COMPUTERS & STRUCTURES, INC.

STRUCTURAL AND EARTHQUAKE ENGINEERING SOFTWARE

SAFE® 2016

Design of Slabs, Beams and Foundations
Reinforced and Post-Tensioned Concrete

Reinforced Concrete Design

COMPUTERS & STRUCTURES INC.

SAFE®

DESIGN OF SLABS, BEAMS AND FOUNDATIONS,
REINFORCED AND POST-TENSIONED CONCRETE

Reinforced Concrete Design Manual

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Chapter 1
Introduction

SAFE automates several slab and mat design tasks. Specifically, it integrates slab design moments across design strips and designs the required reinforcement; it checks slab punching shear around column supports and concentrated loads; and it designs beam flexural, shear, and torsion reinforcement. The design procedures are outlined in the chapter entitled "SAFE Design Features" in the *Key Features and Terminology* manual. The actual design algorithms vary based on the specific design code chosen by the user. This manual describes the algorithms used for the various codes.

It should be noted that the design of reinforced concrete slabs is a complex subject and the design codes cover many aspects of this process. SAFE is a tool to help the user in this process. Only the aspects of design documented in this manual are automated by SAFE design capabilities. The user must check the results produced and address other aspects not covered by SAFE.

Chapter 2
Design for ACI 318-08

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code ACI 318-08 [ACI 2008] is selected. Various notations used in this chapter are listed in Table 2-1. For referencing to the pertinent sections or equations of the ACI code in this chapter, a prefix “ACI” followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

2.1 Notations

Table 2-1 List of Symbols Used in the ACI 318-08 Code

A_{cp}	Area enclosed by the outside perimeter of the section, sq-in
A_g	Gross area of concrete, sq-in

Table 2-1 List of Symbols Used in the ACI 318-08 Code

A_l	Area of longitudinal reinforcement for torsion, sq-in
A_o	Area enclosed by the shear flow path, sq-in
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in
A_s	Area of tension reinforcement, sq-in
A'_s	Area of compression reinforcement, sq-in
A_t /s	Area of closed shear reinforcement per unit length of member for torsion, sq-in/in
A_v	Area of shear reinforcement, sq-in
A_v /s	Area of shear reinforcement per unit length, sq-in/in
a	Depth of compression block, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of section, in
b_f	Effective width of flange (flanged section), in
b_o	Perimeter of the punching shear critical section, in
b_w	Width of web (flanged section), in
b_1	Width of the punching shear critical section in the direction of bending, in
b_2	Width of the punching shear critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in
d	Distance from compression face to tension reinforcement, in
d'	Distance from compression face to compression reinforcement, in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, psi
f'_c	Specified compressive strength of concrete, psi

Table 2-1 List of Symbols Used in the ACI 318-08 Code

f'_s	Stress in the compression reinforcement, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{yt}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
h_f	Height of the flange, in
M_u	Factored moment at a section, lb-in
N_u	Factored axial load at a section occurring simultaneously with V_u or T_u, lb
P_u	Factored axial load at a section, lb
p_{cp}	Outside perimeter of concrete cross-section, in
p_h	Perimeter of centerline of outermost closed transverse torsional reinforcement, in
s	Spacing of shear reinforcement along the beam, in
T_{cr}	Critical torsion capacity, lb-in
T_u	Factored torsional moment at a section, lb-in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_s	Shear force resisted by transverse reinforcement, lb
V_u	Factored shear force at a section, lb
α_s	Punching shear scale factor based on column location
β_c	Ratio of the maximum to the minimum dimensions of the punching shear critical section
β_1	Factor for obtaining depth of the concrete compression block
ε_c	Strain in the concrete
$\varepsilon_{c\ \max}$	Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)

Table 2-1 List of Symbols Used in the ACI 318-08 Code

ε_s	Strain in the reinforcement
$\varepsilon_{s,min}$	Minimum tensile strain allowed in the reinforcement at nominal strength for tension controlled behavior (0.005 in/in)
ϕ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
λ	Shear strength reduction factor for light-weight concrete
θ	Angle of compression diagonals, degrees

2.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-08, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (ACI 9.2.1):

$$1.4D \quad \text{(ACI 9-1)}$$

$$1.2D + 1.6L \quad \text{(ACI 9-2)}$$

$$1.2D + 1.6\,(0.75\ PL) \quad \text{(ACI 13.7.6.3, 9-2)}$$

$$0.9D \pm 1.6W \quad \text{(ACI 9-6)}$$
$$1.2D + 1.0L \pm 1.6W \quad \text{(ACI 9-4)}$$

$$0.9D \pm 1.0E \quad \text{(ACI 9-7)}$$
$$1.2D + 1.0L \pm 1.0E \quad \text{(ACI 9-5)}$$

$$1.2D + 1.6L + 0.5S \quad \text{(ACI 9-2)}$$
$$1.2D + 1.0L + 1.6S \quad \text{(ACI 9-3)}$$
$$1.2D + 1.6S \pm 0.8W \quad \text{(ACI 9-3)}$$
$$1.2D + 1.0L + 0.5S \pm 1.6W \quad \text{(ACI 9-4)}$$
$$1.2D + 1.0L + 0.2S \pm 1.0E \quad \text{(ACI 9-5)}$$

These are the default design load combinations in SAFE whenever the ACI 318-08 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

2.3 Limits on Material Strength

The concrete compressive strength, f'_c, should not be less than 2,500 psi (ACI 5.1.1). If the input f'_c is less than 2,500 psi, SAFE continues to design the members based on the input f'_c and does not warn the user about the violation of the code. The user is responsible for ensuring that the minimum strength is satisfied.

2.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied to the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

ϕ = 0.90 for flexure (tension controlled) (ACI 9.3.2.1)

ϕ = 0.75 for shear and torsion (ACI 9.3.2.3)

These values can be overwritten; however, caution is advised.

2.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

2.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

2.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

2.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, f_y, is limited to 80 ksi (ACI 9.4), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 2-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 10.3.4) when the concrete in compression reaches its assumed strain limit of 0.003. When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed $\left(0.1 f'_c A_g\right)$ (ACI 10.3.5), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $\left(0.1 f'_c A_g\right)$ and axial tensions are always included in flexural and shear design.

2.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M_u (i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 2-1), where,

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f'_c \phi b}} \qquad \text{(ACI 10.2)}$$

and the value of ϕ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, $c_{\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{s\min}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} d \qquad \text{(ACI 10.2.2)}$$

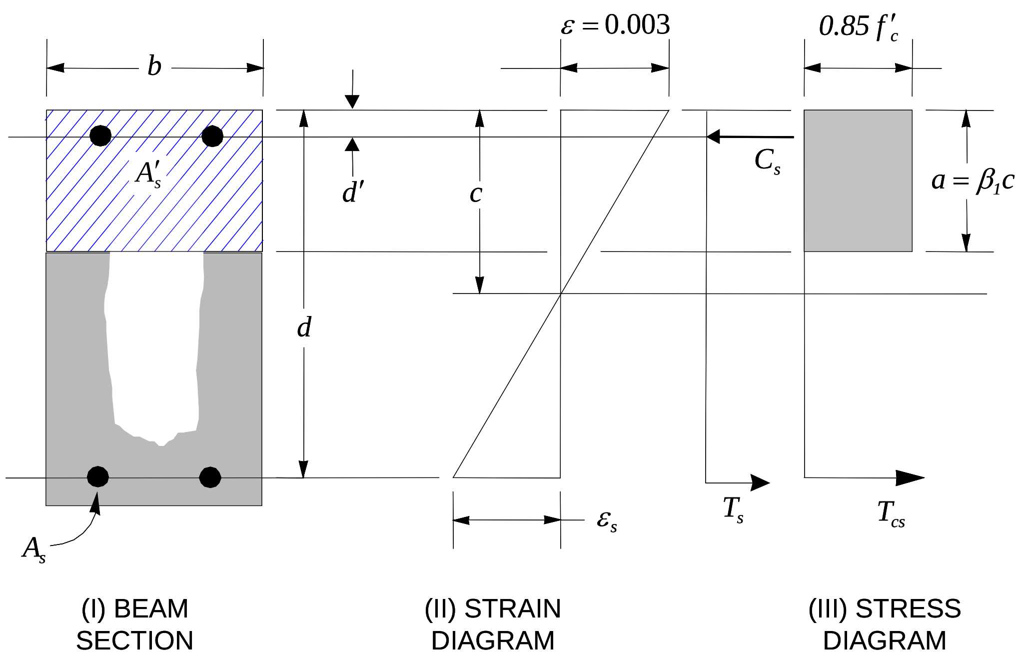


Figure 2-1 Rectangular Beam Design

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le a_{max}$ (ACI 10.3.4), the area of tension reinforcement is then given by:

$$A_s = \frac{M_u}{\phi\, f_y\left(d - \frac{a}{2}\right)}$$

This reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

- If $a > a_{\max}$, compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

  − The compressive force developed in the concrete alone is given by:

  $$C = 0.85 f'_c\, b a_{\max} \qquad \text{(ACI 10.2.7.1)}$$

  and the moment resisted by concrete compression and tension reinforcement is:

  $$M_{uc} = \phi C\left(d - \frac{a_{\max}}{2}\right) \qquad \text{(ACI 9.3.2.1)}$$

  − Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

  $$M_{us} = M_u - M_{uc}$$

  − The required compression reinforcement is given by:

  $$A'_s = \frac{M_{us}}{\phi\left(f'_s - 0.85 f'_c\right)\left(d - d'\right)}, \text{ where}$$

  $$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y \qquad \text{(ACI 10.2.2, 10.2.3, 10.2.4)}$$

  − The required tension reinforcement for balancing the compression in the concrete is:

  $$A_{s1} = \frac{M_{uc}}{\phi f_y\left[d - \frac{a_{\max}}{2}\right]}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{us}}{\phi f_y \left(d - d'\right)}$$

Therefore, the total tension reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

2.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 2-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 2-1 is assumed on the compression side.

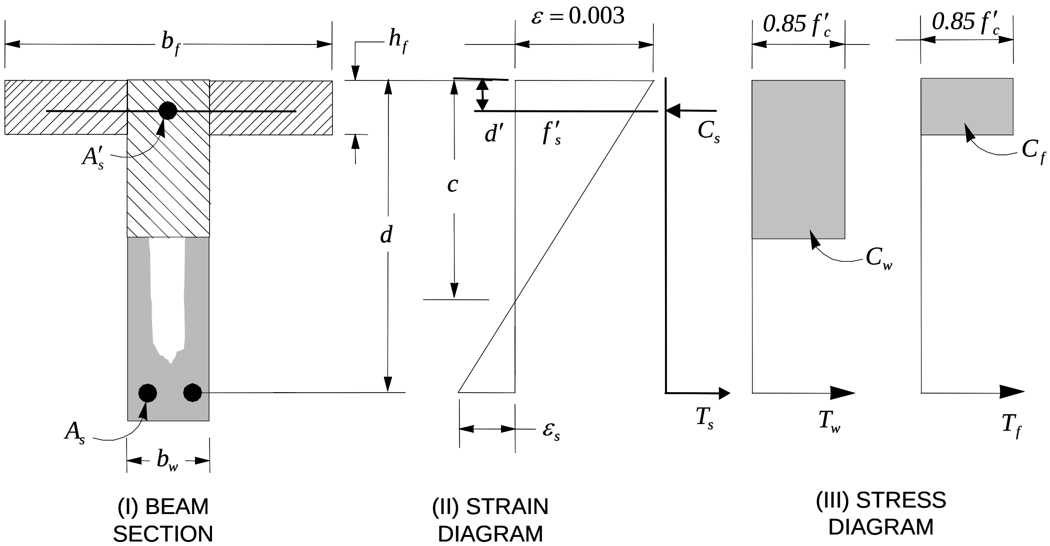


Figure 2-2 T-Beam Design

2.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

2.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85 f'_c \phi b_f}} \qquad \text{(ACI 10.2)}$$

where, the value of ϕ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, $c_{\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{s\min}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} d \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{c\max} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{s\min} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \beta_1 c_{\max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_{max}$.

- If $a > h_f$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 2-2. C_f is given by:

$$C_f = 0.85 f'_c \left(b_f - b_w\right) \min\left(h_f, a_{\max}\right) \qquad \text{(ACI 10.2.7.1)}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of M_u that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min\left(h_f, a_{\max}\right)}{2} \right) \qquad \text{(ACI 9.3.2.1)}$$

Again, the value for ϕ is 0.90 by default. Therefore, the balance of the moment, M_u, to be carried by the web is:

$$M_{uw} = M_u - M_{uf}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi b_w}} \qquad \text{(ACI 10.2)}$$

- If $a_1 \le a_{\max}$ (ACI 10.3.4), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_{\max}$, compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

  − The compressive force in the web concrete alone is given by:

$$C_w = 0.85 f'_c b_w a_{\max} \qquad \text{(ACI 10.2.7.1)}$$

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{uc} = C_w \left(d - \frac{a_{\max}}{2} \right) \phi$$

and the moment resisted by compression and tension reinforcement is:

$$M_{us} = M_{uw} - M_{uc}$$

Therefore, the compression reinforcement is computed as:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c \right)\left(d - d' \right) \phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max} \left[\frac{c_{\max} - d'}{c_{\max}} \right] \le f_y \qquad \text{(ACI 10.2.2, 10.2.3, 10.2.4)}$$

The tension reinforcement for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{uc}}{f_y \left[d - \frac{a_{\max}}{2} \right] \phi}$$

and the tension reinforcement for balancing the compression reinforcement is:

$$A_{s3} = \frac{M_{us}}{f_y \left(d - d' \right) \phi}$$

The total tension reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

2.5.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the two following limits:

$$A_{s,\min} = \max\left(\frac{3\sqrt{f'_c}}{f_y} b_w d, \frac{200}{f_y} b_w d\right) \qquad \text{(ACI 10.5.1)}$$

$$A_s \geq \frac{4}{3} A_{s(\text{required})} \qquad \text{(ACI 10.5.3)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

2.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

2.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

2.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = 2\lambda\sqrt{f'_c}\,b_w d \qquad \text{(ACI 11.2.1.2, 11.2.1.2, 11.2.2.3)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as $f'_c \le 100$ (ACI 11.1.2)

The value of λ should be specified in the material property definition.

2.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = V_c + \left(8\sqrt{f'_c}\right)b_w d \qquad \text{(ACI 11.4.7.9)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3). The flexural reinforcement strength, f_{yt}, is limited to 60 ksi (ACI 11.5.2) even if the material property is defined with a higher value.

- If $V_u \le 0.5\phi V_c$,

$$\frac{A_v}{s} = 0 \qquad \text{(ACI 11.5.6.1)}$$

- If $0.5\phi V_c < V_u \le \phi V_{\max}$,

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi\, f_{yt} d} \qquad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared. (ACI 11.4.7.9)

If V_u exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size (ACI 11.4.7.9).

Note that if torsion design is considered and torsion reinforcement is required, the equation given in ACI 11.5.6.3 does not need to be satisfied independently. See the subsequent section *Design of Beam Torsion Reinforcement* for details.

