}$</td>
<td>Area of closed shear reinforcement per unit length of member for torsion, sq-in/in</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Area of shear reinforcement, sq-in</td>
</tr>
<tr>
<td>$A_{v/s}$</td>
<td>Area of shear reinforcement per unit length, sq-in/in</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, in</td>
</tr>
<tr>
<td>$a_{max}$</td>
<td>Maximum allowed depth of compression block, in</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of section, in</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Effective width of flange (flanged section), in</td>
</tr>
<tr>
<td>$b_o$</td>
<td>Perimeter of the punching shear critical section, in</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), in</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Width of the punching shear critical section in the direction of bending, in</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Width of the punching shear critical section perpendicular to the direction of bending, in</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, in</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, in</td>
</tr>
<tr>
<td>$d'$</td>
<td>Distance from compression face to compression reinforcement, in</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, psi</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, psi</td>
</tr>
<tr>
<td>$f'c$</td>
<td>Specified compressive strength of concrete, psi</td>
</tr>
</tbody>
</table>
### Table 16-1 List of Symbols Used in the ACI 318-14 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_c$</td>
<td>Stress in the compression reinforcement, psi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Specified yield strength of flexural reinforcement, psi</td>
</tr>
<tr>
<td>$f_{sy}$</td>
<td>Specified yield strength of shear reinforcement, psi</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section, in</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Height of the flange, in</td>
</tr>
<tr>
<td>$M_u$</td>
<td>Factored moment at a section, lb-in</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Factored axial load at a section occurring simultaneously with $V_u$ or $T_u$, lb</td>
</tr>
<tr>
<td>$P_u$</td>
<td>Factored axial load at a section, lb</td>
</tr>
<tr>
<td>$p_{cp}$</td>
<td>Outside perimeter of concrete cross-section, in</td>
</tr>
<tr>
<td>$p_h$</td>
<td>Perimeter of centerline of outermost closed transverse torsional reinforcement, in</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of shear reinforcement along the beam, in</td>
</tr>
<tr>
<td>$T_{cr}$</td>
<td>Critical torsion capacity, lb-in</td>
</tr>
<tr>
<td>$T_u$</td>
<td>Factored torsional moment at a section, lb-in</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear force resisted by concrete, lb</td>
</tr>
<tr>
<td>$V_{max}$</td>
<td>Maximum permitted total factored shear force at a section, lb</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Shear force resisted by transverse reinforcement, lb</td>
</tr>
<tr>
<td>$V_u$</td>
<td>Factored shear force at a section, lb</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>Punching shear scale factor based on column location</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching shear critical section</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Factor for obtaining depth of the concrete compression block</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in the concrete</td>
</tr>
<tr>
<td>$\varepsilon_{c_{max}}$</td>
<td>Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)</td>
</tr>
</tbody>
</table>
Table 16-1 List of Symbols Used in the ACI 318-14 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in the reinforcement</td>
</tr>
<tr>
<td>$\varepsilon_{s,\text{min}}$</td>
<td>Minimum tensile strain allowed in the reinforcement at nominal strength for tension controlled behavior (0.005 in/in)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Strength reduction factor</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Fraction of unbalanced moment transferred by flexure</td>
</tr>
<tr>
<td>$\gamma_v$</td>
<td>Fraction of unbalanced moment transferred by eccentricity of shear</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shear strength reduction factor for light-weight concrete</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of compression diagonals, degrees</td>
</tr>
</tbody>
</table>

16.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-14, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (ACI 5.3.1):

1. $1.4D$ (ACI Eqn. 5.3.1a)
2. $1.2D + 1.6L + 0.5L_r$ (ACI Eqn.5.3.1b)
3. $1.2D + 1.0L + 1.6L_r$ (ACI Eqn.5.3.1c)
4. $1.2D + 1.6(0.75 \text{ PL}) + 0.5L_r$ (ACI Eqn.5.3.1b, 6.4)
5. $1.2D + 1.6L + 0.5S$ (ACI Eqn.5.3.1b)
6. $1.2D + 1.0L + 1.6S$ (ACI Eqn.5.3.1c)
7. $0.9D \pm 1.0W$ (ACI Eqn.5.3.1f)
8. $1.2D + 1.0L + 0.5L_r \pm 1.0W$ (ACI Eqn.5.3.1d)
9. $1.2D + 1.6L_r \pm 0.5W$ (ACI Eqn.5.3.1c)
10. $1.2D + 1.6S \pm 0.5W$ (ACI Eqn.5.3.1c)
11. $1.2D + 1.0L + 0.5S \pm 1.0W$ (ACI Eqn.5.3.1d)
These are the default design load combinations in SAFE whenever the ACI 318-14 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

16.3 Limits on Material Strength

The concrete compressive strength, $f'_c$, should not be less than 2,500 psi (ACI 19.2.1, Table 19.2.1.1). The upper limit of the reinforcement yield strength, $f_y$, is taken as 80 ksi (ACI 20.2.2.4a, Table 20.2.2.4a) and the upper limit of the reinforcement shear strength, $f_{yt}$, is taken as 60 ksi (ACI 21.2.2.4a, Table 21.2.2.4a).

If the input $f'_c$ is less than 2,500 psi, SAFE continues to design the members based on the input $f'_c$ and does not warn the user about the violation of the code. The user is responsible for ensuring that the minimum strength is satisfied.

16.4 Strength Reduction Factors

The strength reduction factors, $\phi$, are applied to the specified strength to obtain the design strength provided by a member. The $\phi$ factors for flexure, shear, and torsion are as follows:

\[ \phi = 0.90 \text{ for flexure (tension controlled)} \]  
\[ \phi = 0.75 \text{ for shear and torsion} \]

These values can be overwritten; however, caution is advised.

16.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments,
shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

### 16.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

#### 16.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.
16.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, $f_y$, is limited to 80 ksi (ACI 20.2.2.4a), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 16-1 (ACI 22.2.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 21.2.2, Table 21.2.2) when the concrete in compression reaches its assumed strain limit of 0.003. When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed $0.1 f'_c A_g$ (ACI 9.5.2.1), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $0.1 f'_c A_g$ and axial tensions are always included in flexural and shear design.

16.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, $M_u$ (i.e., designing top or bottom reinforcement), the depth of the compression block is given by $a$ (see Figure 16-1), where,

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f'_c \phi b}}$$  \hspace{1cm} (ACI 22.2.2)

and the value of $\phi$ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.
The maximum depth of the compression zone, \( c_{\text{max}} \), is calculated based on the limitation that the tension reinforcement strain shall not be less than \( \varepsilon_{\text{min}} \), which is equal to 0.005 for tension controlled behavior (ACI 21.2.2, Table 21.2.2):

\[
\frac{c_{\text{max}}}{c_{\text{max}} + \varepsilon_{\text{min}}} d = \varepsilon_{\text{cmax}}
\]

\[\text{(ACI 22.2.1.2)}\]

\[\varepsilon_{\text{cmax}} = 0.003 \quad \text{(ACI 21.2.2, Fig R21.2)}\]

\[\varepsilon_{\text{min}} = 0.005 \quad \text{(ACI 21.2.2, Fig R21.2.26)}\]

The maximum allowable depth of the rectangular compression block, \( a_{\text{max}} \), is given by:

\[a_{\text{max}} = \beta f_{\text{c}} c_{\text{max}}\]

\[\text{(ACI 22.2.2.4.1)}\]
where \( \beta_t \) is calculated as:

\[
\beta_t = 0.85 - 0.05 \left( \frac{f' - 4000}{1000} \right), \quad 0.65 \leq \beta_t \leq 0.85 \quad \text{(ACI 22.2.2.4.3)}
\]

- If \( a \leq a_{max} \) (ACI 10.3.4), the area of tension reinforcement is then given by:

\[
A_y = \frac{M_u}{\phi f' \left( d - \frac{a}{2} \right)}
\]

This reinforcement is to be placed at the bottom if \( M_u \) is positive, or at the top if \( M_u \) is negative.

- If \( a > a_{max} \), compression reinforcement is required (ACI 9.3.3.1, 21.2.2, Fig 21.2.26, 22.2.2.4.1) and is calculated as follows:

  - The compressive force developed in the concrete alone is given by:

\[
C = 0.85 f' c b a_{max}
\]

and the moment resisted by concrete compression and tension reinforcement is:

\[
M_{uc} = \phi C \left( d - \frac{a_{max}}{2} \right)
\]

  - Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

\[
M_{st} = M_u - M_{uc}
\]

  - The required compression reinforcement is given by:

\[
A'_{st} = \frac{M_{us}}{\phi (f' + 0.85 f' c)(d - d')}, \quad \text{where}
\]

\[
f' = E_y E_{c_{max}} \left[ \frac{c_{max} - d'}{c_{max}} \right] \leq f' \quad \text{(ACI 9.2.1.2, 9.5.2.1, 20.2.2, 22.2.1.2)}
\]
The required tension reinforcement for balancing the compression in the concrete is:

\[ A_{s1} = \frac{M_{uc}}{\phi f_y \left[ d - \frac{a_{max}}{2} \right]} \]

and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A_{s2} = \frac{M_{uv}}{\phi f_y (d - d')} \]

Therefore, the total tension reinforcement is \( A_s = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A'_c \). \( A_s \) is to be placed at the bottom and \( A'_c \) is to be placed at the top if \( M_o \) is positive, and vice versa if \( M_o \) is negative.