If the beam depth h is

$h \le 10"$ for rectangular,

$h \le \min\left\{24", \max\left(2.5h_f, \frac{b}{2}\right)\right\}$ for T-beam,

the minimum shear reinforcement given by ACI 11.4.6.3 is not enforced (ACI 11.4.6.1).

$$\frac{A_v}{s} \ge \max\left(\frac{0.75\lambda\sqrt{f'_c}}{f_{yt}} b_w, \quad \frac{50b_w}{f_{yt}}\right) \qquad \text{(ACI 11.4.6.3)}$$

The maximum of all of the calculated A_v/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

2.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_u.
- Determine special section properties.

- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

2.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 11.6.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with the code (ACI 11.6.2.2). However, the program does not automatically redistribute the internal forces and reduce T_u. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

2.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{cp}, A_{oh}, A_o, p_{cp}, and p_h, are calculated. These properties are described in the following (ACI 2.1).

A_{cp} = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_{cp} = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to a 1.5 inch clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{cp} = bh \qquad \text{(ACI 11.5.1, 2.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1, R11.5.3.6(b))}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(ACI 11.5.3.6, 2.1)}$$

$$p_{cp} = 2b + 2h \qquad \text{(ACI 11.5.1, 2.1)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1)}$$

where, the section dimensions b, h, and c are shown in Figure 2-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_{cp} = b_w h + (b_f - b_w)h_f \qquad \text{(ACI 11.5.1, 2.1)}$$

$$A_{oh} = (b_w - 2c)(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1, R11.5.3.6(b))}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(ACI 11.5.3.6, 2.1)}$$

$$p_{cp} = 2b_f + 2h \qquad \text{(ACI11.5.1, 2.1)}$$

$$p_h = 2(h - 2c) + 2(b_w - 2c) \qquad \text{(ACI 11.5.3.1, 2.1)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 2-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h - h_f)$ (ACI 13.2.4).

2.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \phi\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1 + \frac{N_u}{4A_g\lambda\sqrt{f'_c}}} \qquad \text{(ACI 11.5.1(c))}$$

where A_{cp} and p_{cp} are the area and perimeter of the concrete cross-section as described in the previous section, N_u is the factored axial force (compression positive), ϕ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 9.3.2.3), and f'_c is the specified concrete compressive strength.

2.5.3.4 Determine Torsion Reinforcement

If the factored torsion T_u is less than the threshold limit, T_{cr}, torsion can be safely ignored (ACI 11.6.1). In that case, the program reports that no torsion reinforcement is required. However, if T_u exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI R11.6.3.6). Note that the longitudinal reinforcement strength, f_y, is limited to 80 ksi (ACI 9.4) and the transverse reinforcement strength, f_{yt}, is limited to 60 ksi, even if the material property is defined with a higher value.

If $T_u > T_{cr}$, the required closed stirrup area per unit spacing, A_t /s, is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan\theta}{\phi 2 A_o f_{yt}} \qquad \text{(ACI 11.5.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{T_u p_h}{\phi 2 A_o f_y \tan\theta} \qquad \text{(ACI 11.5.3.7, 11.5.3.6)}$$

where, the minimum value of A_t /s is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{yt}} b_w \qquad \text{(ACI 11.5.5.3)}$$

and the minimum value of A_l is taken as:

$$A_l = \frac{5\lambda\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yt}}{f_y}\right) \qquad \text{(ACI 11.5.5.3)}$$

In the preceding expressions, θ is taken as 45 degrees. The code allows any value between 30 and 60 degrees (ACI 11.5.3.6).

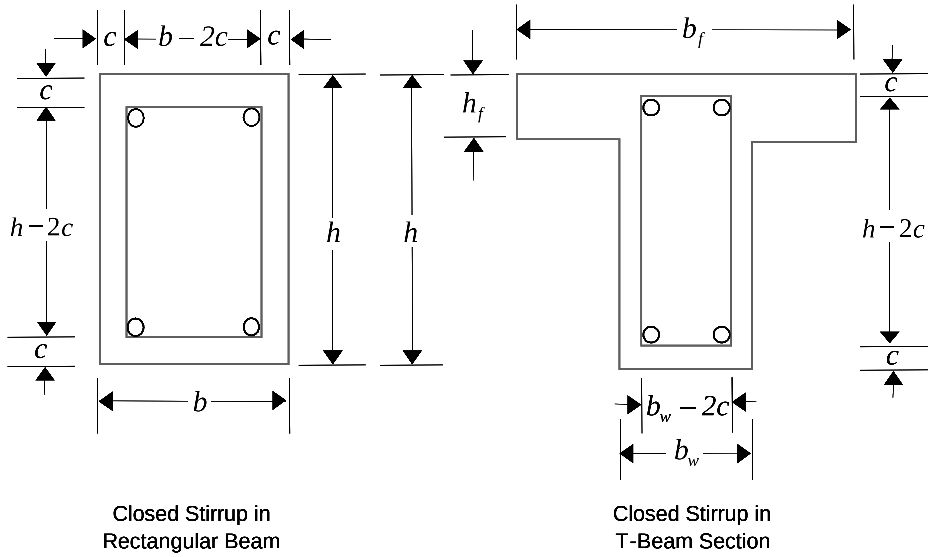


Figure 2-3 Closed stirrup and section dimensions for torsion design

An upper limit of the combination of V_u and T_u that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad \text{(ACI 11.5.3.1)}$$

For rectangular sections, b_w is replaced with b. If the combination of V_u and T_u exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s} + 2\frac{A_t}{s}\right) \geq \max\left\{0.75\lambda\frac{\sqrt{f'_c}}{f_{yt}}b_w, \frac{50b_w}{f_y}\right\} \qquad \text{(ACI 11.5.5.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition. In that case, A_v/s does not need to satisfy the ACI Section 11.4.6.3 independently.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

2.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (ACI 318-08) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

2.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.

- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

2.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

2.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

2.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2):

$$A_{s,\min} = 0.0020\, bh \text{ for } f_y = 40 \text{ ksi or } 50 \text{ ksi} \qquad \text{(ACI 7.12.2.1(a))}$$

$$A_{s,\min} = 0.0018\, bh \text{ for } f_y = 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(b))}$$

$$A_{s,\min} = \frac{0.0018 \times 60000}{f_y} bh \text{ for } f_y > 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(c))}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

2.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following sections.

2.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (ACI 11.11.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.11.1.3). Figure 2-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

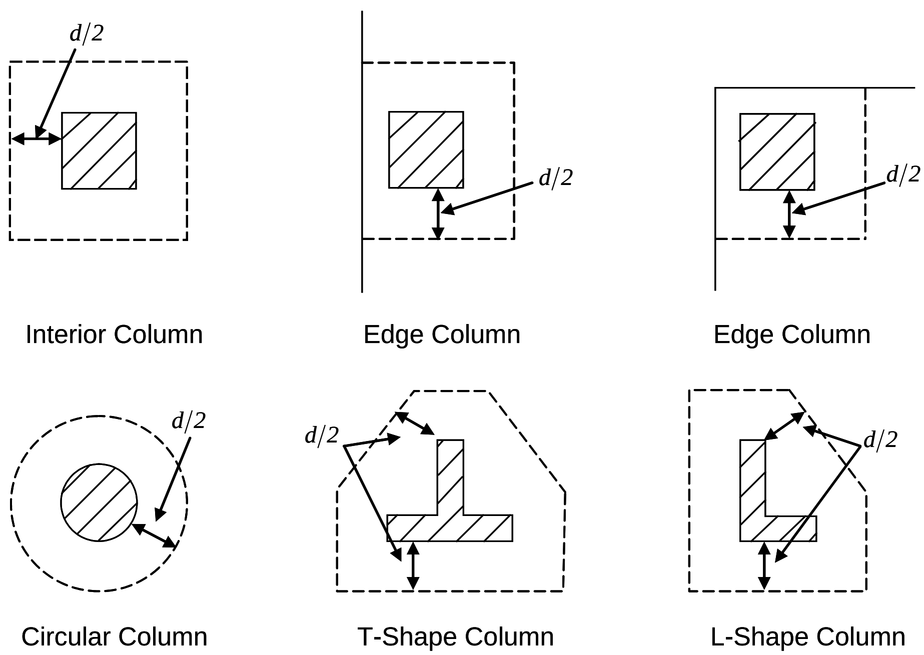


Figure 2-4 Punching Shear Perimeters

2.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$.

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \quad \text{(ACI 13.5.3.2)}$$

$$\gamma_v = 1 - \gamma_f \quad \text{(ACI 13.5.3.1)}$$

For flat plates, γ_v is determined from the following equations taken from ACI 421.2R-07 *Seismic Design of Punching Shear Reinforcement in Flat Plates* [ACI 2007].

For interior columns,

$$\gamma_{vx} = 1 - \frac{1}{1+(2/3)\sqrt{l_y/l_x}} \quad \text{(ACI 421.2 C-11)}$$

$$\gamma_{vy} = 1 - \frac{1}{1+(2/3)\sqrt{l_x/l_y}} \quad \text{(ACI 421.2 C-12)}$$

For edge columns,

γ_{vx} = same as for interior columns (ACI 421.2 C-13)

$$\gamma_{vy} = 1 - \frac{1}{1+(2/3)\sqrt{l_x/l_y - 0.2}} \quad \text{(ACI 421.2 C-14)}$$

$\gamma_{vy} = 0$ when $l_x/l_y \le 0.2$

For corner columns,

$\gamma_{vx} = 0.4$ (ACI 421.2 C-15)

γ_{vy} = same as for edge columns (ACI 421.2 C-16)

NOTE: Program uses ACI 421.2-12 and ACI 421.2-15 equations in lieu of ACI 421.2 C-14 and ACI 421.2 C-16 which are currently NOT enforced.

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span. The values l_x and l_y are the projections of the shear-critical section onto its principal axes, x and y, respectively.

2.6.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \phi\left(2+\dfrac{4}{\beta_c}\right)\lambda\sqrt{f'_c} \\ \phi\left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda\sqrt{f'_c} \\ \phi 4\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

where, β_c is the ratio of the maximum to the minimum dimensions of the critical section, b_o is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as:

$$\sqrt{f'_c} \le 100 \qquad \text{(ACI 11.1.2)}$$

2.6.2.4 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

$$v_U = \frac{V_U}{b_0 d} + \frac{\gamma_{V2}[M_{U2} - V_U(y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}{}^2} - \frac{\gamma_{V3}[M_{U3} - V_U(x_3 - x_1)][I_{22}(x_4 - x_3) - I_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}{}^2} \quad \text{Eq. 1}$$

$I_{22} = \sum_{sides=1}^{n} \overline{I}_{22}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 2

$I_{33} = \sum_{sides=1}^{n} \overline{I}_{33}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 3

$I_{23} = \sum_{sides=1}^{n} \overline{I}_{23}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 4

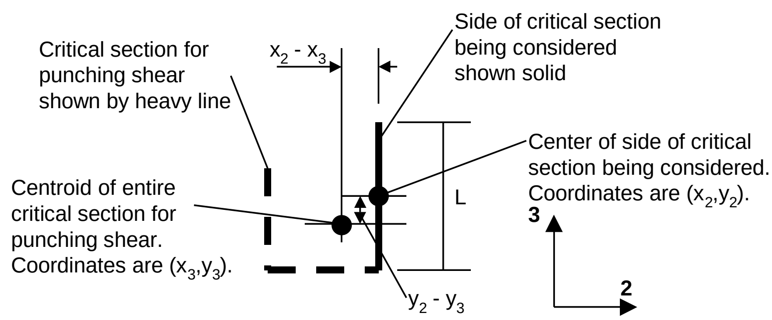
The equations for $\overline{I}_{22}, \overline{I}_{33}$, and $\overline{I}_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figure 2-5.

$\overline{I}_{22} = Ld(y_2 - y_3)^2$, for the side of the critical section parallel to the 2-axis Eq. 5a

$\overline{I}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2$, for the side of the critical section parallel to the 3-axis Eq. 5b

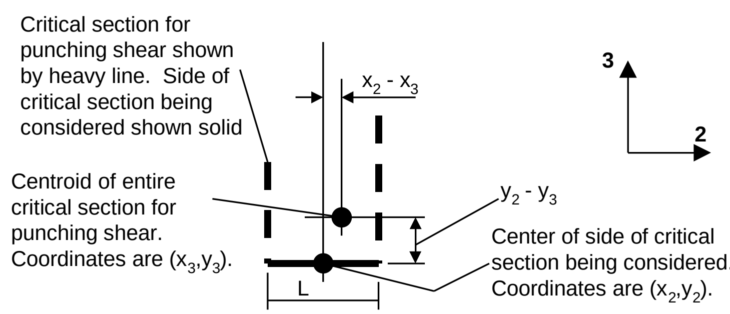
$\overline{I}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2$, for the side of the critical section parallel to the 2-axis Eq. 6a

$\overline{I}_{33} = Ld(x_2 - x_3)^2$, for the side of the critical section parallel to the 3-axis Eq. 6b



Plan View For Side of Critical Section Parallel to 3-Axis

Work This Sketch With Equations 5b, 6b and 7



Plan View For Side of Critical Section Parallel to 2-Axis

Work This Sketch With Equations 5a, 6a and 7

Figure 2-5 Shear Stress Calculations at Critical Sections

$$\overline{I}_{23} = Ld(x_2 - x_3)(y_2 - y_3),$$ for side of critical section parallel to 2-axis or 3-axis Eq. 7

NOTE: $\overline{I}_{23}$ is explicitly set to zero for corner condition.

where,

b_0 = Perimeter of the critical section for punching shear

d = Effective depth at the critical section for punching shear based on the average of d for 2 direction and d for 3 direction

I_{22} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis

I_{33} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis

I_{23} = Product of the inertia of the critical section for punching shear with respect to the 2 and 3 planes

L = Length of the side of the critical section for punching shear currently being considered

M_{U2} = Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)

M_{U3} = Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)

v_U = Punching shear stress

V_U = Shear at the center of the column (positive upward)

x_1, y_1 = Coordinates of the column centroid

x_2, y_2 = Coordinates of the center of one side of the critical section for punching shear

x_3, y_3 = Coordinates of the centroid of the critical section for punching shear

x_4, y_4 = Coordinates of the location where stress is being calculated

γ_{V2} = Percent of M_{U2} resisted by shear

γ_{V3} = Percent of M_{U3} resisted by shear

2.6.2.5 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

2.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 11.11.3). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

2.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

$$v_c \leq \phi 2\lambda\sqrt{f'_c} \text{ for shear links} \qquad \text{(ACI 11.11.3.1)}$$

$$v_c \leq \phi 3\lambda\sqrt{f'_c} \text{ for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

2.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{max} = 6\sqrt{f'_c}\, b_o d \text{ for shear links} \qquad \text{(ACI 11.11.3.2)}$$

$$V_{max} = 8\sqrt{f'_c}\, b_o d \text{ for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

Given V_u, V_c, and V_{max}, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} \qquad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

$$\frac{A_v}{s} \geq \frac{2\sqrt{f'_c}\, b_o}{f_y} \quad \text{for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared. (ACI 11.11.3.2)
- If V_u exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size.

2.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 2-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

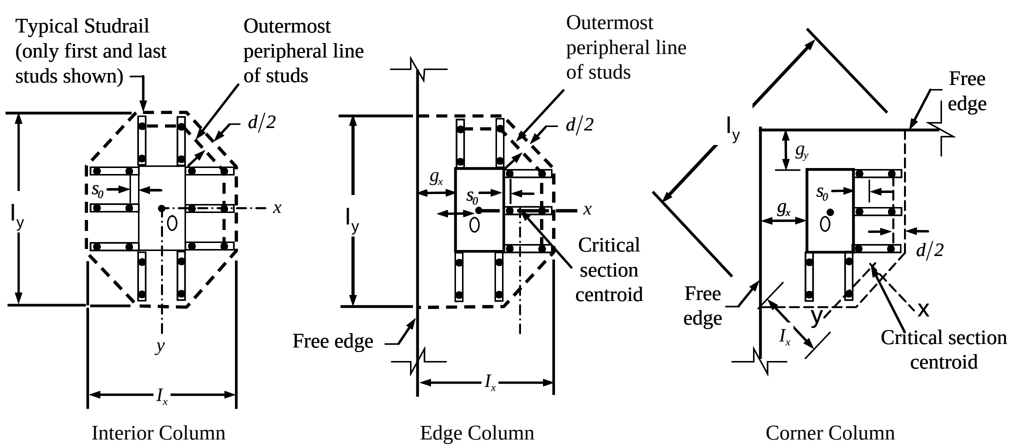


Figure 2-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$(ACI R11.3.3, 11.11.5.2. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

2.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 2*d*, and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.5.2)}$$

$$s \leq \begin{cases} 0.75d & \text{for} \quad v_u \leq 6\phi\lambda\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.5.2)}$$

$$g \leq 2d \qquad \text{(ACI 11.11.5.3)}$$

The limits of s_o and the spacing, *s*, between for the links are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.3)}$$

$$s \leq 0.50d \qquad \text{(ACI 11.11.3)}$$

Chapter 3
Design for AS 3600-01

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Australian code AS 3600-2001 [AS 2001] is selected. Various notations used in this chapter are listed in Table 3-1. For referencing to the pertinent sections of the AS code in this chapter, a prefix "AS" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

3.1 Notations

Table 3-1 List of Symbols Used in the AS 3600-2001 Code

A_g	Gross area of concrete, mm^2
A_l	Area of longitudinal reinforcement for torsion, mm^2
A_s	Area of tension reinforcement, mm^2

Table 3-1 List of Symbols Used in the AS 3600-2001 Code

A_{sc}	Area of compression reinforcement, mm^2
A_{st}	Area of tension reinforcement, mm^2
$A_{s(\text{required})}$	Area of required tension reinforcement, mm^2
A_{sv}	Area of shear reinforcement, mm^2
$A_{sv,\min}$	Minimum area of shear reinforcement, mm^2
A_{sv}/s	Area of shear reinforcement per unit length, mm^2 /mm
A_{sw}/s	Area of shear reinforcement per unit length consisting of closed ties, mm^2/mm
A_t	Area of a polygon with vertices at the center of longitudinal bars at the corners of a section, mm^2
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
$a_{\max}$	Maximum allowed depth of compression block, mm
b	Width of member, mm
b_{ef}	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
c	Depth to neutral axis, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to compression reinforcement, mm
d_o	Distance from the extreme compression fiber to the centroid of the outermost tension reinforcement, mm
d_{om}	Mean value of d_o, averaged around the critical shear perimeter, mm
D	Overall depth of a section, mm
D_s	Thickness of slab (flanged section), mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, MPa
f'_c	Specified compressive strength of concrete, MPa
f'_{cf}	Characteristic flexural tensile strength of concrete, MPa

Table 3-1 List of Symbols Used in the AS 3600-2001 Code

f_{cv}	Concrete shear strength, MPa
f_{sy}	Specified yield strength of flexural reinforcement, MPa
$f_{sy,f}$	Specified yield strength of shear reinforcement, MPa
f'_s	Stress in the compression reinforcement, MPa
J_t	Torsional modulus, mm^3
k_u	Ratio of the depth to the neutral axis from the compression face, to the effective depth, d
M_{ud}	Reduced ultimate strength in bending without axial force, N-mm
M^*	Factored moment at section, N-mm
N^*	Factored axial load at section, N
s	Spacing of shear reinforcement along the beam, mm
T_{uc}	Torsional strength of section without torsional reinforcement, N-mm
$T_{u.\max}$	Maximum permitted total factored torsion at a section, N-mm
T_{us}	Torsion strength of section with torsion reinforcement, N-mm
T^*	Factored torsional moment at a section, N-mm
u_t	Perimeter of the polygon defined by A_t, mm
V^*	Factored shear force at a section, N
$V_{u.\max}$	Maximum permitted total factored shear force at a section, N
$V_{u.\min}$	Shear strength provided by minimum shear reinforcement, N
V_{uc}	Shear force resisted by concrete, N
V_{us}	Shear force resisted by reinforcement, N
γ_1	Factor for obtaining depth of compression block in concrete
β_h	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber, (0.003 mm/mm)

Table 3-1 List of Symbols Used in the AS 3600-2001 Code

ε_s	Strain in reinforcement
ϕ	Strength reduction factor
θ_t	Angle of compression strut for torsion, degrees
θ_v	Angle of compression strut for shear, degrees

3.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For AS 3600-01, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be defined (AS 3.3.1):

1.35D	(AS/NZS 1170.0-02, 4.2.2(a))
1.2D + 1.5L	(AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 1.5(0.75 PL)	(AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 0.4L + 1.0S	(AS/NZS 1170.0-02, 4.2.2(g))
0.9D ± 1.0W	(AS/NZS 1170.0-02, 4.2.2(e))
1.2D ± 1.0W	(AS/NZS 1170.0-02, 4.2.2(d))
1.2D + 0.4L ± 1.0W	(AS/NZS 1170.0-02, 4.2.2(d))
1.0D ± 1.0E	(AS/NZS 1170.0-02, 4.2.2(f))
1.0D + 0.4L ± 1.0E	(AS/NZS 1170.0-02, 4.2.2(f))

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in SAFE whenever the AS 3600-2001 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

3.3 Limits on Material Strength

The upper and lower limits of f'_c are 65 MPa and 20 MPa, respectively, for all framing types (AS 6.1.1.1(b)).

$$f'_c \leq 65\,\text{MPa} \qquad \text{(AS 6.1.1.1)}$$

$$f'_c \geq 20\,\text{MPa} \qquad \text{(AS 6.1.1.1)}$$

The upper limit of f_{sy} is 500 MPa for all frames (AS 6.2.1, Table 6.2.1).

The code allows use of f'_c and f_{sy} beyond the given limits, provided special care is taken regarding the detailing and ductility (AS 6.1.1, 6.2.1, 19.2.1.1).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

3.4 Strength Reduction Factors

The strength reduction factor, ϕ, is defined as given in AS 2.3(c), Table 2.3:

$\phi = 0.80$ for flexure (tension controlled) (AS 2.3(c))

$\phi = 0.70$ for shear and torsion (AS 2.3(c))

These values can be overwritten; however, caution is advised.

3.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

3.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

3.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

3.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, f_y, is limited to 500MPa (AS 6.2.1), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 3-1 (AS 8.1.2.2).

The following assumptions are used for the stress block used to compute the flexural bending capacity of rectangular sections (AS 8.1.2.2).

- The maximum strain in the extreme compression fiber is taken as 0.003.
- A uniform compressive stress of $0.85f'_c$ acts on an area bounded by:
  - The edges of the cross-sections.
  - A line parallel to the neutral axis at the strength limit under the loading concerned, and located at a distance $\gamma k_u d$ from the extreme compression fiber.

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by

$$a_{\max} = \gamma k_u d \quad \text{where,} \qquad \text{(AS 8.1.3)}$$

$$\gamma = \left[0.85 - 0.007\left(f'_c - 28\right)\right]$$

$$0.65 \leq \gamma \leq 0.85 \qquad \text{(AS 8.1.2.2)}$$

$$k_u = 0.4$$

The design procedure used by SAFE for both rectangular and flanged sections (L- and T-beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed ($A_{sc}f_{sy} > 0.15N^*$) (AS 10.7.1a); hence, all beams are designed for major direction flexure, shear, and torsion only.

3.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M^*(i.e., designing top or bottom reinforcement), the depth of the compression block is given by *a* (see Figure 3-1), where,

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{0.85 f'_c \phi b}} \qquad \text{(AS 8.1.2.2)}$$

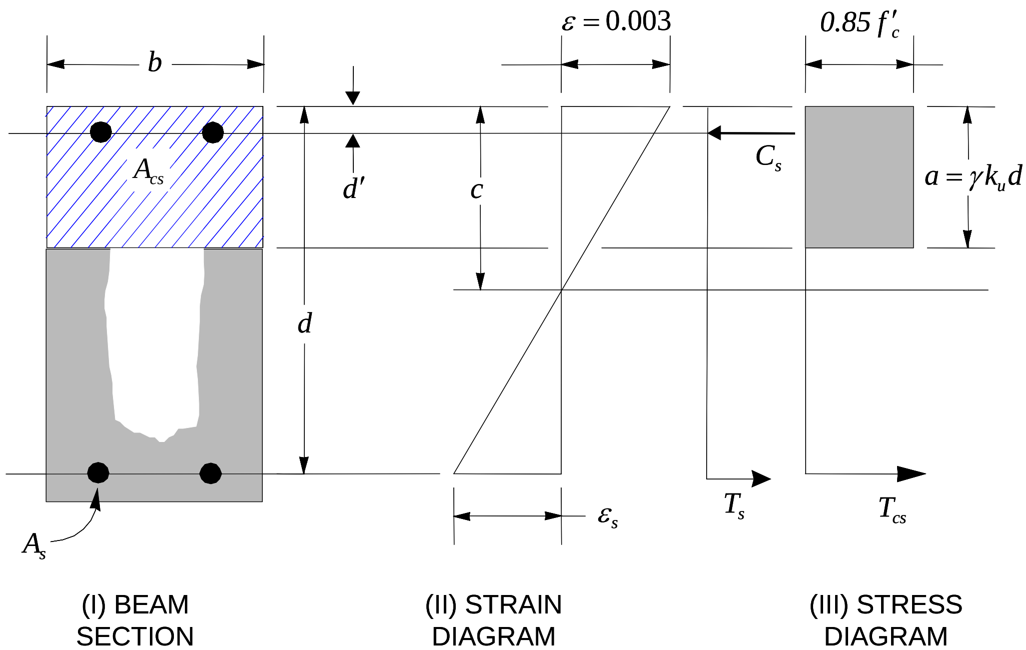


Figure 3-1 Rectangular Beam Design

where, the value of ϕ is taken as that for a tension controlled section ($k_u \le 0.4$), which by default is 0.80 (AS 2.3) in the preceding and following equations.

- If $a \le a_{\max}$, the area of tension reinforcement is then given by:

$$A_{st} = \frac{M^*}{\phi f_{sy}\left(d - \frac{a}{2}\right)}$$

This reinforcement is to be placed at the bottom if M^* is positive, or at the top if M^* is negative.

- If $a > a_{\max}$, i.e., $k_u > 0.4$, compression reinforcement is required (AS 8.1.3) and is calculated as follows:

The compressive force developed in the concrete alone is given by:

$$C = 0.85 f'_c\, b a_{\max} \qquad \text{(AS 8.1.2.2)}$$

and the moment resisted by concrete compression and tension reinforcement is:

$$M_{uc} = C\left(d - \frac{a_{\max}}{2}\right)\phi$$

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M^* - M_{uc}$$

The required compression reinforcement is given by:

$$A_{sc} = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d - d'\right)\phi}\text{, where}$$

$$f'_s = 0.003 E_s\left[\frac{c - d'}{c}\right] \le f_{sy} \qquad \text{(AS 8.1.2.1, 6.2.2)}$$

The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_{uc}}{f_{sy}\left[d - \frac{a_{\max}}{2}\right]\phi}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{us}}{f_{sy}\left(d - d'\right)\phi}$$

Therefore, the total tension reinforcement is $A_{st} = A_{s1} + A_{s2}$, and the total compression reinforcement is A_{sc}. A_{st} is to be placed at the bottom and A_{sc} is to be placed at the top if M^* is positive, and vice versa if M^* is negative.

3.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 3-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 3-1 is assumed on the compression side (AS 8.1.3).

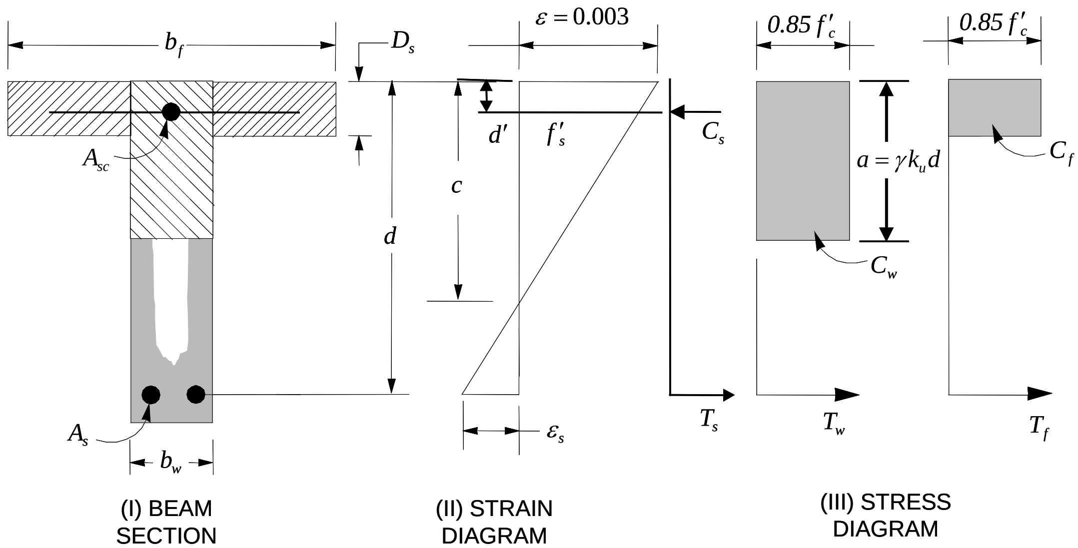


Figure 3-2 T-Beam Design

3.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M^* (i.e., designing top reinforcement), the calculation of the reinforcement is exactly the same as above, i.e., no flanged beam data is used.

3.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M^* > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85 f'_c \phi b_f}}$$

where, the value of ϕ is taken as that for $k_u \leq 0.4$, which is 0.80 by default (AS 2.3) in the preceding and the following equations.

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \gamma k_u d \text{ where, } k_u = 0.4 \qquad \text{(AS 8.1.3)}$$

- If $a \leq D_s$, the subsequent calculations for A_{st} are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{\max}$.