### 16.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 16-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 16-1 is assumed on the compression side.
16.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, $M_u$ (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

16.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85f'_c \phi b_f}}$$

(ACI 22.2)

where, the value of $\phi$ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 21.2.1, 21.2.2, Table 21.2.1, Table 21.2.2) in the preceding and the following equations.

The maximum depth of the compression zone, $c_{max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{min}$, which is equal to 0.005 for tension controlled behavior (ACI 9.3.3.1, 21.2.2, Fig 21.2.26):
where,
\[
\varepsilon_{c_{\text{max}}} = 0.003 \quad \text{(ACI 21.2.2, Fig 21.2.26)}
\]
\[
\varepsilon_{s_{\text{min}}} = 0.005 \quad \text{(ACI 21.2.2, Fig 21.2.26)}
\]

The maximum allowable depth of the rectangular compression block, \(a_{\text{max}}\), is given by:
\[
a_{\text{max}} = \beta_1 c_{\text{max}} \quad \text{(ACI 22.2.2.4.11)}
\]

where \(\beta_1\) is calculated as:
\[
\beta_1 = 0.85 - 0.05 \left( \frac{f^{'c} - 4000}{1000} \right), \quad 0.65 \leq \beta_1 \leq 0.85 \quad \text{(ACI 22.2.2.4.3)}
\]

- If \(a \leq h_f\), the subsequent calculations for \(A_s\) are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as \(b_f\). Compression reinforcement is required if \(a > a_{\text{max}}\).

- If \(a > h_f\), the calculation for \(A_s\) has two parts. The first part is for balancing the compressive force from the flange, \(C_f\), and the second part is for balancing the compressive force from the web, \(C_w\), as shown in Figure 16-2. \(C_f\) is given by:
\[
C_f = 0.85 f^{'c} (b_f - b_w) \min(h_f, a_{\text{max}}) \quad \text{(ACI 22.2.2.4.1)}
\]

Therefore, \(A_{s1} = \frac{C_f}{f_y}\) and the portion of \(M_u\) that is resisted by the flange is given by:
\[
M_{uf} = \phi C_f \left( d - \frac{\min(h_f, a_{\text{max}})}{2} \right)
\]

Again, the value for \(\phi\) is 0.90 by default. Therefore, the balance of the moment, \(M_w\), to be carried by the web is:

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\[ M_{uw} = M_u - M_{uf} \]

The web is a rectangular section with dimensions \( b_w \) and \( d \), for which the design depth of the compression block is recalculated as:

\[ a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi b_w}} \]  

(ACI 22.2)

- If \( a_1 \leq a_{max} \) (ACI 9.3.3.1, 21.2.2), the area of tension reinforcement is then given by:

\[ A_{s2} = \frac{M_{uw}}{\phi f_y \left( d - \frac{a_1}{2} \right)} \]

And

\[ A_s = A_{s1} + A_{s2} \]

This reinforcement is to be placed at the bottom of the flanged beam.

- If \( a_1 > a_{max} \), compression reinforcement is required (ACI 9.3.3.1, 21.2.2, Fig 21.2.2b, 22.2.2.4.1) and is calculated as follows:

  - The compressive force in the web concrete alone is given by:

\[ C_w = 0.85 f'_c b_w a_{max} \]  

(ACI 22.2.2.4.1)

Therefore the moment resisted by the concrete web and tension reinforcement is:

\[ M_{wc} = C_w \left( d - \frac{a_{max}}{2} \right) \phi \]

and the moment resisted by compression and tension reinforcement is:

\[ M_{as} = M_{uw} - M_{wc} \]

Therefore, the compression reinforcement is computed as:

\[ A'_s = \frac{M_{as}}{(f'_s - 0.85 f'_c) (d - d') \phi}, \]  

where
The tension reinforcement for balancing compression in the web concrete is:

\[ A_{s2} = \frac{M_{uc}}{f_y \left( d - \frac{a_{\text{max}}}{2} \right) \phi} \]

and the tension reinforcement for balancing the compression reinforcement is:

\[ A_{s3} = \frac{M_{uc}}{f_y (d - d') \phi} \]

The total tension reinforcement is \( A_s = A_{s1} + A_{s2} + A_{s3} \), and the total compression reinforcement is \( A' \). \( A_s \) is to be placed at the bottom and \( A' \) is to be placed at the top.

**16.5.1.2.3 Minimum and Maximum Reinforcement**

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the two following limits:

\[ A_{s,\text{min}} = \max \left( \frac{3\sqrt{f'_c}}{f_y} b_w d, \frac{200}{f_y} b_w d \right) \]  

\( A_s \geq \frac{4}{3} A_{s(\text{required})} \)

For T-beam in negative moment \( b_w \) in the above expression is substituted by \( b'_w \), where:

\[ b'_w = \min \{b_f, 2b_w\} \]
An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

\[
A_s \leq \begin{cases} 
0.4bd & \text{Rectangular beam} \\
0.4b_w d & \text{Flanged beam}
\end{cases}
\]

\[
A'_{s} \leq \begin{cases} 
0.4bd & \text{Rectangular beam} \\
0.4b_w d & \text{Flanged beam}
\end{cases}
\]

### 16.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, $V_u$.
- Determine the shear force, $V_c$, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

#### 16.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

#### 16.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_c$, is calculated as:

\[
V_c = 2\lambda \sqrt{f' c b_w d} \quad \text{(ACI 22.5.5.1)}
\]

A limit is imposed on the value of $\sqrt{f' c}$ as $f' c \leq 100$ \quad \text{(ACI 22.5.3.1)}
The value of $\lambda$ should be specified in the material property definition.

### 16.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\text{max}} = V_c + \left( 8 \sqrt{f'_c} \right) b_u d$$

(ACI 22.5.1.2)

Given $V_u$, $V_c$, and $V_{\text{max}}$, the required shear reinforcement is calculated as follows where, $\phi$, the strength reduction factor, is 0.75 (ACI 9.3.2.3). The flexural reinforcement strength, $f_{yt}$, is limited to 60 ksi (ACI 11.5.2) even if the material property is defined with a higher value.

- If $V_u \leq 0.5\phi V_c$,

$$\frac{A_v}{s} = 0$$

(ACI 9.6.3.1)

- If $0.5\phi V_c < V_u \leq \phi V_{\text{max}}$, 

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{yt} d}$$

(ACI 22.5.1.1, 22.5.10.1, 20.5.10.5.3)

- If $V_u > \phi V_{\text{max}}$, a failure condition is declared. 

(ACI 22.5.1.2)

If $V_u$ exceeds the maximum permitted value of $\phi V_{\text{max}}$, the concrete section should be increased in size (ACI 22.5.1.2).

Note that if torsion design is considered and torsion reinforcement is required, the equation given in ACI 9.6.3.3 does not need to be satisfied independently. See the subsequent section Design of Beam Torsion Reinforcement for details.

If the beam depth $h$ is

- $h \leq 10''$ for rectangular,

- $h \leq \min \left\{ 24'', \max \left( 2.5h_{ft}, \frac{b}{2} \right) \right\}$ for T-beam,
the minimum shear reinforcement given by ACI 9.6.3.3 does not need to be enforced.

\[ \frac{A_v}{s} \geq \max \left( \frac{0.75 \sqrt{f'_c} b_w}{f_y}, \frac{50 b_w}{f_y} \right) \]  

(ACI 9.6.3.3, Table 9.6.3.3)

The maximum of all of the calculated \( A_v/s \) values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

### 16.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, \( T_u \).
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

#### 16.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 9.4.4.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the
design $T_u$ is permitted to be reduced in accordance with the code (ACI 22.7.3.3). However, the program does not automatically redistribute the internal forces and reduce $T_u$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

### 16.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as $A_{cp}$, $A_{oh}$, $A_o$, $p_{cp}$, and $p_h$, are calculated. These properties are described in the following (ACI 2.2).

$$A_{cp} = \text{Area enclosed by outside perimeter of concrete cross-section}$$

$$A_{oh} = \text{Area enclosed by centerline of the outermost closed transverse torsional reinforcement}$$

$$A_o = \text{Gross area enclosed by shear flow path}$$

$$p_{cp} = \text{Outside perimeter of concrete cross-section}$$

$$p_h = \text{Perimeter of centerline of outermost closed transverse torsional reinforcement}$$