- If $a > D_s$, the calculation for A_{st} has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 3-2. C_f is given by:

$$C_f = 0.85 f'_c \left(b_{ef} - b_w\right) \times \min\left(D_s, a_{\max}\right) \qquad \text{(AS 8.1.2.2)}$$

Therefore, $A_{s1} = \frac{C_f}{f_{sy}}$ and the portion of M^* that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min\left(D_s, a_{\max}\right)}{2}\right)$$

Therefore, the balance of the moment, M^* to be carried by the web is:

$$M_{uw} = M^* - M_{uf}$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi\, b_w}}$$

- If $a_1 \le a_{max}$, the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi\, f_{sy}\left(d - \frac{a_1}{2}\right)}, \text{ and}$$

$$A_{st} = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_{max}$, compression reinforcement is required and is calculated as follows:

The compression force in the web concrete alone is given by:

$$C_w = 0.85 f'_c\, b_w a_{max} \qquad \text{(AS 8.1.2.2)}$$

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{uc} = C_w\left(d - \frac{a_{max}}{2}\right)\phi$$

and the moment resisted by compression and tension reinforcement is:

$$M_{us} = M_{uw} - M_{uc}$$

Therefore, the compression reinforcement is computed as:

$$A_{sc} = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d - d'\right)\phi}, \text{ where}$$

$$f'_s = 0.003 E_s \left[\frac{c_{\max} - d'}{c_{\max}}\right] \leq f_{sy} \qquad \text{(AS 8.1.2.1, 6.2.2)}$$

The tension reinforcement for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{uc}}{f_{sy}\left[d - \frac{a_{\max}}{2}\right]\phi}$$

and the tension reinforcement for balancing the compression reinforcement is:

$$A_{s3} = \frac{M_{us}}{f_{sy}\left(d - d'\right)\phi}$$

The total tensile reinforcement is $A_{st} = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A_{sc}. A_{st} is to be placed at the bottom and A_{sc} is to be placed at the top.

3.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following limit:

$$A_{st.\min} = 0.22\left(\frac{D}{d}\right)^2 \frac{f'_{cf}}{f_{sy}} bd, \text{ where} \qquad \text{(AS 8.1.4.1)}$$

$$f'_{cf} = 0.6\sqrt{f'_c} \qquad \text{(AS 6.1.1.2)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_{st} \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A_{sc} \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

3.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V^*.
- Determine the shear force, V_{uc}, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

3.5.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

3.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_{uc}, is calculated as:

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_w d_o \left[\frac{A_{st} f'_c}{b_w d_o} \right]^{1/3} \quad \text{(AS 8.2.7.1)}$$

where,

$$\beta_1 = 1.1\left(1.6 - \frac{d_o}{1000}\right) \geq 1.1 \qquad \text{(AS 8.2.7.1)}$$

$\beta_2 = 1$, or (AS 8.2.7.1)

$= 1 - \left(\dfrac{N^*}{3.5A_g}\right) \geq 0$ for members subject to significant axial tension, or

$= 1 + \left(\dfrac{N^*}{14A_g}\right)$ for members subject to significant axial compression.

$\beta_3 = 1$

3.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to:

$$V_{u.\min} = V_{uc} + 0.6 b_v d_o \qquad \text{(AS 8.2.9)}$$

$$V_{u.\max} = 0.2 f'_c \, b d_o \qquad \text{(AS 8.2.6)}$$

Given V^*, V_{uc}, and $V_{u.\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.6 by default (AS 2.3).

- If $V^* \leq \phi V_{uc} / 2$,

  $\dfrac{A_{sv}}{s} = 0$, if $D \leq 750$ mm; otherwise $A_{sv.\min}$ shall be provided. (AS 8.2.5).

- If $\left(\phi V_{uc} / 2\right) < V^* \leq \phi V_{u.\min}$,

  $\dfrac{A_{sv}}{s} = 0$, if $D < b_w/2$ or 250 mm, whichever is greater (AS 8.2.5(c)(i)); otherwise $A_{sv.\min}$ shall be provided.

- If $\phi V_{u.\min} < V^* \leq \phi V_{u.\max}$,

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{\phi f_{sy.f} d_o \cot \theta_v}, \quad \text{(AS 8.2.10)}$$

and greater than $A_{sv.\min}$, defined as:

$$\frac{A_{sv.\min}}{s} = \left(0.35 \frac{b_w}{f_{sy.f}}\right) \quad \text{(AS 8.2.8)}$$

θ_v = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $V^*=\phi V_{u.\min}$ to 45 degrees when $V^*=\phi V_{u,\max}$.

- If $V^* > \phi V_{\max}$, a failure condition is declared. (AS 8.2.6)
- If V^* exceeds its maximum permitted value $\phi V_{\max}$, the concrete section size should be increased (AS 8.2.6).

Note that if torsion design is considered and torsion reinforcement is required, the calculated shear reinforcement is ignored. Closed stirrups are designed for combined shear and torsion according to AS 8.3.4(b).

The maximum of all of the calculated A_{sv}/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

3.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T^*.

- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

3.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T^* is permitted to be reduced in accordance with the code (AS 8.3.2). However, the program does not automatically redistribute the internal forces and reduce T^*. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

3.5.3.2 Determine Special Section Properties

For torsion design, special section properties such as A_t, J_t, and u_t are calculated. These properties are described in the following (AS 8.3).

A_t = Area of a polygon with vertices at the center of longitudinal bars at the corners of the cross-section

u_t = Perimeter of the polygon defined by A_t

J_t = Torsional modulus

In calculating the section properties involving reinforcement, such as A_{sw}/s and A_l, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm-diameter stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{uc} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$A_t = (b - 2c)(h - 2c),$ (AS 8.3.5)

$u_t = 2(b - 2c) + 2(h - 2c),$ (AS 8.3.6)

$J_t = 0.4x^2y$ (AS 8.3.3)

where, the section dimensions b, h and, c are as shown in Figure 3-3. Similarly, the special section properties for a flanged beam section are given as:

$A_t = (b_w - 2c)(h - 2c),$ (AS 8.3.5)

$u_t = 2(h - 2_c) + 2(b_w - 2_c),$ (AS 8.3.6)

$J_t = 0.4\Sigma x^2y$ (AS 8.3.3)

where the section dimensions b_w, h, and c for a flanged beam are as shown in Figure 3-3. The values x and y refer to the smaller and larger dimensions of a component rectangle, respectively.

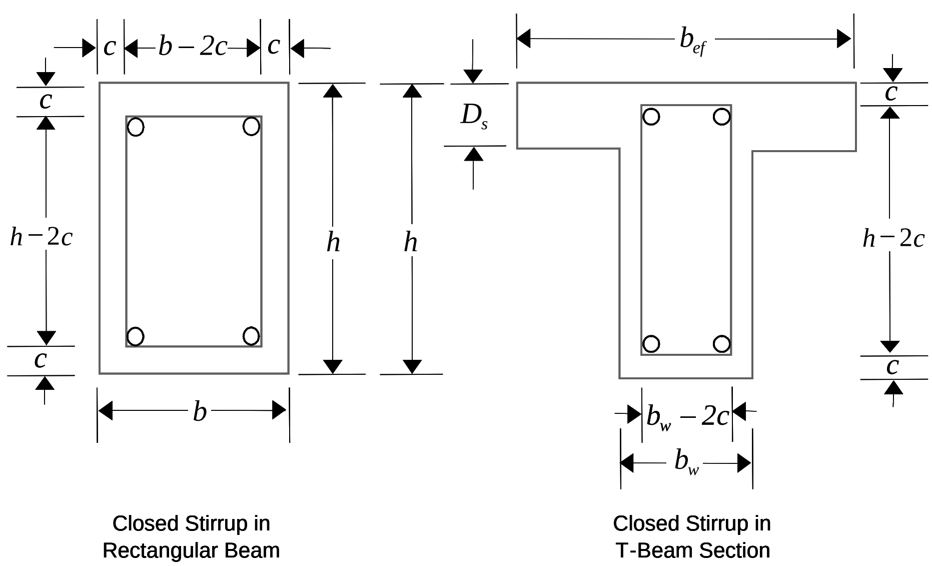


Figure 3-3 Closed stirrup and section dimensions for torsion design

3.5.3.3 Determine Torsion Reinforcement

The torsional strength of the section without torsion reinforcement, T_{uc}, is calculated as:

$$T_{uc} = 0.3 J_t \sqrt{f'_c} \qquad \text{(AS 8.3.5)}$$

where J_t is the torsion modulus of the concrete cross-section as described in detail in the previous section.

Torsion reinforcement also can be ignored if any of the following is satisfied:

$$T^* \leq 0.25\phi T_{uc} \qquad \text{(AS 8.3.4(a)(i))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 0.5 \qquad \text{(AS 8.3.4(a)(ii))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1 \text{ and } D \leq \max(250\text{mm}, b/2) \qquad \text{(AS 8.3.4(a)(iii))}$$

If the factored torsion T^* alone or in combination with V^* does not satisfy any of the three conditions in the preceding description, torsion reinforcement is needed. It is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (AS 8.3).

- If $T^* > T_{cr}$, the required closed stirrup area per unit spacing, A_{sw} /s, is calculated as:

$$\frac{A_{sw}}{s} = \frac{T^* \tan\theta_t}{\phi 2 f_{sy.f} A_t} \qquad \text{(AS 8.3.5(b))}$$

where, the minimum value of A_{sw} /s is taken as follows:

$$\frac{A_{sw.\min}}{s} = \frac{0.35 b_w}{f_{sy.f}} \qquad \text{(AS 8.2.8)}$$

The value θ_t is the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $T^* = \phi T_{uc}$ to 45 degrees when $T^* = \phi T_{u,\max}$.

The following equation shall also be satisfied for combined shear and torsion by adding additional shear stirrups.

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} \le 1.0 \qquad \text{(AS 8.3.4(b))}$$

where,

$$T_{us} = f_{sy.f}\left(\frac{A_{sw}}{s}\right) 2A_t \cot\theta_t \qquad \text{(AS 8.3.5(b))}$$

$$V_{us} = \left(A_{sv} f_{sy.f} d_o / s\right)\cot\theta_v \qquad \text{(AS 8.2.10(a))}$$

The required longitudinal rebar area is calculated as:

$$A_l = \frac{0.5 f_{sy.f}\left(\frac{A_{sw}}{s}\right) u_t \cot^2\theta_t}{f_{sy}} \qquad \text{(AS 8.3.6(a))}$$

An upper limit of the combination of V^* and T^* that can be carried by the section is also checked using the equation:

$$\frac{T^*}{\phi T_{u.\max}} + \frac{V^*}{\phi V_{u.\max}} \le 1.0 \qquad \text{(AS 8.3.3)}$$

where,

$$V_{u.\max} = 0.2 f'_c b_w d_o \qquad \text{(AS 8.2.6)}$$

$$T_{u.\max} = 0.2 f'_c J_t \qquad \text{(AS 8.3.5(a))}$$

For rectangular sections, b_w is replaced with b. If the combination of V^* and T^* exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T^* > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups satisfy the following limit.

$$\left(\frac{A_{sv}}{s}+2\frac{A_{sw}}{s}\right)\geq\frac{0.35b}{f_{sy.f}} \qquad \text{(AS 8.3.7, 8.2.8)}$$

If this equation is not satisfied with the originally calculated A_{sv}/s and A_{sw}/s, A_{sv}/s is increased to satisfy this condition. In that case, A_{sv}/s does not need to satisfy AS Section 8.2.8 independently.

The maximum of all the calculated A_l and A_{sw}/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

3.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (AS 3600-2001) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

3.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the following subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

3.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

3.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

3.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (AS 9.1.1):

$A_s \geq 0.0025\, bh$ for flat slabs (AS 9.1.1(a))

$A_s \geq 0.0020\, bh$ for slabs supported by beams/walls and slab footings

(AS 9.1.1(b))

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

3.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

3.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d_{om}/2$ from the face of the support (AS 9.2.1.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (AS 9.2.1.3). Figure 3-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

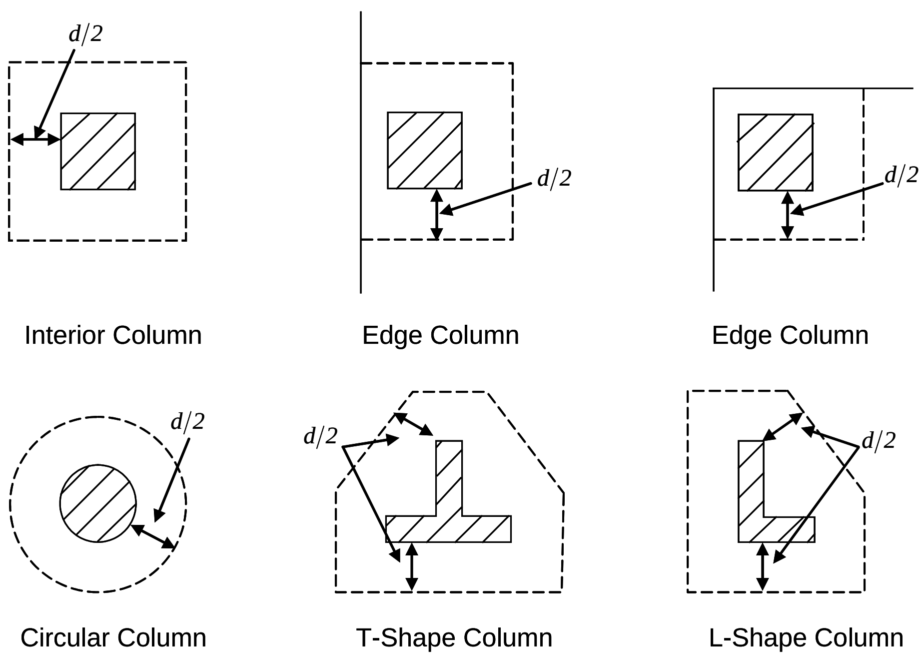


Figure 3-4 Punching Shear Perimeters

3.6.2.2 Determine Concrete Capacity

The shear capacity, f_{cv}, is calculated based on the minimum of the two expressions from AS 3600-01 equation 11-35, as shown, with the d_{om} and u terms removed to convert force to stress.

$$f_{cv} = \min \begin{cases} 0.17\left(1+\dfrac{2}{\beta_h}\right)\sqrt{f'_c} \\ 0.34\sqrt{f'_c} \end{cases} \qquad \text{(AS 9.2.3(a))}$$

where, β_h is the ratio of the longest dimension to the shortest dimension of the critical section.

3.6.2.3 Determine Maximum Shear Stress

The maximum design shear stress is computed along the major and minor axis of column separately using the following equation:

$$v_{\max} = \frac{V^*}{ud_{\text{om}}}\left[1.0 + \frac{uM_v}{8V^* ad_{\text{om}}}\right] \qquad \text{(AS 9.2.4(a))}$$

3.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

3.6.3 Design Punching Shear Reinforcement

The design guidelines for shear links or shear studs are not available in AS 3600-2001. SAFE uses the NZS 3101-06 guidelines to design shear studs or shear links.

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

3.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

3.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = 3\ V_{u,\min} = 3^*V_u \qquad \text{(AS 92.2.4(a), (d))}$$

where V_u is computed from AS 9.2.3 or 9.2.4. Given V^*, V_u, and $V_{u,\max}$, the required shear reinforcement is calculated as follows, where, ϕ is the strength reduction factor.

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_u\right)}{f_{sy} d_{\text{om}}}, \qquad \text{(AS 8.2.10)}$$

Minimum punching shear reinforcement should be provided such that:

$$V_s \geq \frac{1}{16}\sqrt{f'_c}\, u d_{\text{om}} \qquad \text{(NZS 12.7.4.3)}$$

- If $V^* > \phi V_{\max}$, a failure condition is declared. (NZS 12.7.3.4)
- If V^* exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size.

3.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 3-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (NZS 12.7.4.4).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

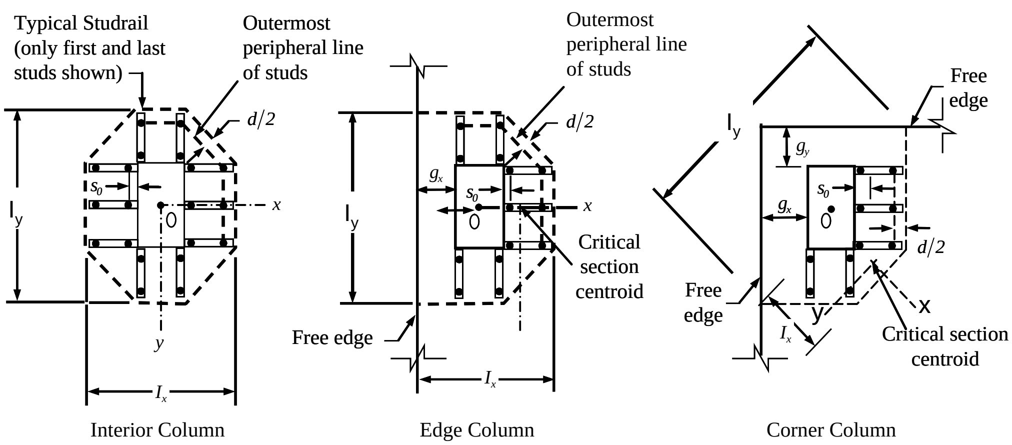


Figure 3-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

3.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 2*d* and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(NZS 12.7.4.4)}$$

$$s \leq 0.5d \qquad \text{(NZS 12.7.4.4)}$$

$$g \leq 2d \qquad \text{(NZS 12.7.4.4)}$$

Chapter 4
Design for BS 8110-97

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the British code BS 8110-1997 [BSI 1997] is selected. For light-weight concrete and torsion, reference is made to BS 8110-2:1985 [BSI 1985]. Various notations used in this chapter are listed in Table 4-1. For referencing to the pertinent sections of the British code BS 8110-1997 in this chapter, a prefix “BS” followed by the section number is used.

The design is based on user-specified loading combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

4.1 Notations

Table 4-1 List of Symbols Used in the BS 8110-1997 Code

A_g	Gross area of cross-section, mm^2

Table 4-1 List of Symbols Used in the BS 8110-1997 Code

A_l	Area of longitudinal reinforcement for torsion, mm^2
A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
$A_{sv,t}$	Total cross-sectional area of closed links for torsion, mm^2
A_{sv} / s_v	Area of shear reinforcement per unit length, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
C	Torsional constant, mm^4
d	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f	Punching shear factor considering column location
f_{cu}	Characteristic cube strength at 28 days, MPa
f'_s	Stress in the compression reinforcement, MPa
f_y	Characteristic strength of reinforcement, MPa
f_{yv}	Characteristic strength of shear reinforcement, MPa
h	Overall depth of a section in the plane of bending, mm

Table 4-1 List of Symbols Used in the BS 8110-1997 Code

h_f	Flange thickness, mm
$h_{\min}$	Smaller dimension of a rectangular section, mm
$h_{\max}$	Larger dimension of a rectangular section, mm
K	Normalized design moment, $\frac{M_u}{bd^2 f_{cu}}$
K'	Maximum $\frac{M_u}{bd^2 f_{cu}}$ for a singly reinforced concrete section, taken as 0.156 by assuming that moment redistribution is limited to 10%.
k_1	Shear strength enhancement factor for support compression
k_2	Concrete shear strength factor, $\left[f_{cu}/25\right]^{1/3}$
k_3	Shear strength reduction factor for light-weight concrete
M	Design moment at a section, N-mm
M_{single}	Limiting moment capacity as singly reinforced beam, N-mm
s_v	Spacing of the links along the length of the beam, mm
T	Design torsion at ultimate design load, N-mm
u	Perimeter of the punching critical section, mm
V	Design shear force at ultimate design load, N
v	Design shear stress at a beam cross-section or at a punching critical section, MPa
v_c	Design concrete shear stress capacity, MPa
$v_{\max}$	Maximum permitted design factored shear stress, MPa

Table 4-1 List of Symbols Used in the BS 8110-1997 Code

v_t	Torsional shear stress, MPa
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm
β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain, 0.0035
ε_s	Strain in tension reinforcement
ε_s'	Strain in compression reinforcement

4.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (BS 2.4.1.3). For BS 8110-1997, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered (BS 2.4.3).

1.4D
1.4D + 1.6L (BS 2.4.3)

1.4D + 1.6(0.75PL) (BS 2.4.3)

1.0D ± 1.4W
1.4D ± 1.4W (BS 2.4.3)
1.2D + 1.2L ± 1.2W

1.4D + 1.6L + 1.6S
1.2D + 1.2S ± 1.2W
1.2D + 1.2L + 1.2S ± 1.2W (BS 2.4.3)

These are also the default design load combinations in SAFE whenever the BS 8110-1997 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

4.3 Limits on Material Strength

The concrete compressive strength, f_{cu}, should not be less than 25 MPa (BS 3.1.7.2). SAFE does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsibility to use the proper strength values while defining the materials.

4.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the following table, as taken from BS Table 2.2 (BS 2.4.4.1):

Values of γ_m for the Ultimate Limit State	
Reinforcement	1.15
Concrete in flexure and axial load	1.50
Concrete shear strength without shear reinforcement	1.25

These factors are already incorporated into the design equations and tables in the code. Note that for reinforcement, the default factor of 1.15 is for Grade 500 reinforcement. If other grades are used, this value should be overwritten as necessary. Changes to the partial safety factors are carried through the design equations where necessary, typically affecting the material strength portions of the equations.

4.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

4.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

4.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

4.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 4-1 (BS 3.4.4.4). Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$; BS 3.4.4.4). The code also places a limitation on the neutral axis depth, $x/d \leq 0.5$, to safeguard against non-ductile failures (BS 3.4.4.4). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams) is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed ($0.1f_{cu}\,A_g$) (BS 3.4.4.1); hence, all beams are designed for major direction flexure, shear, and torsion only.

4.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam, M_{single}, is first calculated for a section. The reinforcement is determined based on M being greater than, less than, or equal to M_{single}. See Figure 4-1.

- Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

$$M_{\text{single}} = K' f_{cu}\, bd^2, \text{ where} \qquad \text{(BS 3.4.4.4)}$$

$$K' = 0.156$$

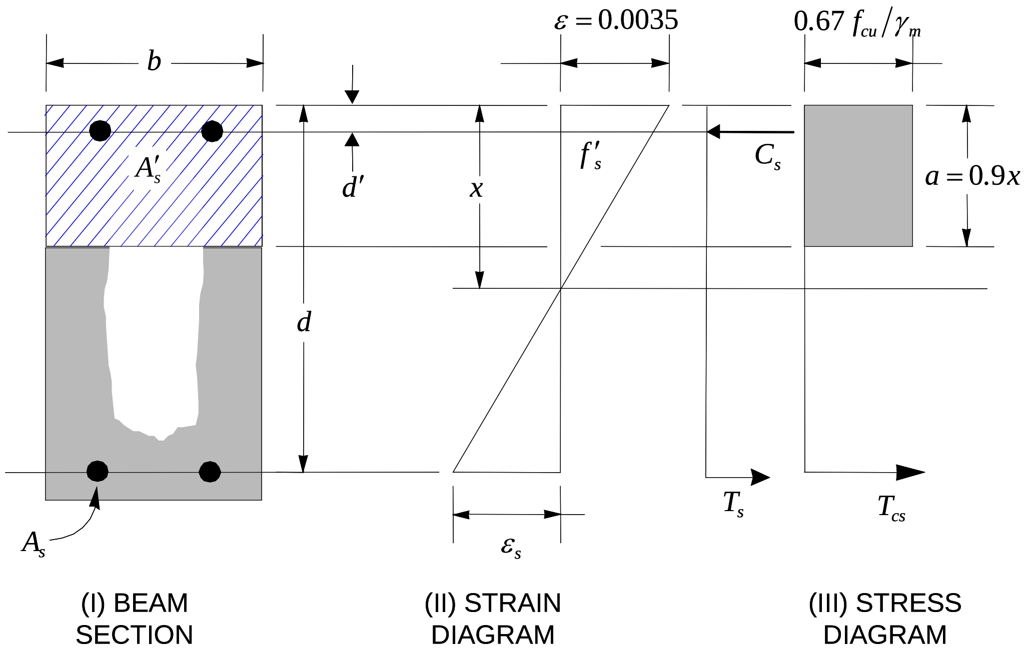


Figure 4-1 Rectangular Beam Design

- If $M \leq M_{\text{single}}$, the area of tension reinforcement, A_s, is given by:

$$A_s = \frac{M}{0.87 f_y z},\text{, where} \qquad \text{(BS 3.4.4.4)}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \leq 0.95d \qquad \text{(BS 3.4.4.4)}$$

$$K = \frac{M}{f_{cu} b d^2} \qquad \text{(BS 3.4.4.4)}$$

This reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

- If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$A'_s = \frac{M - M_{\text{single}}}{\left(f'_s - \dfrac{0.67 f_{cu}}{\gamma_c} \right)\left(d - d' \right)} \qquad \text{(BS 3.4.4.4)}$$

where *d*' is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = 0.87 f_y \text{ if } d'/d \le \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(BS 3.4.4.1, 2.5.3, Fig 2.2)}$$

$$f'_s = E_s \varepsilon_c \left[1 - \frac{2d'}{d}\right] \text{ if } d'/d > \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(BS 3.4.4.1, 2.5.3, Fig 2.2)}$$

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$A_s = \frac{M_{\text{single}}}{0.87 f_y z} + \frac{M - M_{\text{single}}}{0.87 f_y \left(d - d' \right)}, \text{ where} \qquad \text{(BS 3.4.4.4)}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \right) = 0.777d \qquad \text{(BS 3.4.4.4)}$$

4.5.1.2.2 Design of Flanged Beams

4.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, *M* (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

4.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located in the flange. Based on this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f.

If the stress block extends beyond the flange depth, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 4-2.

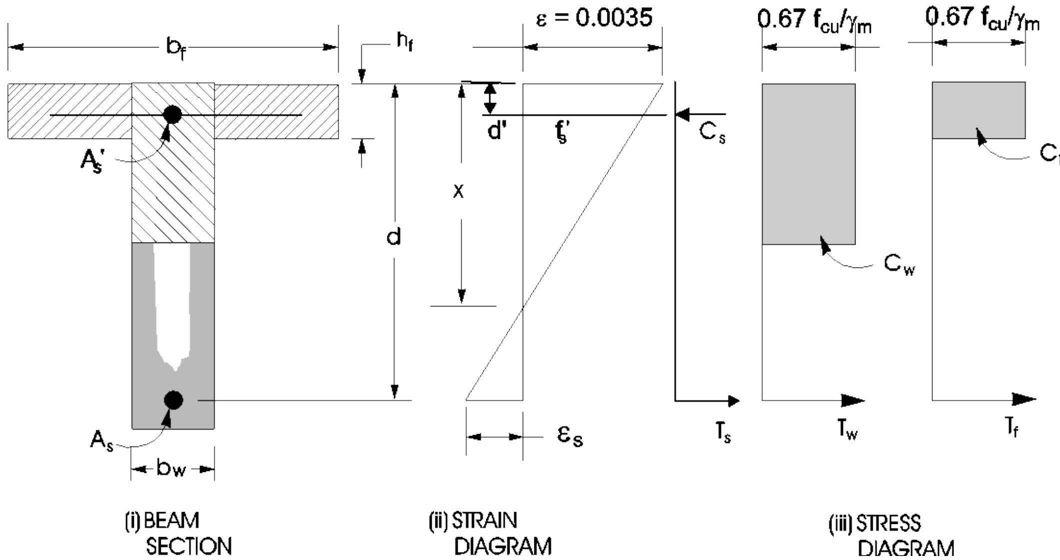


Figure 4-2 Design of a T-Beam Section

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

$$K = \frac{M}{f_{cu}\, b_f\, d^2} \qquad \text{(BS 3.4.4.4)}$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d \qquad \text{(BS 3.4.4.4)}$$

the depth of the neutral axis is computed as:

$$x = \frac{1}{0.45}(d - z) \qquad \text{(BS 3.4.4.4)}$$

and the depth of the compression block is given by:

$$a = 0.9x \qquad \text{(BS 3.4.4.4)}$$

- If $a \leq h_f$, the subsequent calculations for A_S are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as b_f. Compression reinforcement is required when $K > K'$.

- If $a > h_f$, when $M \leq \beta_f f_{cu} bd^2$ and $h_f \leq 0.45d$, then

$$A_s = \frac{M + 0.1 f_{cu} bd\left(0.45d - h_f\right)}{0.87 f_y \left(d - 0.5h_f\right)}, \text{ where} \qquad \text{(BS 3.4.4.5)}$$

$$\beta_f = 0.45\frac{h_f}{d}\left(1 - \frac{b_w}{b}\right)\left(1 - \frac{h_f}{2d}\right) + 0.15\frac{b_w}{b} \qquad \text{(BS 3.4.4.5)}$$

Otherwise the calculation for A_S has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 4-2.