In calculating the section properties involving reinforcement, such as $A_{oh}$, $A_o$, and $p_h$, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to a 1.5 inch clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during $T_c$ calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{cp} = bh$$  \hspace{1cm} (ACI 2.2, R22.7.5)

$$A_{oh} = (b - 2c)(h - 2c)$$  \hspace{1cm} (ACI 2.2, R22.7.7, Fig R22.7.6.1.1)

$$A_o = 0.85 A_{oh}$$  \hspace{1cm} (ACI 22.7.6.1.1, Fig R22.7.6.1.1)

$$p_{cp} = 2b + 2h$$  \hspace{1cm} (ACI 2.2, R22.7.5)

$$p_h = 2(b - 2c) + 2(h - 2c)$$  \hspace{1cm} (ACI 22.7.6.1.1, Fig R22.7.6.1.1)
where, the section dimensions $b$, $h$, and $c$ are shown in Figure 16-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_{cp} = b_w h + (b_f - b_w) h_f$$

$$A_{oh} = (b_w - 2c)(h - 2c)$$

$$A_o = 0.85 A_{oh}$$

$$p_{cp} = 2b_f + 2h$$

$$p_h = 2(h - 2c) + 2(b_w - 2c)$$

where the section dimensions $b_f$, $b_w$, $h$, $h_f$, and $c$ for a flanged beam are shown in Figure 16-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h - h_f)$.

16.5.3.3 Determine Critical Torsion Capacity

The threshold torsion limit, $T_{th}$, and the cracking torsion limits, $T_{cr}$, for which the torsion in the section can be ignored is calculated as:

$$T_{th} = \lambda \sqrt{f_c'} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f_c'}}}$$

$$T_{th} = 4\lambda \sqrt{f_c'} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f_c'}}}$$

where $A_{cp}$ and $p_{cp}$ are the area and perimeter of the concrete cross-section as described in detail in the previous section; $f_{pc}$ is the concrete compressive stress at the centroid of the section; $\phi$ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 21.2.1g, Table 21.2.1c); and $f_c'$ is the specified concrete compressive strength.

16.5.3.4 Determine Torsion Reinforcement

If the factored torsion $T_u$ is less than the threshold limit, $T_{cr}$, torsion can be safely ignored (ACI 22.7.1.1, 9.6.4.1). In that case, the program reports that no torsion reinforcement is required. However, if $T_u$ exceeds the threshold limit,
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$T_{cr}$, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI 22.7.1, 22.7.6.1). Note that the longitudinal reinforcement strength, $f_y$, and the transverse reinforcement strength, $f_{ty}$, are limited to 60 ksi (ACI 20.2.2.4a), even if the material property is defined with a higher value.

If $T_u > T_{cr}$, the required closed stirrup area per unit spacing, $A_t/s$, is calculated as:

$$\frac{A_t}{s} = \frac{T_u}{\phi 2 A_o f_{ty} \tan \theta}$$

(ACI 22.7.6.1)

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{T_u p_h}{\phi 2 A_o f_{ty} \tan \theta}$$

(ACI 22.7.6.1)

where, the minimum value of $A_t/s$ is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{ty}} b_w$$

(ACI 22.7.6.1)

the minimum value of $A_l$ is taken as the least of the following:

$$A_{l,\text{min}} \geq \frac{5 \sqrt{f_{ty}} A_{yp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \left( \frac{f_{ys}}{f_y} \right)$$

(ACI 9.6.4.3a)

$$A_{l,\text{min}} = \frac{5 \sqrt{f_{ty}} A_{yp}}{f_y} - \left( \frac{25}{f_{ys}} b_w \right) p_h \left( \frac{f_{ys}}{f_y} \right)$$

(ACI 9.6.4.3b)

In the preceding expressions, $\theta$ is taken as 45 degrees. The code allows any value between 30 and 60 degrees.
Figure 16-3 Closed stirrup and section dimensions for torsion design

An upper limit of the combination of $V_u$ and $T_u$ that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$$

(ACI 22.7.7.1a)

For rectangular sections, $b_w$ is replaced with $b$. If the combination of $V_u$ and $T_u$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s} + 2\frac{A_t}{s}\right) \geq \max \left\{0.75 \frac{\sqrt{f'_c}}{f_{ys}} b_w, \frac{50}{f_y} b_w\right\}$$

(ACI 9.6.4.2)
If this equation is not satisfied with the originally calculated $A'_{v}/s$ and $A'_{t}/s$, $A'_{v}/s$ is increased to satisfy this condition. In that case, $A'_{v}/s$ does not need to satisfy the ACI Section 9.6.6.3 independently.

The maximum of all of the calculated $A_{v}$ and $A_{t}/s$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

16.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (ACI 318-14) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the Key Features and Terminology manual.

16.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

16.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

16.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

16.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.6.1.1, 8.6.1.1):

\[
A_{s,\text{min}} = 0.0020 \times bh \text{ for } f_y < 60 \text{ ksi} \quad \text{(ACI Table 7.6.1.1, Table 8.6.1.1)}
\]

\[
A_{s,\text{min}} = 0.0018 \times bh \text{ for } f_y = 60 \text{ ksi} \quad \text{(ACI Table 7.6.1.1, Table 8.6.1.1)}
\]

\[
A_{s,\text{min}} = \frac{0.0018 \times 600000}{f_y} bh \text{ for } f_y > 60 \text{ ksi}(\text{ACI Table 7.6.1.1, Table 8.6.1.1})
\]
In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

16.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following sections.

16.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of \(\frac{d}{2}\) from the face of the support (ACI 22.6.4.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 22.6.4.3). Figure 16-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge or corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 16-4 Punching Shear Perimeters](image)

Figure 16-4 Punching Shear Perimeters
16.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be \( \gamma_f M_{sc} \) and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be \( \gamma_v M_{sc} \).

\[
\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \quad \text{(ACI 8.4.2.3)}
\]

\[
\gamma_v = 1 - \gamma_f \quad \text{(ACI 8.4.4.2.2)}
\]

For reinforced concrete slabs, \( \gamma_f \) is permitted to increase to the maximum modified values provided in ACI Table 8.4.2.3.4 provided that the limitations on \( \nu_{ug} \) and \( \varepsilon_t \) given in ACI Table 8.4.2.3.4 are satisfied.

<table>
<thead>
<tr>
<th>Column Location</th>
<th>Span Direction</th>
<th>( \nu_{ug} )</th>
<th>( \varepsilon_t )</th>
<th>Maximum modified ( \gamma_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner column</td>
<td>Either direction</td>
<td>( \leq 0.5\phi v_c )</td>
<td>( \geq 0.004 )</td>
<td>1.0</td>
</tr>
<tr>
<td>Edge column</td>
<td>Perpendicular to the edge</td>
<td>( \leq 0.75\phi v_c )</td>
<td>( \geq 0.004 )</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Parallel to the edge</td>
<td>( \leq 0.4\phi v_c )</td>
<td>( \geq 0.010 )</td>
<td>( \gamma_f = \frac{1.25}{1 + (2/3)\sqrt{b_1/b_2}} \leq 1.0 )</td>
</tr>
<tr>
<td>Interior column</td>
<td>Either direction</td>
<td>( \leq 0.4\phi v_c )</td>
<td>( \geq 0.010 )</td>
<td>( \gamma_f = \frac{1.25}{1 + (2/3)\sqrt{b_1/b_2}} \leq 1.0 )</td>
</tr>
</tbody>
</table>

where \( b_1 \) is the width of the critical section measured in the direction of the span and \( b_2 \) is the width of the critical section measured in the direction perpendicular to the span.

16.6.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:
SAFE Reinforced Concrete Design

\[
v_c = \min \left\{ \frac{2 + \frac{4}{\beta_c}}{\lambda \sqrt{f'_c}} \lambda \sqrt{f'_c} + \frac{\alpha c d}{b_o} \lambda \sqrt{f'_c} + 4 \lambda \sqrt{f'_c} \right\}
\]

(ACI 22.6.5.2)

where, \( \beta_c \) is the ratio of the maximum to the minimum dimensions of the critical section, \( b_o \) is the perimeter of the critical section, and \( \alpha_c \) is a scale factor based on the location of the critical section.

\[
\alpha_s = \begin{cases} 
40 & \text{for interior columns,} \\
30 & \text{for edge columns, and} \\
20 & \text{for corner columns.}
\end{cases}
\]

(ACI 22.6.65.3)

A limit is imposed on the value of \( \sqrt{f'_c} \) as:

\[
\sqrt{f'_c} \leq 100
\]

(ACI 22.5.3.1)

### 16.6.2.4 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

\[
v_U = \frac{V_U}{b o d} + \frac{\gamma v_2 [M_{U2} - V_U (y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^2} - \frac{\gamma v_3 [M_{U3} - V_U (x_3 - x_1)][J_{22}(x_4 - x_3) - J_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}^2}
\]

Eq. 1

\[
I_{22} = \sum_{sides=1}^{n} I_{22}, \text{ where "sides" refers to the sides of the critical section}
\]

for punching shear

Eq. 2

16 - 26 Slab Design
\[ \sum_{\text{sides}=1}^{n} T_{33}, \quad \text{where } "\text{sides}" \text{ refers to the sides of the critical section for punching shear} \quad \text{Eq. 3} \]

\[ \sum_{\text{sides}=1}^{n} T_{23}, \quad \text{where } "\text{sides}" \text{ refers to the sides of the critical section for punching shear} \quad \text{Eq. 4} \]

The equations for \( T_{22}, T_{33}, \) and \( T_{23} \) are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figure 16-5.

\[ T_{22} = Ld(y_2 - y_3)^2, \quad \text{for the side of the critical section parallel to the 2-axis} \quad \text{Eq. 5a} \]

\[ T_{22} = \frac{Ld^3}{12} + \frac{dL}{12} + Ld(y_2 - y_3)^2, \quad \text{for the side of the critical section parallel to the 3-axis} \quad \text{Eq. 5b} \]

\[ T_{33} = \frac{Ld^3}{12} + \frac{dL}{12} + Ld(x_2 - x_3)^2, \quad \text{for the side of the critical section parallel to the 2-axis} \quad \text{Eq. 6a} \]

\[ T_{33} = Ld(x_2 - x_3)^2, \quad \text{for the side of the critical section parallel to the 3-axis} \quad \text{Eq. 6b} \]
Figure 16-5 Shear Stress Calculations at Critical Sections

\[ T_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \]

Eq. 7

**NOTE:** \( T_{23} \) is explicitly set to zero for corner condition.

where,

- \( b_0 \) = Perimeter of the critical section for punching shear
- \( d \) = Effective depth at the critical section for punching shear based on the average of \( d \) for 2 direction and \( d \) for 3 direction
- \( I_{22} \) = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis
Chapter 16 - Design for ACI 318-14

\[ I_{33} = \text{Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis} \]

\[ I_{23} = \text{Product of the inertia of the critical section for punching shear with respect to the 2 and 3 planes} \]

\[ L = \text{Length of the side of the critical section for punching shear currently being considered} \]

\[ M_{U2} = \text{Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)} \]

\[ M_{U3} = \text{Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)} \]

\[ v_U = \text{Punching shear stress} \]

\[ V_U = \text{Shear at the center of the column (positive upward)} \]

\[ x_1, y_1 = \text{Coordinates of the column centroid} \]

\[ x_2, y_2 = \text{Coordinates of the center of one side of the critical section for punching shear} \]

\[ x_3, y_3 = \text{Coordinates of the centroid of the critical section for punching shear} \]

\[ x_4, y_4 = \text{Coordinates of the location where stress is being calculated} \]

\[ \gamma_{V2} = \text{Percent of } M_{U2} \text{ resisted by shear} \]

\[ \gamma_{V3} = \text{Percent of } M_{U3} \text{ resisted by shear} \]

**16.6.2.5 Determine Capacity Ratio**

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.
16.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 22.6.7.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The Critical Section for Punching Shear and Transfer of Unbalanced Moment as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

16.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

\[ v_c \leq 2\lambda \sqrt{f'_c} \] for shear links \hspace{1cm} (ACI 22.6.6.1)

\[ v_c \leq 3\lambda \sqrt{f'_c} \] for shear studs \hspace{1cm} (ACI 22.6.6.1)

16.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

\[ V_{\text{max}} = 6\sqrt{f'_c} b_o d \] for shear links \hspace{1cm} (ACI 22.6.6.2)

\[ V_{\text{max}} = 8\sqrt{f'_c} b_o d \] for shear studs \hspace{1cm} (ACI 22.6.6.2)

Given \( V_u \), \( V_c \), and \( V_{\text{max}} \), the required shear reinforcement is calculated as follows, where, \( \phi \), the strength reduction factor, is 0.75 (ACI 9.3.2.3).

\[ \frac{A}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} \] \hspace{1cm} (ACI 22.5.1.1, 22.5.10.1, 20.5.10.5.3)
\[ \frac{A_v}{s} \geq 2\sqrt{\frac{f_y b_o}{f_y}} \] for shear studs

- If \( V_u > \phi V_{\text{max}} \), a failure condition is declared. (ACI 22.5.1.2)
- If \( V_u \) exceeds the maximum permitted value of \( \phi V_{\text{max}} \), the concrete section should be increased in size.

### 16.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 16-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed \( d/2 \) (ACI 8.7.6.3, Table 8.7.6.3) and the spacing between shear reinforcement shall not exceed \( d/2 \) (ACI 8.7.6.3, Table 8.7.6.3). The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed \( 2d \) measured in a direction parallel to the column face (ACI 8.7.6.3, Table 8.7.6.3).
Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

16.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in 20.6.1.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, \( s_o \), between the column face and the first peripheral line of shear studs should not be smaller than 0.5\( d \). The spacing between adjacent shear studs, \( g \), at the first peripheral line of studs shall not exceed 2\( d \), and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of \( s_o \) and the spacing, \( s \), between the peripheral lines are specified as:

\[
\begin{align*}
  s_o & \leq 0.5d \\
  s & \leq \begin{cases} 
  0.75d & \text{for } v_u \leq 6\phi \sqrt{f'_c} \\
  0.50d & \text{for } v_u > 6\phi \sqrt{f'_c} 
\end{cases}
\end{align*}
\]

(ACI 8.7.7.1.2)

\[
  g \leq 2d
\]

(ACI 8.7.7.1.2)

The limits of \( s_o \) and the spacing, \( s \), between for the links are specified as:

\[
\begin{align*}
  s_o & \leq 0.5d \\
  s & \leq 0.50d
\end{align*}
\]

(ACI 8.7.6.3)
Chapter 17
Design for CSA A23.3-14

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Canadian code CSA A23.3-14 [CSA 14] is selected. Various notations used in this chapter are listed in Table 5-1. For referencing to the pertinent sections of the Canadian code in this chapter, a prefix “CSA” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

17.1 Notations

<table>
<thead>
<tr>
<th>Table 17-1 List of Symbols Used in the CSA A23.3-14 Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
</tr>
<tr>
<td>$A_{ct}$</td>
</tr>
</tbody>
</table>
### Table 17-1 List of Symbols Used in the CSA A23.3-14 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_l$</td>
<td>Area of longitudinal reinforcement for torsion, sq-mm</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Gross area enclosed by shear flow path, sq-mm</td>
</tr>
<tr>
<td>$A_{oh}$</td>
<td>Area enclosed by centerline of outermost closed transverse torsional reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Area of tension reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>Area of compression reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_{s(\text{required})}$</td>
<td>Area of steel required for tension reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_{i/s}$</td>
<td>Area of closed shear reinforcement for torsion per unit length, sq-mm/mm</td>
</tr>
<tr>
<td>$A_{v}$</td>
<td>Area of shear reinforcement, sq-mm</td>
</tr>
<tr>
<td>$A_{v/s}$</td>
<td>Area of shear reinforcement per unit length, sq-mm/mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Depth of compression block, mm</td>
</tr>
<tr>
<td>$a_b$</td>
<td>Depth of compression block at balanced condition, mm</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of member, mm</td>
</tr>
<tr>
<td>$b_f$</td>
<td>Effective width of flange (flanged section), mm</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web (flanged section), mm</td>
</tr>
<tr>
<td>$b_0$</td>
<td>Perimeter of the punching critical section, mm</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Width of the punching critical section in the direction of bending, mm</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Width of the punching critical section perpendicular to the direction of bending, mm</td>
</tr>
<tr>
<td>$c$</td>
<td>Depth to neutral axis, mm</td>
</tr>
<tr>
<td>$c_b$</td>
<td>Depth to neutral axis at balanced conditions, mm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from compression face to tension reinforcement, mm</td>
</tr>
<tr>
<td>$d_v$</td>
<td>Effective shear depth, mm</td>
</tr>
<tr>
<td>$d'$</td>
<td>Distance from compression face to compression reinforcement, mm</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Thickness of slab (flanged section), mm</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete, MPa</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of reinforcement, assumed as 200,000 MPa</td>
</tr>
<tr>
<td>$f'c$</td>
<td>Specified compressive strength of concrete, MPa</td>
</tr>
<tr>
<td>$f's$</td>
<td>Stress in the compression reinforcement, psi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Specified yield strength of flexural reinforcement, MPa</td>
</tr>
<tr>
<td>$f_{yt}$</td>
<td>Specified yield strength of shear reinforcement, MPa</td>
</tr>
<tr>
<td>$h$</td>
<td>Overall depth of a section, mm</td>
</tr>
<tr>
<td>$I_g$</td>
<td>Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Factored moment at section, N-mm</td>
</tr>
<tr>
<td>$N_f$</td>
<td>Factored axial force at section, N</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Outside perimeter of concrete cross-section, mm</td>
</tr>
<tr>
<td>$p_h$</td>
<td>Perimeter of area $A_{oh}$, mm</td>
</tr>
<tr>
<td>$s$</td>
<td>Spacing of the shear reinforcement along the length of the beam, mm</td>
</tr>
<tr>
<td>$s_z$</td>
<td>Crack spacing parameter</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Factored torsion at section, N-mm</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear resisted by concrete, N</td>
</tr>
<tr>
<td>$V_{c,\text{max}}$</td>
<td>Maximum permitted total factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_f$</td>
<td>Factored shear force at a section, N</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Shear force at a section resisted by steel, N</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Ratio of average stress in rectangular stress block to the specified concrete strength</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Factor accounting for shear resistance of cracked concrete</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Factor for obtaining depth of compression block in concrete</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Ratio of the maximum to the minimum dimensions of the punching critical section</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Strain in concrete</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Strain in reinforcing steel</td>
</tr>
</tbody>
</table>
Table 17-1 List of Symbols Used in the CSA A23.3-14 Code