In that case, the ultimate resistance moment of the flange is given by:

$$M_f = 0.45 f_{cu} \left(b_f - b_w\right) h_f \left(d - 0.5h_f\right) \qquad \text{(BS 3.4.4.5)}$$

The moment taken by the web is computed as:

$$M_w = M - M_f$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2} \qquad \text{(BS 3.4.4.4)}$$

- If $K_w \leq 0.156$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{0.87 f_y \left(d - 0.5h_f\right)} + \frac{M_w}{0.87 f_y z}, \text{ where}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \leq 0.95d$$

- If $K_w > K'$ (BS 3.4.4.4), compression reinforcement is required and is calculated as follows:

  The ultimate moment of resistance of the web only is given by:

  $$M_{uw} = K' f_{cu} b_w d^2 \qquad \text{(BS 3.4.4.4)}$$

  The compression reinforcement is required to resist a moment of magnitude $M_w - M_{uw}$. The compression reinforcement is computed as:

  $$A'_s = \frac{M_w - M_{uw}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')} \qquad \text{(BS 3.4.4.4)}$$

  where, d' is the depth of the compression reinforcement from the concrete compression face, and

  $$f'_s = 0.87 f_y \text{ if } d'/d \leq \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(BS 3.4.4.1, 2.5.3, Fig 2.2)}$$

  $$f'_s = E_s \varepsilon_c \left[1 - \frac{2d'}{d}\right] \text{ if } d'/d > \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(BS 3.4.4.1, 2.5.3, Fig 2.2)}$$

  The area of tension reinforcement is obtained from equilibrium as:

  $$A_s = \frac{M_f}{0.87 f_y (d - 0.5h_f)} + \frac{M_{uw}}{0.87 f_f (0.777d)} + \frac{M_w - M_{uw}}{0.87 f_y (d - d')}$$

4.5.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from BS Table 3.25 (BS 3.12.5.3) with interpolation for reinforcement of intermediate strength.

Section	Situation	Definition of percentage	Minimum percentage	
			f_y = 250 MPa	f_y = 500 MPa
Rectangular	—	$100 \frac{A_s}{bh}$	0.24	0.13
T- or L-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100 \frac{A_s}{b_w h}$	0.32	0.18
	$\frac{b_w}{b_f} \geq 0.4$	$100 \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100 \frac{A_s}{b_w h}$	0.48	0.26
L-Beam with web in compression	—	$100 \frac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from BS Table 3.25 (BS 3.12.5.3).

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100 \frac{A_s'}{bh}$	0.20
T- or L-Beam	Web in tension	$100 \frac{A_s'}{b_f h_f}$	0.40
	Web in compression	$100 \frac{A_s'}{b_w h}$	0.20

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows (BS 3.12.6.1):

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

4.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

4.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors. The shear stress is then calculated as:

$$v = \frac{V}{b_w d} \qquad \text{(BS 3.4.5.2)}$$

The maximum allowable shear stress, v_{max} is defined as:

$$v_{max} = \min(0.8\sqrt{f_{cu}}\text{, 5 MPa}) \qquad \text{(BS 3.4.5.2)}$$

For light-weight concrete, v_{max} is defined as:

$$v_{max} = \min(0.63\sqrt{f_{cu}}\text{, 4 MPa}) \qquad \text{(BS 8110-2:1985 5.4)}$$

4.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, v_c, is calculated as:

$$v'_c = v_c + 0.6\frac{NVh}{A_c M} \leq v_c\sqrt{1+\frac{N}{A_c v_c}} \qquad \text{(BS 3.4.5.12)}$$

$$v_c = \frac{0.79 k_1 k_2 k_3}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} \qquad \text{(BS 3.4.5.4, Table 3.8)}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 (BS 3.4.5.8)

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3},\ 1 \leq k_2 \leq \left(\frac{40}{25}\right)^{1/3} \qquad \text{(BS 3.4.5.4, Table 3.8)}$$

$$\gamma_m = 1.25 \qquad \text{(BS 2.4.4.1)}$$

However, the following limits also apply:

$$0.15 \leq \frac{100\ A_s}{bd} \leq 3 \qquad \text{(BS 3.4.5.4, Table 3.8)}$$

$$\left(\frac{400}{d}\right)^{1/4} \geq 0.67 \text{ (unreinforced) or } \geq 1 \text{ (reinforced)} \qquad \text{(BS 3.4.5.4, Table 3.8)}$$

$$f_{cu} \leq 40 \text{ MPa (for calculation purposes only)} \qquad \text{(BS 3.4.5.4)}$$

$$\frac{Vh}{M} \leq 1 \qquad \text{(BS 3.4.5.12)}$$

A_s is the area of tension reinforcement.

4.5.2.3 Determine Required Shear Reinforcement

Given v, v_c, and v_{max}, the required shear reinforcement is calculated as follows (BS Table 3.8, BS 3.4.5.3):

- If $v \leq (v'_c + 0.4)$,

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87 f_{yv}}$$ (BS 3.4.5.3, Table 3.7)

- If $(v'_c + 0.4) < v \le v_{max}$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v'_c) b_w}{0.87 f_{yv}}$$ (BS 3.4.5.3, Table 3.7)

- If $v > v_{max}$, a failure condition is declared. (BS 3.4.5.2)

In the preceding expressions, a limit is imposed on f_{yv} as:

$f_{yv} \le 500$ MPa. (BS 3.4.5.1)

The maximum of all of the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

4.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t.
- Determine special section properties.
- Determine critical torsion stress.
- Determine the torsion reinforcement required.

Note that this section refers to BS 8110-2:1985 instead of BS 8110-1997 code.

4.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following sections (BS 8110-2:85 3.4.5.13).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2\left(h_{\max} - h_{\min}/3\right)} \qquad \text{(BS 8110-2:85 2.4.4.1)}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{\text{seg}} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(BS 8110-2:85 2.4.4.2)}$$

$h_{\max}$ = Larger dimension of a rectangular section

$h_{\min}$ = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

$$v_t \leq \min(0.8\sqrt{f_{cu}}, 5\text{N/mm}^2) \times \frac{y_1}{550} \qquad \text{(BS 8110-2:85 2.4.5)}$$

4.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.4\text{N/mm}^2\right) \qquad \text{(BS 8110-2:85 2.4.6)}$$

where f_{cu} is the specified concrete compressive strength.

For light-weight concrete, $v_{t,\min}$ is defined as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.4\text{N/mm}^2\right) \times 0.8 \qquad \text{(BS 8110-2:85 5.5)}$$

4.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (BS 8110-2:85 2.4.6). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (BS 8110-2:85 2.4.6).

If $v_t > v_{t,\min}$ the required closed stirrup area per unit spacing, $A_{sv,t}/s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8 x_1 y_1 \left(0.87 f_{yv}\right)} \qquad \text{(BS 8110-2:85 2.4.7)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t} f_{yv}\left(x_1 + y_1\right)}{s_v f_y} \qquad \text{(BS 8110-2:85 2.4.7)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link, and y_1 is the larger center-to-center dimension of the closed link.

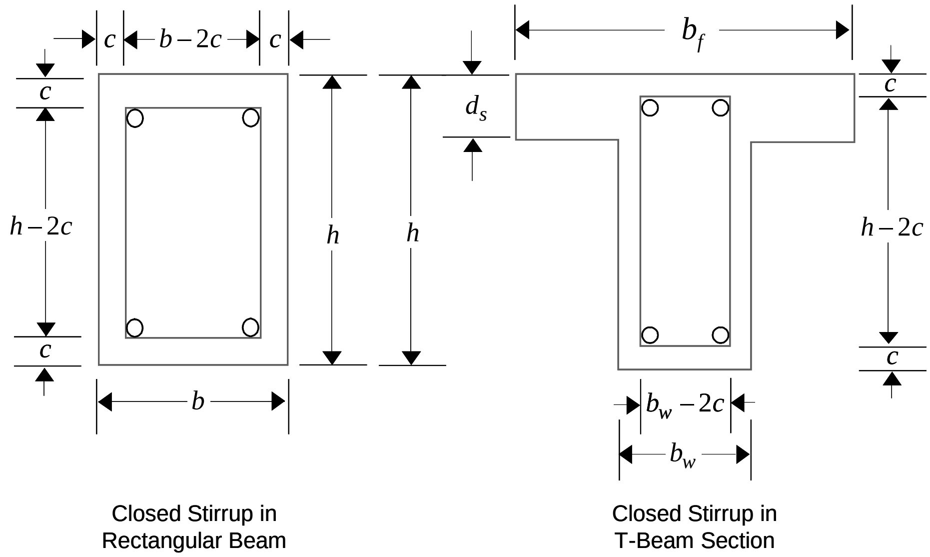


Figure 4-3 Closed stirrup and section dimensions for torsion design

An upper limit of the combination of v and v_t that can be carried by the section is also checked using the equation:

$$v + v_t \leq \min\left(0.8\sqrt{f_{cu}},\ 5\text{N/mm}^2\right) \qquad \text{(BS 8110-2:85 2.4.5)}$$

For light-weight concrete, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.63\sqrt{f_{cu}},\ 4\ \text{MPa}) \qquad \text{(BS 8110-2:85 5.4)}$$

If the combination of shear stress, v, and torsional shear stress, v_t, exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

4.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method (BS 8110-97) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

4.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

4.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

4.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

4.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (BS 3.12.5.3, BS Table 3.25) with interpolation for reinforcement of intermediate strength:

$$A_s \geq \begin{cases} 0.0024bh & \text{if} \quad f_y = 250\text{MPa} \\ 0.0013bh & \text{if} \quad f_y = 500\text{MPa} \end{cases} \qquad \text{(BS 3.12.5.3)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

4.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

4.6.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (BS 3.7.6.4) and at a critical section at a distance of 1.5*d* from the face of the support (BS 3.7.7.6). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (BS 3.7.7.1). Figure 4-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

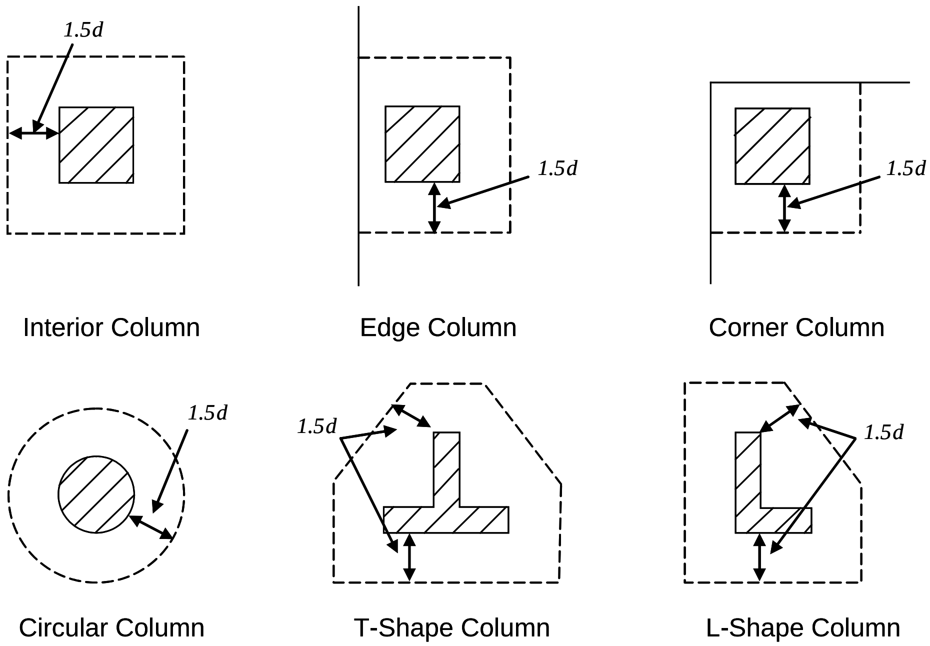


Figure 4-4 Punching Shear Perimeters

4.6.2.2 Determine Concrete Capacity

The concrete punching shear factored strength is taken as (BS 3.7.7.4, 3.7.7.6):

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} \qquad \text{(BS 3.4.5.4, Table 3.8)}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 (BS 3.4.5.8)

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3}, \; 1 \le k_2 \le \left(\frac{40}{25}\right)^{1/3} \qquad \text{(BS 3.4.5.4, Table 3.8)}$$

$\gamma_m = 1.25$ (BS 3.4.5.2)

However, the following limitations also apply:

$$0.15 \le \frac{100\ A_s}{bd} \le 3$$ (BS 3.4.5.4, Table 3.8)

$$\left(\frac{400}{d}\right)^{1/4} \ge 0.67 \text{ (unreinforced) or } \ge 1 \text{ (reinforced)}$$ (BS 3.4.5.4)

$$v \le \min(0.8\sqrt{f_{cu}}, 5\text{MPa})$$ (BS 3.7.6.4)

For light-weight concrete, $v_{\max}$ is defined as:

$$v \le \min(0.63\sqrt{f_{cu}}, 4 \text{ MPa})$$ (BS 8110-2:1985 5.4)

$f_{cu} \le 40$ MPa (for calculation purpose only) (BS 3.4.5.4)

A_s = area of tension reinforcement, which is taken as the average tension reinforcement of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement is taken as zero in the current implementation.

4.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, $v_{\max}$, is calculated as:

$$V_{eff,x} = V\left(f + \frac{1.5M_x}{Vy}\right)$$ (BS 3.7.6.2, 3.7.6.3)

$$V_{eff,y} = V\left(f + \frac{1.5M_y}{Vx}\right)$$ (BS 3.7.6.2, 3.7.6.3)

$$v_{\max} = \max \begin{cases} \dfrac{V_{eff,x}}{u\,d} \\ \dfrac{V_{eff,y}}{u\,d} \end{cases} \qquad \text{(BS 3.7.7.3)}$$

where,

u is the perimeter of the critical section

x and y are the lengths of the sides of the critical section parallel to the axis of bending

M_x and M_y are the design moments transmitted from the slab to the column at the connection

V is the total punching shear force

f is a factor to consider the eccentricity of punching shear force and is taken as:

$$f = \begin{cases} 1.00 & \text{for interior columns} \\ 1.25 & \text{for edge columns} \\ 1.25 & \text{for corner columns} \end{cases} \qquad \text{(BS 3.7.6.2, 3.7.6.3)}$$

4.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

4.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (BS 3.7.7.5). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as explained in the subsections that follow.

4.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

4.6.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

$$v_{\max} = 2v_c \qquad \text{(BS 3.7.7.5)}$$

Given v, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows (BS 3.7.7.5).

- If $v \leq 1.6v_c$,

$$\frac{A_v}{s} = \frac{(v - v_c)ud}{0.87\, f_{yv}} \geq \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(BS 3.7.7.5)}$$

- If $1.6v_c \leq v < 2.0v_c$,

$$\frac{A_v}{s} = \frac{5(0.7v - v_c)ud}{0.87\, f_{yv}} \geq \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(BS 3.7.7.5)}$$

- If $v > v_{\max}$, a failure condition is declared. (BS 3.7.7.5)

If v exceeds the maximum permitted value of $v_{\max}$, the concrete section should be increased in size.

4.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 4-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed *d*/2. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5*d* measured in a direction parallel to the column face (BS 3.7.7.6).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

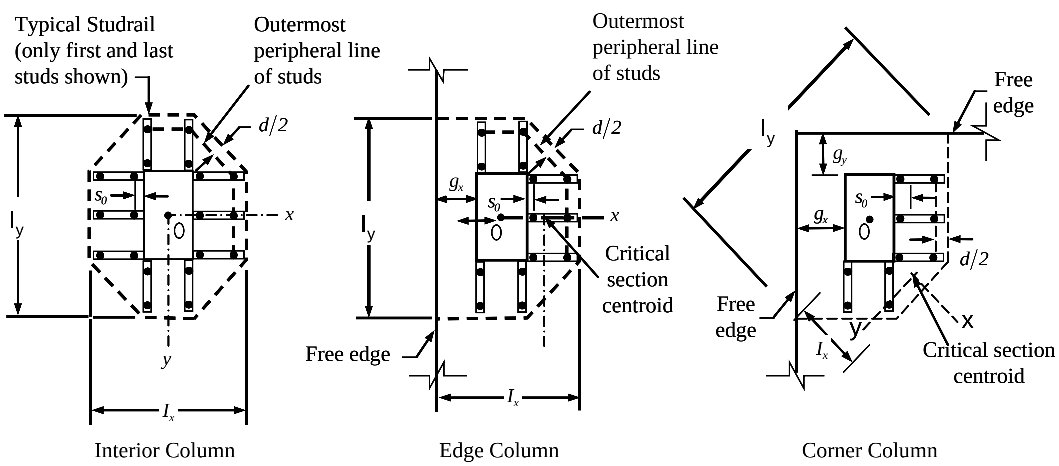


Figure 4-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

4.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in BS 3.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 1.5*d*. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(BS 3.7.7.6)}$$

$$s \leq 0.75d \qquad \text{(BS 3.7.7.6)}$$

$$g \leq 1.5d \qquad \text{(BS 3.7.7.6)}$$

Chapter 5
Design for CSA A23.3-04

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Canadian code CSA A23.3-04 [CSA 04] is selected. Various notations used in this chapter are listed in Table 5-1. For referencing to the pertinent sections of the Canadian code in this chapter, a prefix “CSA” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

5.1 Notations

Table 5-1 List of Symbols Used in the CSA A23.3-04 Code

A_c	Area enclosed by outside perimeter of concrete cross-section, sq-mm
A_{ct}	Area of concrete on flexural tension side, sq-mm

Table 5-1 List of Symbols Used in the CSA A23.3-04 Code

A_l	Area of longitudinal reinforcement for torsion, sq-mm
A_o	Gross area enclosed by shear flow path, sq-mm
A_{oh}	Area enclosed by centerline of outermost closed transverse torsional reinforcement, sq-mm
A_s	Area of tension reinforcement, sq-mm
A'_s	Area of compression reinforcement, sq-mm
$A_{s(\text{required})}$	Area of steel required for tension reinforcement, sq-mm
A_t / s	Area of closed shear reinforcement for torsion per unit length, sq-mm/mm
A_v	Area of shear reinforcement, sq-mm
A_v / s	Area of shear reinforcement per unit length, sq-mm/mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
b	Width of member, mm
b_f	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
c_b	Depth to neutral axis at balanced conditions, mm
d	Distance from compression face to tension reinforcement, mm
d_v	Effective shear depth, mm
d'	Distance from compression face to compression reinforcement, mm
h_s	Thickness of slab (flanged section), mm
E_c	Modulus of elasticity of concrete, MPa

Table 5-1 List of Symbols Used in the CSA A23.3-04 Code

E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f'_c	Specified compressive strength of concrete, MPa
f'_s	Stress in the compression reinforcement, psi
f_y	Specified yield strength of flexural reinforcement, MPa
f_{yt}	Specified yield strength of shear reinforcement, MPa
h	Overall depth of a section, mm
I_g	Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.
M_f	Factored moment at section, N-mm
N_f	Factored axial force at section, N
p_c	Outside perimeter of concrete cross-section, mm
p_h	Perimeter of area A_{oh}, mm
s	Spacing of the shear reinforcement along the length of the beam, mm
s_z	Crack spacing parameter
T_f	Factored torsion at section, N-mm
V_c	Shear resisted by concrete, N
$V_{r,\max}$	Maximum permitted total factored shear force at a section, N
V_f	Factored shear force at a section, N
V_s	Shear force at a section resisted by steel, N
α_1	Ratio of average stress in rectangular stress block to the specified concrete strength
β	Factor accounting for shear resistance of cracked concrete
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
ε_s	Strain in reinforcing steel

Table 5-1 List of Symbols Used in the CSA A23.3-04 Code

ε_x	Longitudinal strain at mid-depth of the section
ϕ_c	Strength reduction factor for concrete
ϕ_s	Strength reduction factor for steel
ϕ_m	Strength reduction factor for member
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
θ	Angle of diagonal compressive stresses, degrees
λ	Shear strength factor

5.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For CSA A23.3-04, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (CSA 8.3.2, Table C.1)

1.4D (CSA 8.3.2, Table C.1 Case 1)

1.25D + 1.5L
1.25D + 1.5L + 0.5S
1.25D + 1.5L ± 0.4W
0.9D + 1.5L (CSA 8.3.2, Table C.1 Case 2)
0.9D + 1.5L + 0.5S
0.9D + 1.5L ± 0.4W

1.25D + 1.5(0.75 PL) (CSA 13.8.4.3)

1.25D + 1.5S
1.25D + 1.5S + 0.5L
1.25D + 1.5S ± 0.4W (CSA 8.3.2, Table C.1 Case 3)
0.9D + 1.5S
0.9D + 1.5S + 0.5L

$0.9D + 1.5S \pm 0.4W$

$1.25D \pm 1.4W$
$1.25D + 0.5L \pm 1.4W$
$1.25D + 0.5S \pm 1.4W$
$0.9D \pm 1.4W$ (CSA 8.3.2, Table C.1 Case 4)
$0.9D + 0.5L \pm 1.4W$
$0.9D + 0.5S \pm 1.4W$

$1.0D \pm 1.0E$
$1.0D + 0.5L \pm 1.0E$
$1.0D + 0.25S \pm 1.0E$ (CSA 8.3.2, Table C.1 Case 5)
$1.0D + 0.5L + 0.25S \pm 1.0E$

These are also the default design load combinations in SAFE whenever the CSA A23.3-04 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

5.3 Limits on Material Strength

The upper and lower limits of f'_c are 80 MPa and 20 MPa, respectively, for all framing types (CSA 8.6.1.1).

$$20 \text{ MPa} \leq f'_c \leq 80 \text{ MPa} \qquad \text{(CSA 8.6.1.1)}$$

The upper limit of f_y is 500 MPa for all frames (CSA 8.5.1).

$$f_y \leq 500 \text{ MPa} \qquad \text{(CSA 8.5.1)}$$

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

5.4 Strength Reduction Factors

The strength reduction factors, ϕ, are material dependent and defined as:

ϕ_c = 0.65 for concrete (CSA 8.4.2)

ϕ_s = 0.85 for reinforcement (CSA 8.4.3a)

These values can be overwritten; however, caution is advised.

5.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

5.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

5.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom reinforcement. In such cases the beam may be designed as a rectangular or flanged beam. Negative beam moments produce top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

5.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 5-1 (CSA 10.1.7). Furthermore, it is assumed that the compression carried by the concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed (0.1 $f'_c A_g$), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than 0.1 $f'_c A_g$ and axial tensions are always included in flexural and shear design.

5.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M_f (i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 5-1), where,

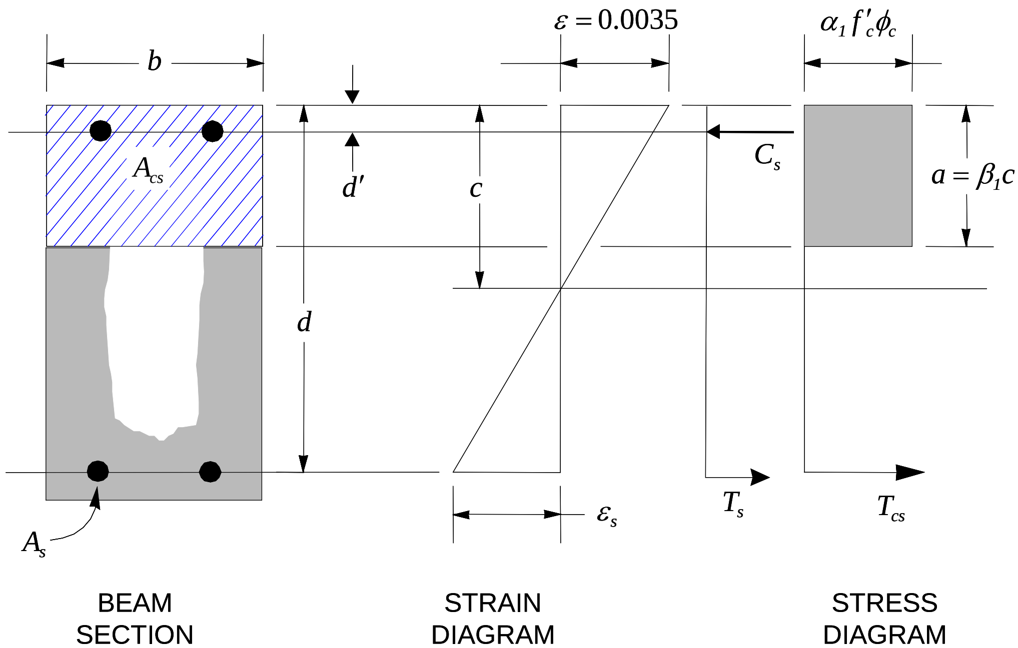


Figure 5-1 Rectangular Beam Design

$$a = d - \sqrt{d^2 - \frac{2\left|M_f\right|}{\alpha_1 f'_c \phi_c b}} \qquad \text{(CSA 10.1)}$$

where the value of ϕ_c is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters α_1, β_1, and c_b are calculated as:

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d \qquad \text{(CSA 10.5.2)}$$

The balanced depth of the compression block is given by:

$$a_b = \beta_1 c_b \qquad \text{(CSA 10.1.7)}$$

- If $a \le a_b$ (CSA 10.5.2), the area of tension reinforcement is given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2} \right)}$$

This reinforcement is to be placed at the bottom if M_f is positive, or at the top if M_f is negative.

- If $a > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The factored compressive force developed in the concrete alone is given by:

$$C = \phi_c \alpha_1 f'_c b a_b \qquad \text{(CSA 10.1.7)}$$

and the factored moment resisted by concrete compression and tension reinforcement is:

$$M_{fc} = \mathrm{C}\left(d - \frac{a_b}{2} \right)$$

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{fs} = M_f - M_{fc}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{fs}}{\left(\phi_s f'_s - \phi_c \alpha_1 f'_c \right)\left(d - d' \right)}, \text{ where}$$

$$f'_s = 0.0035\, E_s \left[\frac{c - d'}{c} \right] \le f_y \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{2}\right)\phi_s}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{fs}}{f_y\left(d - d'\right)\phi_s}$$

Therefore, the total tension reinforcement, $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa if M_f is negative.

5.5.1.2.2 Design of Flanged Beams

5.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_f (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

5.5.1.2.2.2 Flanged Beam Under Positive Moment

- If $M_f > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_f}{\alpha_1 f'_c \phi_c b_f}} \qquad \text{(CSA 10.1)}$$

where, the value of ϕ_c is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters α_1, β_1, and c_b are calculated as:

$$\alpha_1 = 0.85 - 0.0015\, f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025\, f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y}\, d \qquad \text{(CSA 10.5.2)}$$

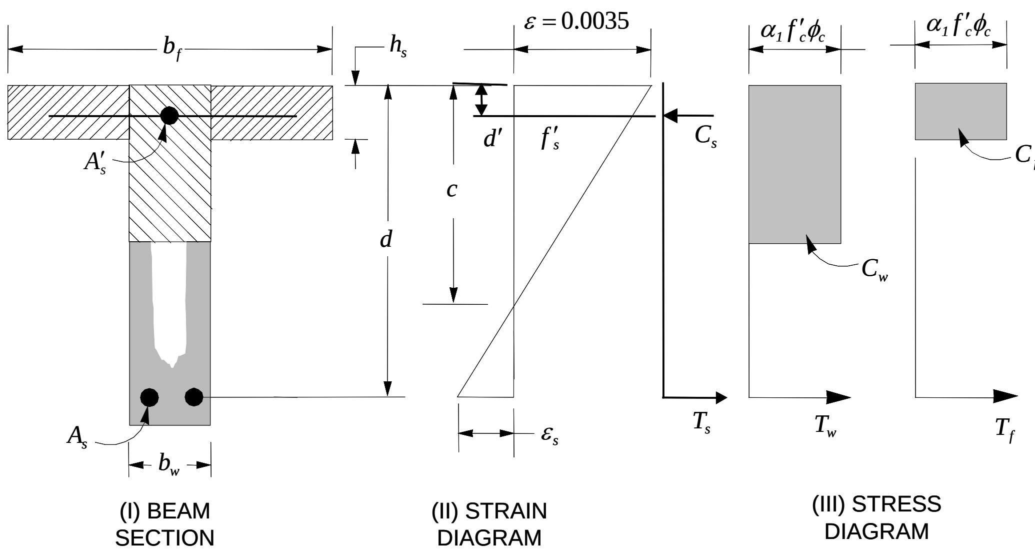


Figure 5-2 Design of a T-Beam Section

The balanced depth of the compression block is given by:

$$a_b = \beta_1 c_b \qquad \text{(CSA 10.1.4,10.1.7)}$$

- If $a \le h_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_b$.

- If $a > h_s$, calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w as shown in Figure 5-2. C_f is given by:

$$C_f = \alpha_1 f'_c \left(b_f - b_w\right) \min\left(h_s, a_b\right) \qquad \text{(CSA 10.1.7)}$$

Therefore, $A_{s1} = \dfrac{C_f \phi_c}{f_y \phi_s}$ and the portion of M_f that is resisted by the flange is given by:

$$M_{ff} = C_f\left(d - \frac{\min(h_s, a_b)}{2}\right)\phi_c$$

Therefore, the balance of the moment, M_f to be carried by the web is:

$$M_{fw} = M_f - M_{ff}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f'_c \phi_c b_w}} \qquad \text{(CSA 10.1)}$$

- If $a_1 \le a_b$ (CSA 10.5.2), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{fw}}{\phi_s f_y\left(d - \frac{a_1}{2}\right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The compressive force in the web concrete alone is given by:

$$C = \phi_c \alpha_1 f'_c b_w a_b \qquad \text{(CSA 10.1.7)}$$

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{fc} = C\left(d - \frac{a_b}{2}\right)$$

and the moment resisted by compression and tension reinforcement is:

$$M_{fs} = M_{fw} - M_{fc}$$

Therefore, the compression reinforcement is computed as:

$$A'_s = \frac{M_{fs}}{\left(\phi_s f'_c - \phi_c \alpha_1 f'_c\right)\left(d - d'\right)}, \text{ where}$$

$$f'_s = \varepsilon_c E_s \left[\frac{c - d'}{c}\right] \leq f_y \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The tension reinforcement for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{fc}}{f_y \left(d - \frac{a_b}{2}\right) \phi_s}$$

and the tension reinforcement for balancing the compression reinforcement is:

$$A_{s3} = \frac{M_{fs}}{f_y \left(d - d'\right) \phi_s}$$

The total tension reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

5.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the following two limits:

$$A_s \geq \frac{0.2\sqrt{f'_c}}{f_y} b_w h \qquad \text{(CSA 10.5.1.2)}$$

$$A_s \geq \frac{4}{3} A_{s(\text{required})} \qquad \text{(CSA 10.5.1.3)}$$

In addition, the minimum flexural tension reinforcement provided in a flanged beam with the flange under tension in an ordinary moment resisting frame is given by the limit:

$$A_s \geq 0.004\ (b - b_w)\ h_s \qquad \text{(CSA 10.5.3.1)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

5.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_f.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

5.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

5.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v \qquad \text{(CSA 11.3.4)}$$

$$\sqrt{f'_c} \leq 8 \text{ MPa} \qquad \text{(CSA 11.3.4)}$$

ϕ_c is the resistance factor for concrete. By default it is taken as 0.65 (CSA 8.4.2).