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_x$</td>
<td>Longitudinal strain at mid-depth of the section</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Strength reduction factor for concrete</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Strength reduction factor for steel</td>
</tr>
<tr>
<td>$\phi_m$</td>
<td>Strength reduction factor for member</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Fraction of unbalanced moment transferred by flexure</td>
</tr>
<tr>
<td>$\gamma_e$</td>
<td>Fraction of unbalanced moment transferred by eccentricity of shear</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of diagonal compressive stresses, degrees</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shear strength factor</td>
</tr>
</tbody>
</table>

17.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For CSA A23.3-14, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (CSA 8.3.2, Table C.1a)

1.4D  
1.25D + 1.5L  
1.25D + 1.5L + 1.0S  
1.25D + 1.5L ± 0.4W  
0.9D + 1.5L  
0.9D + 1.5L + 1.0S  
0.9D + 1.5L ± 0.4W  
0.9D + 1.5L + 0.5L  
0.9D + 1.5L ± 0.4W  
1.25D + 1.5(0.75 PL)  
1.25D + 1.5S  
1.25D + 1.5S + 0.5L  
1.25D + 1.5S ± 0.4W  
0.9D + 1.5S  
0.9D + 1.5S + 0.5L
0.9D + 1.5S ± 0.4W
1.25D ± 1.4W
1.25D + 0.5L ± 1.4W
1.25D + 0.5S ± 1.4W
0.9D ± 1.4W
0.9D + 0.5L ± 1.4W
0.9D + 0.5S ± 1.4W
1.25D + 0.5L ± 1.4W
1.25D + 0.5S ± 1.4W
0.9D + 0.5L ± 1.4W
0.9D + 0.5S ± 1.4W
1.0D ± 1.0E
1.0D + 0.5L ± 1.0E
1.0D + 0.25S ± 1.0E
1.0D + 0.5L + 0.25S ± 1.0E

These are also the default design load combinations in SAFE whenever the CSA A23.3-14 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

17.3 Limits on Material Strength

The upper and lower limits of $f'_c$ are 80 MPa and 20 MPa, respectively, for all framing types (CSA 8.6.1.1).

\[ 20 \text{ MPa} \leq f'_c \leq 80 \text{ MPa} \quad (\text{CSA 8.6.1.1}) \]

The upper limit of $f_y$ is 500 MPa for all frames (CSA 8.5.1).

\[ f_y \leq 500 \text{ MPa} \quad (\text{CSA 8.5.1}) \]

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

17.4 Strength Reduction Factors

The strength reduction factors, $\phi$, are material dependent and defined as:
\[ \phi_c = 0.65 \text{ for concrete} \quad \text{(CSA 8.4.2)} \]
\[ \phi_s = 0.85 \text{ for reinforcement} \quad \text{(CSA 8.4.3a)} \]

These values can be overwritten; however, caution is advised.

### 17.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

#### 17.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

#### 17.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by
factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom reinforcement. In such cases the beam may be designed as a rectangular or flanged beam. Negative beam moments produce top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

17.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 17-1 (CSA 10.1.7). Furthermore, it is assumed that the compression carried by the concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed \((0.1 f'_{c} A_{g})\), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than \(0.1 f'_{c} A_{g}\) and axial tensions are always included in flexural and shear design.

17.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, \(M_{f}\) (i.e., designing top or bottom reinforcement), the depth of the compression block is given by \(a\) (see Figure 17-1), where,
Figure 17-1 Rectangular Beam Design

\[ a = d - \sqrt{d^2 - \frac{2|M_f|}{\alpha_i f' \phi_b b}} \]  

where the value of \( \phi_k \) is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters \( \alpha_i, \beta_i, \) and \( c_b \) are calculated as:

\[ \alpha_i = 0.85 - 0.0015 f' c \geq 0.67, \]  

(CSA 10.1.7)

\[ \beta_i = 0.97 - 0.0025 f' c \geq 0.67, \]  

(CSA 10.1.7)

\[ c_b = \frac{700}{700 + f_y} d \]  

(CSA 10.5.2)

The balanced depth of the compression block is given by:

\[ a_b = \beta_i c_b \]  

(CSA 10.1.7)
If $a \leq a_b$ (CSA 10.5.2), the area of tension reinforcement is given by:

$$A_t = \frac{M_f}{\phi_s f_y \left( d - \frac{a}{2} \right)}$$

This reinforcement is to be placed at the bottom if $M_f$ is positive, or at the top if $M_f$ is negative.

If $a > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The factored compressive force developed in the concrete alone is given by:

$$C = \phi c \alpha f'_c ba_b$$  \hspace{1cm} (CSA 10.1.7)

and the factored moment resisted by concrete compression and tension reinforcement is:

$$M_{fc} = C \left( d - \frac{a_b}{2} \right)$$

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{fs} = M_f - M_{fc}$$

The required compression reinforcement is given by:

$$A'_c = \frac{M_{fs}}{(\phi_s f'_s - \phi_s \alpha f'_t)(d - d')}, \text{ where}$$

$$f'_s = 0.0035 E_s \left[ \frac{c - d'}{c} \right] \leq f_y$$  \hspace{1cm} (CSA 10.1.2, 10.1.3)

The required tension reinforcement for balancing the compression in the concrete is:
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\[ A_{s1} = \frac{M_{fc}}{f_y \left( d - \frac{a_p}{2} \right) \phi_s} \]

and the tension reinforcement for balancing the compression reinforcement is given by:

\[ A_{s2} = \frac{M_{fc}}{f_y \left( d - d' \right) \phi_s} \]

Therefore, the total tension reinforcement, \( A_s = A_{s1} + A_{s2} \), and the total compression reinforcement is \( A'_s \). \( A_s \) is to be placed at the bottom and \( A'_s \) is to be placed at the top if \( M_f \) is positive, and vice versa if \( M_f \) is negative.

### 17.5.1.2.2 Design of Flanged Beams

#### 17.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, \( M_f \) (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

#### 17.5.1.2.2.2 Flanged Beam Under Positive Moment

- If \( M_f > 0 \), the depth of the compression block is given by:

\[ a = d - \sqrt{d^2 - \frac{2M_f}{\alpha f'c' \beta_i b_f}} \]  

(CSA 10.1)

where, the value of \( \phi_c \) is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters \( \alpha_t, \beta_t \), and \( c_b \) are calculated as:

\[ \alpha_t = 0.85 - 0.0015 f'_c \geq 0.67, \]  

(CSA 10.1.7)

\[ \beta_t = 0.97 - 0.0025 f'_c \geq 0.67, \]  

(CSA 10.1.7)

\[ c_b = \frac{700}{700 + f_y} d \]  

(CSA 10.5.2)
The balanced depth of the compression block is given by:

\[ a_b = \beta v c_b \]  

(CSA 10.1.4, 10.1.7)

- If \( a \leq h_s \), the subsequent calculations for \( A_s \) are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as \( b_f \). Compression reinforcement is required when \( a > a_b \).

- If \( a > h_s \), calculation for \( A_s \) has two parts. The first part is for balancing the compressive force from the flange, \( C_f \), and the second part is for balancing the compressive force from the web, \( C_w \) as shown in Figure 17-2. \( C_f \) is given by:

\[ C_f = \alpha f'_s (b_f - b_w) \min(h_s, a_b) \]  

(CSA 10.1.7)

Therefore, \( A_{si} = \frac{C_f \phi_c}{f_y \phi_s} \) and the portion of \( M_f \) that is resisted by the flange is given by:
Therefore, the balance of the moment, $M_f$ to be carried by the web is:

$$M_{fw} = M_f - M_{ff}$$

The web is a rectangular section with dimensions $b_w$ and $d$, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f_c' \phi b_w}}$$  \hspace{1cm} (CSA 10.1)

- If $a_1 \leq a_b$ (CSA 10.5.2), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{fw}}{\phi_b f_y (d - a_1/2)}$$

$$A_s = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The compressive force in the web concrete alone is given by:

$$C = \phi_c \alpha_1 f_c' b_w a_b$$  \hspace{1cm} (CSA 10.1.7)

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{fc} = C \left( d - \frac{a_b}{2} \right)$$

and the moment resisted by compression and tension reinforcement is:

$$M_{fc} = M_{fw} - M_{fc}$$
Therefore, the compression reinforcement is computed as:

\[ A'_c = \frac{M_{fs}}{(\phi f'_c - \phi \alpha f'_c)(d - d')} \], where

\[ f'_c = \varepsilon E_s \left[ \frac{c - d''}{c} \right] \leq f_y \]  

(CSA 10.1.2, 10.1.3)

The tension reinforcement for balancing compression in the web concrete is:

\[ A_{s2} = \frac{M_{fc}}{f'_y \left( d - \frac{a_b}{2} \right) \phi_s} \]

and the tension reinforcement for balancing the compression reinforcement is:

\[ A_{s3} = \frac{M_{fs}}{f'_y (d - d') \phi_s} \]

The total tension reinforcement is \( A_s = A_{s1} + A_{s2} + A_{s3} \), and the total compression reinforcement is \( A'_c \). \( A_s \) is to be placed at the bottom and \( A'_c \) is to be placed at the top.

### 17.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the following two limits:

\[ A_s \geq \frac{0.2 \sqrt{f'_c}}{f_y} b_w h \]  

(CSA 10.5.1.2)

\[ A_s \geq \frac{4}{3} A_s(\text{required}) \]  

(CSA 10.5.1.3)

In addition, the minimum flexural tension reinforcement provided in a flanged beam with the flange under tension in an ordinary moment resisting frame is given by the limit:
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\[ A_s \geq 0.004 (b - b_w) h_s \]  
(CSA 10.5.3.1)

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

\[
\begin{align*}
    A_s \leq & \begin{cases} 
    0.04bd & \text{Rectangular beam} \\
    0.04b_wd & \text{Flanged beam} 
    \end{cases} \\
    A'_s \leq & \begin{cases} 
    0.04bd & \text{Rectangular beam} \\
    0.04b_wd & \text{Flanged beam} 
    \end{cases}
\end{align*}
\]

17.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, \( V_f \).
- Determine the shear force, \( V_c \), that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sub-sections describe in detail the algorithms associated with these steps.

17.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

17.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, \( V_c \), is calculated as:

\[ V_c = \phi \lambda \beta \sqrt{f'_c b_w d_v} \]  
(CSA 11.3.4)

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\[
\sqrt{f'_c} \leq 8 \text{ MPa} \quad \text{(CSA 11.3.4)}
\]

\(\phi_c\) is the resistance factor for concrete. By default it is taken as 0.65 (CSA 8.4.2).

\(\lambda\) is the strength reduction factor to account for low density concrete (CSA 2.2). For normal density concrete, its value is 1 (CSA 8.6.5), which is taken by the program as the default value. For concrete using lower density aggregate, the user can change the value of \(\lambda\) in the material property data. The recommended value for \(\lambda\) is as follows (CSA 8.6.5):

\[
\lambda = \begin{cases} 
1.00, & \text{for normal density concrete}, \\
0.85, & \text{for semi-low-density concrete in which all of the fine aggregate is natural sand}, \\
0.75, & \text{for semi-low-density concrete in which none of the fine aggregate is natural sand}. 
\end{cases} \quad \text{(CSA 8.6.5)}
\]

\(\beta\) is the factor for accounting for the shear resistance of cracked concrete (CSA 2.2) and should be greater or equal to 0.05. Its value is normally between 0.1 and 0.4. It is determined according to CSA 11.3.6, and described further in the following sections.

\(b_w\) is the effective web width. For rectangular beams, it is the width of the beam. For flanged beams, it is the width of the web of the beam.

\(d_v\) is the effective shear depth. It is taken as the greater of 0.9\(d\) or 0.72\(h\) (CSA 2.3), where \(d\) is the distance from the extreme compression fiber to the centroid of the tension reinforcement, and \(h\) is the overall depth of the cross-section in the direction of the shear force (CSA 2.3).

The value of \(\beta\) is preferably taken as the special value (CSA 11.3.6.2) or it is determined using the simplified method (CSA 11.3.6.3), if applicable. When the conditions of the special value or simplified method do not apply, the general method is used (CSA 11.3.6.4).

If the overall beam depth, \(h\), is less than 250 mm or if the depth of a flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, \(\beta\) is taken as 0.21 (CSA 11.3.6.2).
When the specified yield strength of the longitudinal reinforcing $f_y$ does not exceed 400 MPa, the specified concrete strength $f'_{c}$ does not exceed 60 MPa, and the tensile force is negligible, $\beta$ is determined in accordance with the simplified method, as follows (CSA 11.3.6.3):

- When the section contains at least the minimum transverse reinforcement, $\beta$ is taken as 0.18 (CSA 11.6.3.3a).

$$\beta = 0.18 \quad \text{(CSA 11.3.6.3(a))}$$

- When the section contains no transverse reinforcement, $\beta$ is determined based on the specified maximum nominal size of coarse aggregate, $a_g$.

For a maximum size of coarse aggregate not less than 20 mm, $\beta$ is taken as:

$$\beta = \frac{230}{1000 + d_v} \quad \text{(CSA 11.3.6.3(b))}$$

where $d_v$ is the effective shear depth expressed in millimeters.

For a maximum size of coarse aggregate less than 20 mm, $\beta$ is taken as:

$$\beta = \frac{230}{1000 + s_{ze}} \quad \text{(CSA 11.3.6.3 c)}$$

where

$$s_{ze} = \frac{35s_z}{15 + a_g} \geq 0.85s_z \quad \text{(CSA 11.3.6.3.c)}$$

In the preceding expression, the crack spacing parameter, $s_{ze}$, shall be taken as the minimum of $d_v$ and the maximum distance between layers of distributed longitudinal reinforcement. However, $s_{ze}$ is conservatively taken as equal to $d_v$.

In summary, for simplified cases, $\beta$ can be expressed as follows:
When the specified yield strength of the longitudinal reinforcing \( f_y \) is greater than 400 MPa, the specified concrete strength \( f' c \) is greater than 60 MPa, or tension is not negligible, \( \beta \) is determined in accordance with the general method as follows (CSA 11.3.6.1, 11.3.6.4):

\[
\beta = \frac{0.40}{(1+1500\varepsilon_x)} \left( \frac{1300}{(1000+S_{ze})} \right)
\]  

(CSA 11.3.6.4)

In the preceding expression, the equivalent crack spacing parameter, \( s_{ze} \) is taken equal to 300 mm if minimum transverse reinforcement is provided (CSA 11.3.6.4). Otherwise it is determined as stated in the simplified method.

\[
S_{ze} = \begin{cases} 
300 & \text{if minimum transverse reinforcement is provided,} \\
\frac{35}{15+a_g}S_z \geq 0.85S_z & \text{otherwise.}
\end{cases}
\]  

(CSA 11.3.6.3, 11.3.6.4)

The value of \( a_g \) in the preceding equations is taken as the maximum aggregate size for \( f' c \) of 60 MPa, is taken as zero for \( f' c \) of 70 MPa, and linearly interpolated between these values (CSA 11.3.6.4).

The longitudinal strain, \( \varepsilon_x \) at mid-depth of the cross-section is computed from the following equation:

\[
\varepsilon_x = \frac{M_f / d_e + V_f + 0.5N_f}{2(E_A)}
\]  

(CSA 11.3.6.4)

In evaluating \( \varepsilon_x \) the following conditions apply:

- \( \varepsilon_x \) is positive for tensile action.
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- $V_f$ and $M_f$ are taken as positive quantities. (CSA 11.3.6.4(a))
- $M_f$ is taken as a minimum of $V_f d_v$. (CSA 11.3.6.4(a))
- $N_f$ is taken as positive for tension. (CSA 2.3)

$A_r$ is taken as the total area of longitudinal reinforcement in the beam. It is taken as the envelope of the reinforcement required for all design load combinations. The actual provided reinforcement might be slightly higher than this quantity. The reinforcement should be developed to achieve full strength (CSA 11.3.6.3(b)).

If the value of $\varepsilon_x$ is negative, it is recalculated with the following equation, in which $A_{ct}$ is the area of concrete in the flexural tensile side of the beam, taken as half of the total area.

$$
\varepsilon_x = \frac{M_f / d_v + V_f + 0.5 N_f}{2(\varepsilon_s A_s + \varepsilon_c A_{ct})}
$$

(6)

$$E_s = 200,000 \text{ MPa} \quad \text{(CSA 8.5.4.1)}$$

$$E_c = 4500\sqrt{f'_c} \text{ MPa} \quad \text{(CSA 8.6.2.3)}$$

If the axial tension is large enough to induce tensile stress in the section, the value of $\varepsilon_x$ is doubled (CSA 11.3.6.4(e)).

For sections closer than $d_v$ from the face of the support, $\varepsilon_x$ is calculated based on $M_f$ and $V_f$ at a section at a distance $d_v$ from the face of the support (CSA 11.3.6.4(d)). This condition currently is not checked by SAFE.

An upper limit on $\varepsilon_x$ is imposed as:

$$\varepsilon_x \leq 0.003 \quad \text{(CSA 11.3.6.4(f))}$$

In both the simplified and general methods, the shear strength of the section due to concrete, $v_c$, depends on whether the minimum transverse reinforcement is provided. To check this condition, the program performs the design in two passes. In the first pass, it assumes that no transverse shear reinforcement is needed. When the program determines that shear reinforcement is required, the
program performs the second pass assuming that at least minimum shear reinforcement is provided.

17.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to $V_{r,\text{max}}$, where:

$$ V_{r,\text{max}} = 0.25\phi f'c b_w d $$

(CSA 11.3.3)

Given $V_f$, $V_c$, and $V_{r,\text{max}}$, the required shear reinforcement is calculated as follows:

- If $V_f \leq V_c$,
  $$ \frac{A_x}{s} = 0 $$
  (CSA 11.3.5.1)

- If $V_c < V_f \leq V_{r,\text{max}}$,
  $$ \frac{A_x}{s} = \frac{(V_f - V_c)\tan\theta}{\phi_f f_{ys} d_y} $$
  (CSA 11.3.3, 11.3.5.1)

- If $V_f > V_{r,\text{max}}$, a failure condition is declared. (CSA 11.3.3)

A minimum area of shear reinforcement is provided in the following regions (CSA 11.2.8.1):

(a) in regions of flexural members where the factored shear force $V_f$ exceeds $V_c$

(b) in regions of beams with an overall depth greater than 750 mm

(c) in regions of beams where the factored torsion $T_f$ exceeds $0.25 T_{cr}$.

Where the minimum shear reinforcement is required by CSA 11.2.8.1, or by calculation, the minimum area of shear reinforcement per unit spacing is taken as:
In the preceding equations, the term $\theta$ is used, where $\theta$ is the angle of inclination of the diagonal compressive stresses with respect to the longitudinal axis of the member (CSA 2.3). The $\theta$ value is normally between 22 and 44 degrees. It is determined according to CSA 11.3.6.

Similar to the $\beta$ factor, which was described previously, the value of $\theta$ is preferably taken as the special value (CSA 11.3.6.2), or it is determined using the simplified method (CSA 11.3.6.3), whenever applicable. The program uses the general method when conditions for the simplified method are not satisfied (CSA 11.3.6.4).

- If the overall beam depth, $h$, is less than 250 mm or if the depth of the flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, $\theta$ is taken as 42 degrees (CSA 11.3.6.2).

- If the specified yield strength of the longitudinal reinforcing $f_y$ does not exceed 400 MPa, or the specified concrete strength $f'_c$ does not exceed 60 MPa, $\theta$ is taken to be 35 degree (CSA 11.3.6.3).

\[
\theta = 35^\circ \text{ for } P_f \leq 0 \text{ or } f_y \leq 400 \text{ MPa or } f'_c \leq 60 \text{ MPa} \quad \text{(CSA11.3.6.3)}
\]

- If the axial force is tensile, the specified yield strength of the longitudinal reinforcing $f_y > 400$ MPa, and the specified concrete strength $f'_c > 60$ MPa, $\theta$ is determined using the general method as follows (CSA 11.3.6.4),

\[
\theta = 29 + 7000\varepsilon_x \text{ for } P_f < 0, \quad f_y > 400 \text{ MPa, } f'_c \leq 60 \text{ MPa} \quad \text{(CSA11.3.6.4)}
\]

where $\varepsilon_x$ is the longitudinal strain at the mid-depth of the cross-section for the factored load. The calculation procedure is described in preceding sections.

The maximum of all of the calculated $A_v/s$ values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup require-
17.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, $T_f$.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

17.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design $T_f$ is permitted to be reduced in accordance with the code (CSA 11.2.9.2). However, the program does not automatically redistribute the internal forces and reduce $T_f$. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

17.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as $A_c$, $A_{oh}$, $A_o$, $p_c$, and $p_h$, are calculated. These properties are described in the following (CSA 2.3).

\[ A_c = \text{Area enclosed by outside perimeter of concrete cross-section} \]
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\( A_{oh} = \) Area enclosed by centerline of the outermost closed transverse torsional reinforcement

\( A_o = \) Gross area enclosed by shear flow path

\( p_c = \) Outside perimeter of concrete cross-section

\( p_h = \) Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as \( A_{oh}, A_o, \) and \( p_h, \) it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 millimeters. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during \( T_{cr} \) calculation. With this assumption, the special properties for a rectangular beam section are given as follows:

\[
A_c = bh \quad \text{(CSA 11.2.9.1)}
\]

\[
A_{oh} = (b - 2c)(h - 2c) \quad \text{(CSA 11.3.10.3)}
\]

\[
A_o = 0.85 A_{oh} \quad \text{(CSA 11.3.10.3)}
\]

\[
p_c = 2b + 2h \quad \text{(CSA 11.2.9.1)}
\]

\[
p_h = 2(b - 2c) + 2(h - 2c) \quad \text{(CSA 11.3.10.4)}
\]

where, the section dimensions \( b, h, \) and \( c \) are shown in Figure 17-3. Similarly, the special section properties for a flanged beam section are given as follows:

\[
A_c = b_u h + (b_f - b_u) h_s \quad \text{(CSA 11.2.9.1)}
\]

\[
A_{oh} = (b_u - 2c)(h - 2c) \quad \text{(CSA 11.3.10.3)}
\]

\[
A_o = 0.85 A_{oh} \quad \text{(CSA 11.3.10.3)}
\]

\[
p_c = 2b_f + 2h \quad \text{(CSA 11.2.9.1)}
\]

\[
p_h = 2(h - 2c) + 2(b_f - 2c) \quad \text{(CSA 11.3.10.4)}
\]
where the section dimensions \( b_f, b_w, h, h_f, \) and \( c \) for a flanged beam are shown in Figure 17-3. Note that the flange width on either side of the beam web is limited to the smaller of \( 6h_s \) or \( 1/12 \) the span length (CSA 10.3.4).

17.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, \( T_{cr} \), for which the torsion in the section can be ignored, is calculated as:

\[
T_{cr} = \frac{0.38 \lambda \phi_c \sqrt{f'_c} \left( \frac{A_p^2}{p_c} \right)}{4}
\]

where \( A_{cp} \) and \( p_c \) are the area and perimeter of the concrete cross-section as described in the previous section, \( \lambda \) is a factor to account for low-density concrete, \( \phi_c \) is the strength reduction factor for concrete, which is equal to 0.65, and \( f'_c \) is the specified concrete compressive strength.

17.5.3.4 Determine Torsion Reinforcement

If the factored torsion \( T_f \) is less than the threshold limit, \( T_{cr} \), torsion can be safely ignored (CSA 11.2.9.1). In that case, the program reports that no torsion reinforcement is required. However, if \( T_f \) exceeds the threshold limit, \( T_{cr} \), it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CSA 11.3).

- If \( T_f > T_{cr} \), the required closed stirrup area per unit spacing, \( A_t/s \), is calculated as:

\[
\frac{A_t}{s} = \frac{T_f \tan \theta}{\phi_c 2A_o f_{yt}}
\]

and the required longitudinal reinforcement is calculated as:

\[
A_t = \frac{M_f + 0.5N_f + \sqrt{(V_f - 0.5V_s)^2 + \left( \frac{0.45p_b T_f}{2A_o} \right)^2 \cot \theta}}{\phi_s f_y}
\]

(CSA 11.3.10.6, 11.3.9)
In the preceding expressions, $\theta$ is computed as previously described for shear, except that if the general method is being used, the value $\varepsilon_x$, calculated as specified in CSA 11.3.6.4, is replaced by:

$$
\varepsilon_x = \frac{M_f}{d_v} + \sqrt{V_f^2 + \left(0.9 p_h T_f \frac{2A_o}{E_o A_y}\right)^2 + 0.5N_f}
$$

(CSA 11.3.10.5)

An upper limit of the combination of $V_u$ and $T_u$ that can be carried by the section also is checked using the equation:

$$
\sqrt{\left(\frac{V_f}{b.w.d_c}\right)^2 + \left(\frac{T_f p_h}{1.7 A_{oh}^2}\right)^2} \leq 0.25 \phi_c f'_c
$$

(CSA 11.3.10.4(b))

For rectangular sections, $b_w$ is replaced with $b$. If the combination of $V_f$ and $T_f$ exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.
When torsional reinforcement is required \((T_f > T_c)\), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit:

\[
\left( \frac{A_v}{s} + 2 \frac{A_t}{s} \right) \geq 0.06 \sqrt{f'_c} \frac{b_w}{f_{yu}}
\]  

(CSA 11.2.8.2)

If this equation is not satisfied with the originally calculated \(A_v/s\) and \(A_t/s\), \(A_v/s\) is increased to satisfy this condition.

The maximum of all of the calculated \(A_t\) and \(A_v/s\) values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

### 17.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is performed based on the ultimate strength design method (CSA A23.3-14) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

#### 17.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.
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The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

17.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

17.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

17.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limit (CSA 13.10.1):
As \[ A_v \geq 0.002 \, bh \] (CSA 7.8.1)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

17.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the Key Features and Terminology manual. Only the code-specific items are described in the following subsections.

17.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of \( d/2 \) from the face of the support (CSA 13.3.3.1 and CSA 13.3.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.3.3.3). Figure 17-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

![Figure 17-4 Punching Shear Perimeters](image)
17.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}}, \quad \text{and}$$

$$\gamma_v = 1 - \frac{1}{1 + (2/3)\sqrt{b_1/b_2}},$$

where $b_1$ is the width of the critical section measured in the direction of the span, and $b_2$ is the width of the critical section measured in the direction perpendicular to the span.

17.6.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_v = \min \left\{ \phi_c \left(1 + \frac{2}{B_c}\right) 0.19\sqrt{f'_c}, \right.\left. \phi_c \left(0.19 + \frac{\alpha_s d}{b_0}\right) \lambda\sqrt{f'_c}, \right.\left. \phi_c 0.38\lambda\sqrt{f'_c} \right\}$$

where, $B_c$ is the ratio of the minimum to the maximum dimensions of the critical section, $b_0$ is the perimeter of the critical section, and $\alpha_s$ is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4, \text{ for interior columns} \\ 3, \text{ for edge columns}, \text{ and} \\ 2, \text{ for corner columns}. \end{cases}$$

(CSA 13.3.4.1(b))
The value of $\sqrt{f'_c}$ is limited to 8 MPa for the calculation of the concrete shear capacity (CSA 13.3.4.2).

If the effective depth, $d$, exceeds 300 mm, the value of $v_c$ is reduced by a factor equal to $1300/(1000 + d)$ (CSA 13.3.4.3).

**17.6.2.4 Determine Maximum Shear Stress**

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

$$v_f = \frac{V_f}{bod} + \frac{\gamma v_2 [M_{f2} - V_f (y_3 - y_1)] [I_{33} (y_4 - y_3) - I_{23} (x_4 - x_3)]}{I_{22} I_{33} - I_{23}^2} - \frac{\gamma v_3 [M_{f3} - V_f (x_3 - x_1)] [I_{22} (x_4 - x_3) - I_{23} (y_4 - y_3)]}{I_{22} I_{33} - I_{23}^2}$$

Eq. 1

$$I_{22} = \sum_{sides=1}^{n} T_{22}, \text{ where "sides" refers to the sides of the critical section for punching shear}$$

Eq. 2

$$I_{33} = \sum_{sides=1}^{n} T_{33}, \text{ where "sides" refers to the sides of the critical section for punching shear}$$

Eq. 3

$$I_{23} = \sum_{sides=1}^{n} T_{23}, \text{ where "sides" refers to the sides of the critical section for punching shear}$$

Eq. 4

The equations for $T_{22}, T_{33},$ and $T_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 5-5.
Figure 17-5 Shear Stress Calculations at Critical Sections

\[ \bar{T}_{22} = Ld(y_2 - y_1)^2, \] for side of critical section parallel to 2-axis \hspace{1cm} \text{Eq. 5a} \\

\[ \bar{T}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_1)^2, \] for side of critical section parallel to 3-axis \hspace{1cm} \text{Eq. 5b} \\

\[ \bar{T}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2, \] for side of critical section parallel to 2-axis \hspace{1cm} \text{Eq. 6a} \\

\[ \bar{T}_{33} = Ld(x_2 - x_3)^2, \] for side of critical section parallel to 3-axis \hspace{1cm} \text{Eq. 6b}
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\[ \bar{T}_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \]

**Eq. 7**

**NOTE:** \( \bar{T}_{23} \) is explicitly set to zero for corner condition.

where,

- \( b_0 \) = Perimeter of the critical section for punching shear
- \( d \) = Effective depth at the critical section for punching shear based on the average of \( d \) for 2 direction and \( d \) for 3 direction
- \( I_{22} \) = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis
- \( I_{33} \) = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis
- \( I_{23} \) = Product of inertia of the critical section for punching shear with respect to the 2 and 3 planes
- \( L \) = Length of the side of the critical section for punching shear currently being considered
- \( M_{f2} \) = Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)
- \( M_{f3} \) = Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)
- \( V_f \) = Punching shear stress
- \( V_f \) = Shear at the center of the column (positive upward)
- \( x_1, y_1 \) = Coordinates of the column centroid
- \( x_2, y_2 \) = Coordinates of the center of one side of the critical section for punching shear
- \( x_3, y_3 \) = Coordinates of the centroid of the critical section for punching shear
- \( x_4, y_4 \) = Coordinates of the location where stress is being calculated
\[ \gamma_2 = \text{Percent of } M_2 \text{ resisted by shear} \]
\[ \gamma_3 = \text{Percent of } M_3 \text{ resisted by shear} \]

**17.6.2.5 Determine Capacity Ratio**

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

**17.6.3 Design Punching Shear Reinforcement**

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CSA 13.2.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed, and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as explained in the subsections that follow.

**17.6.3.1 Determine Concrete Shear Capacity**

The concrete punching shear stress capacity of a section with punching shear reinforcement is taken as:

\[ v_c = 0.28 \lambda \phi_c \sqrt{f_c'} \text{ for shear studs} \]  
(CSA 13.3.8.3)

\[ v_c = 0.19 \lambda \phi_c \sqrt{f_c'} \text{ for shear stirrups} \]  
(CSA 13.3.9.3)

**17.6.3.2 Determine Required Shear Reinforcement**

The shear force is limited to a maximum of \( v_{r,\text{max}} \), where
$v_{r,\text{max}} = 0.75 \lambda f_c \sqrt{f'_c}$ for shear studs \hfill (CSA 13.3.8.2)

$v_{r,\text{max}} = 0.55 \lambda f_c \sqrt{f'_c}$ for shear stirrups \hfill (CSA 13.3.9.2)

Given $v_f$, $v_c$, and $v_{f,\text{max}}$, the required shear reinforcement is calculated as follows, where, $\phi_s$ is the strength reduction factor.

- If $v_f > v_{r,\text{max}}$,

$$A_v = \frac{(v_f - v_c)}{\phi_s f_{fy}} b_o$$ \hfill (CSA 13.3.8.5, 13.3.9.4)

- If $v_f > v_{r,\text{max}}$, a failure condition is declared. \hfill (CSA 13.3.8.2)

- If $v_f$ exceeds the maximum permitted value of $v_{\text{max}}$, the concrete section should be increased in size.

### 17.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 17-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.
Figure 17-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed

- $0.4d$ for shear studs \hspace{1cm} (CSA 13.3.8.6)
- $0.25d$ for shear stirrups \hspace{1cm} (CSA 13.3.8.6)

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

17.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CSA 7.9 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, $s_o$, between the column face and the first peripheral line of shear studs should not be smaller than $0.45d$ to $0.4d$. The limits of $s_o$ and the spacing, $s$, between the peripheral lines are specified as:

$$s_o \leq 0.4d \hspace{1cm} \text{(CSA 13.3.8.6)}$$

$$s \begin{cases} 0.75d & \nu_f \leq 0.56\lambda\phi_c\sqrt{f'c} \\ 0.50d & \nu_f > 0.56\lambda\phi_c\sqrt{f'c} \end{cases} \hspace{1cm} \text{(CSA 13.3.8.6)}$$

For shear stirrups,

$$s_o \leq 0.25d \hspace{1cm} \text{(CSA 13.3.9.5)}$$

$$s \leq 0.25d \hspace{1cm} \text{(CSA 13.3.9.5)}$$

The minimum depth for reinforcement should be limited to 300 mm (CSA 13.3.9.1).
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