λ is the strength reduction factor to account for low density concrete (CSA 2.2). For normal density concrete, its value is 1 (CSA 8.6.5), which is taken by the program as the default value. For concrete using lower density aggregate, the user can change the value of λ in the material property data. The recommended value for λ is as follows (CSA 8.6.5):

$$\lambda = \begin{cases} 1.00, & \text{for normal density concrete,} \\ 0.85, & \text{for semi-low-density concrete} \\ & \text{in which all of the fine aggregate is natural sand,} \\ 0.75, & \text{for semi-low-density concrete} \\ & \text{in which none of the fine aggregate is natural sand.} \end{cases} \qquad \text{(CSA 8.6.5)}$$

β is the factor for accounting for the shear resistance of cracked concrete (CSA 2.2). Its value is normally between 0.1 and 0.4. It is determined according to CSA 11.3.6, and described further in the following sections.

b_w is the effective web width. For rectangular beams, it is the width of the beam. For flanged beams, it is the width of the web of the beam.

d_v is the effective shear depth. It is taken as the greater of 0.9*d* or 0.72*h*(CSA 2.3), where *d* is the distance from the extreme compression fiber to the centroid of the tension reinforcement, and *h* is the overall depth of the cross-section in the direction of the shear force (CSA 2.3).

The value of β is preferably taken as the special value (CSA 11.3.6.2) or it is determined using the simplified method (CSA 11.3.6.3), if applicable. When the conditions of the special value or simplified method do not apply, the general method is used (CSA 11.3.6.4).

If the overall beam depth, *h*, is less than 250 mm or if the depth of a flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, β is taken as 0.21 (CSA 11.3.6.2).

When the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, the specified concrete strength f'_c does not exceed 60 MPa, and the tensile force is negligible, β is determined in accordance with the simplified method, as follows (CSA 11.3.6.3):

- When the section contains at least the minimum transverse reinforcement, β is taken as 0.18 (CSA 11.6.3.3a).

$$\beta = 0.18 \qquad \text{(CSA 11.3.6.3(a))}$$

- When the section contains no transverse reinforcement, β is determined based on the specified maximum nominal size of coarse aggregate, a_g.

For a maximum size of coarse aggregate not less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + d_v} \qquad \text{(CSA 11.3.6.3(b))}$$

where d_v is the effective shear depth expressed in millimeters.

For a maximum size of coarse aggregate less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + s_{ze}} \qquad \text{(CSA 11.3.6.3 c)}$$

$$\text{where, } s_{ze} = \frac{35 s_z}{15 + a_g} \geq 0.85 s_z \qquad \text{(CSA 11.3.6.3.c)}$$

In the preceding expression, the crack spacing parameter, s_{ze}, shall be taken as the minimum of d_v and the maximum distance between layers of distributed longitudinal reinforcement. However, s_{ze} is conservatively taken as equal to d_v.

In summary, for simplified cases, β can be expressed as follows:

$$\beta = \begin{cases} 0.18, & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{230}{1000 + d_v}, & \text{if no transverse reinforcement is provided, and } a_g \geq 20\text{mm,} \\ \dfrac{230}{1000 + S_{ze}}, & \text{if no transverse reinforcement is provided, and } a_g < 20\text{mm.} \end{cases}$$

- When the specified yield strength of the longitudinal reinforcing f_y is greater than 400 MPa, the specified concrete strength f'_c is greater than 60 MPa, or tension is not negligible, β is determined in accordance with the general method as follows (CSA 11.3.6.1, 11.3.6.4):

$$\beta = \frac{0.40}{(1 + 1500\varepsilon_x)} \bullet \frac{1300}{(1000 + S_{ze})} \qquad \text{(CSA 11.3.6.4)}$$

In the preceding expression, the equivalent crack spacing parameter, s_{ze} is taken equal to 300 mm if minimum transverse reinforcement is provided (CSA 11.3.6.4). Otherwise it is determined as stated in the simplified method.

$$S_{ze} = \begin{cases} 300 & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{35}{15 + a_g} S_z \geq 0.85 S_z & \text{otherwise.} \end{cases}$$

(CSA 11.3.6.3, 11.3.6.4)

The value of a_g in the preceding equations is taken as the maximum aggregate size for f'_c of 60 MPa, is taken as zero for f'_c of 70 MPa, and linearly interpolated between these values (CSA 11.3.6.4).

The longitudinal strain, ε_x at mid-depth of the cross-section is computed from the following equation:

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5 N_f}{2(E_s A_s)} \qquad \text{(CSA 11.3.6.4)}$$

In evaluating ε_x the following conditions apply:

- ε_x is positive for tensile action.

- V_f and M_f are taken as positive quantities. (CSA 11.3.6.4(a))
- M_f is taken as a minimum of $V_f\, d_v$. (CSA 11.3.6.4(a))
- N_f is taken as positive for tension. (CSA 2.3)

A_s is taken as the total area of longitudinal reinforcement in the beam. It is taken as the envelope of the reinforcement required for all design load combinations. The actual provided reinforcement might be slightly higher than this quantity. The reinforcement should be developed to achieve full strength (CSA 11.3.6.3(b)).

If the value of ε_x is negative, it is recalculated with the following equation, in which A_{ct} is the area of concrete in the flexural tensile side of the beam, taken as half of the total area.

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5 N_f}{2\left(E_s A_s + E_c A_{ct}\right)} \quad \text{(CSA 11.3.6.4(c))}$$

$$E_s = 200{,}000 \text{ MPa} \quad \text{(CSA 8.5.4.1)}$$

$$E_c = 4500\sqrt{f_c'} \text{ MPa} \quad \text{(CSA 8.6.2.3)}$$

If the axial tension is large enough to induce tensile stress in the section, the value of ε_x is doubled (CSA 11.3.6.4(e)).

For sections closer than d_v from the face of the support, ε_x is calculated based on M_f and V_f at a section at a distance d_v from the face of the support (CSA 11.3.6.4(d)). This condition currently is not checked by SAFE.

An upper limit on ε_x is imposed as:

$$\varepsilon_x \le 0.003 \quad \text{(CSA 11.3.6.4(f))}$$

In both the simplified and general methods, the shear strength of the section due to concrete, v_c depends on whether the minimum transverse reinforcement is provided. To check this condition, the program performs the design in two passes. In the first pass, it assumes that no transverse shear reinforcement is needed. When the program determines that shear reinforcement is required, the program performs the second pass assuming that at least minimum shear reinforcement is provided.

5.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to $V_{r,\max}$ where:

$$V_{r,\max} = 0.25\phi_c f'_c b_w d \qquad \text{(CSA 11.3.3)}$$

Given V_f, V_c, and $V_{r,\max}$, the required shear reinforcement is calculated as follows:

- If $V_f \le V_c$,

$$\frac{A_v}{s} = 0 \qquad \text{(CSA 11.3.5.1)}$$

- If $V_c < V_f \le V_{r,\max}$,

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)\tan\theta}{\phi_s f_{yt} d_v} \qquad \text{(CSA 11.3.3, 11.3.5.1)}$$

- If $V_f > V_{r,\max}$, a failure condition is declared. (CSA 11.3.3)

A minimum area of shear reinforcement is provided in the following regions (CSA 11.2.8.1):

(a) in regions of flexural members where the factored shear force V_f exceeds V_c

(b) in regions of beams with an overall depth greater than 750 mm

(c) in regions of beams where the factored torsion T_f exceeds $0.25T_{cr}$.

Where the minimum shear reinforcement is required by CSA 11.2.8.1, or by calculation, the minimum area of shear reinforcement per unit spacing is taken as:

$$\frac{A_v}{s} \geq 0.06 \frac{\sqrt{f'_c}}{f_{yt}} b_w \qquad \text{(CSA 11.2.8.2)}$$

In the preceding equations, the term θ is used, where θ is the angle of inclination of the diagonal compressive stresses with respect to the longitudinal axis of the member (CSA 2.3). The θ value is normally between 22 and 44 degrees. It is determined according to CSA 11.3.6.

Similar to the β factor, which was described previously, the value of θ is preferably taken as the special value (CSA 11.3.6.2), or it is determined using the simplified method (CSA 11.3.6.3), whenever applicable. The program uses the general method when conditions for the simplified method are not satisfied (CSA 11.3.6.4).

- If the overall beam depth, h, is less than 250 mm or if the depth of the flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, θ is taken as 42 degrees (CSA 11.3.6.2).

- If the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, or the specified concrete strength f'_c does not exceed 60 MPa, θ is taken to be 35 degree (CSA 11.3.6.3).

$$\theta = 35^{\circ} \text{ for } P_f \leq 0 \text{ or } f_y \leq 400 \text{ MPa or } f'_c \leq 60 \text{ MPa} \qquad \text{(CSA11.3.6.3)}$$

- If the axial force is tensile, the specified yield strength of the longitudinal reinforcing f_y > 400 MPa, and the specified concrete strength f'_c > 60 MPa, θ is determined using the general method as follows (CSA 11.3.6.4),

$$\theta = 29 + 7000\varepsilon_x \text{ for } P_f < 0, \quad f_y > 400 \text{ MPa}, f'_c \leq 60 \text{ MPa} \qquad \text{(CSA11.3.6.4)}$$

where ε_x is the longitudinal strain at the mid-depth of the cross-section for the factored load. The calculation procedure is described in preceding sections.

The maximum of all of the calculated A_v/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup require-

ments to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

5.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_f.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

5.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_f is permitted to be reduced in accordance with the code (CSA 11.2.9.2). However, the program does not automatically redistribute the internal forces and reduce T_f. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

5.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_c, A_{oh}, A_o, p_c, and p_h, are calculated. These properties are described in the following (CSA 2.3).

A_c = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_c = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 millimeters. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as follows:

$$A_c = bh \qquad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \qquad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b + 2h \qquad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \qquad \text{(CSA 11.3.10.4)}$$

where, the section dimensions b, h, and c are shown in Figure 5-3. Similarly, the special section properties for a flanged beam section are given as follows:

$$A_c = b_w h + (b_f - b_w)h_s \qquad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b_w - 2c)(h - 2c) \qquad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b_f + 2h \qquad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(h - 2c) + 2(b_w - 2c) \qquad \text{(CSA 11.3.10.4)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 5-3. Note that the flange width on either side of the beam web is limited to the smaller of $6h_s$ or 1/12 the span length (CSA 10.3.4).

5.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored, is calculated as:

$$T_{cr} = \frac{0.38\lambda\phi_c\sqrt{f'_c}\left(\dfrac{A_c^2}{p_c}\right)}{4} \qquad \text{(CSA 11.2.9.1)}$$

where A_{cp} and p_c are the area and perimeter of the concrete cross-section as described in the previous section, λ is a factor to account for low-density concrete, ϕ_c is the strength reduction factor for concrete, which is equal to 0.65, and f'_c is the specified concrete compressive strength.

5.5.3.4 Determine Torsion Reinforcement

If the factored torsion T_f is less than the threshold limit, T_{cr}, torsion can be safely ignored (CSA 11.2.9.1). In that case, the program reports that no torsion reinforcement is required. However, if T_f exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CSA 11.3).

- If $T_f > T_{cr}$, the required closed stirrup area per unit spacing, A_t/s, is calculated as:

$$\frac{A_t}{s} = \frac{T_f \tan\theta}{\phi_s 2A_o f_{yt}} \qquad \text{(CSA 11.3.10.3)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{\dfrac{M_f}{d_v} + 0.5N_f + \sqrt{\left(V_f - 0.5V_s\right)^2 + \left(\dfrac{0.45\,p_h T_f}{2A_o}\right)^2}\cot\theta}{\phi_s f_y}$$

(CSA 11.3.10.6, 11.3.9)

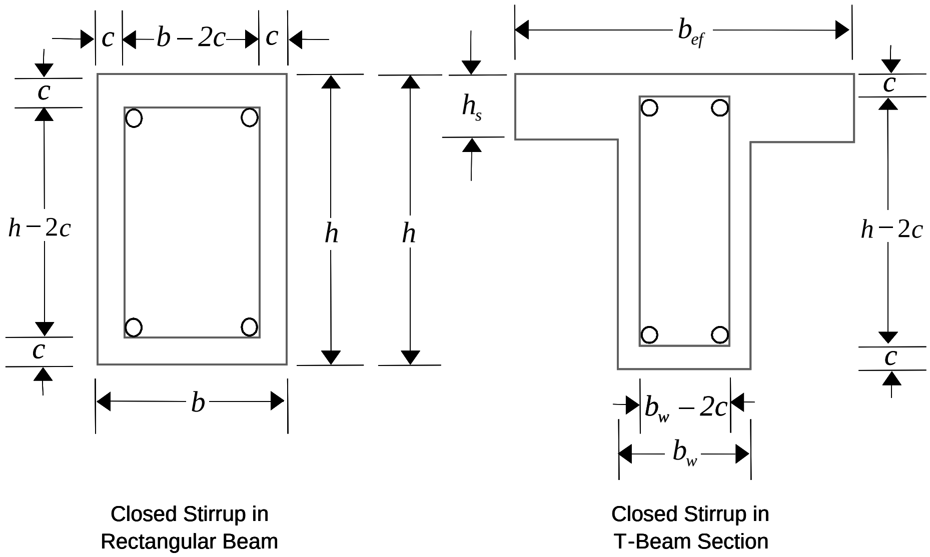


Figure 5-3 Closed stirrup and section dimensions for torsion design

In the preceding expressions, θ is computed as previously described for shear, except that if the general method is being used, the value ε_x, calculated as specified in CSA 11.3.6.4, is replaced by:

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{V_f^{\,2} + \left(\dfrac{0.9 p_h T_f}{2A_o}\right)^2} + 0.5N_f}{2\left(E_s A_s\right)} \qquad \text{(CSA 11.3.10.5)}$$

An upper limit of the combination of V_u and T_u that can be carried by the section also is checked using the equation:

$$\sqrt{\left(\frac{V_f}{b_w d_v}\right)^2 + \left(\frac{T_f p_h}{1.7 A_{oh}^2}\right)^2} \le 0.25\phi_c f'_c \qquad \text{(CSA 11.3.10.4(b))}$$

For rectangular sections, b_w is replaced with b. If the combination of V_f and T_f exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_f > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s}+2\frac{A_t}{s}\right)\geq 0.06\sqrt{f'_c}\frac{b_w}{f_{yt}} \qquad \text{(CSA 11.2.8.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

5.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is performed based on the ultimate strength design method (CSA A23.3-04) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

5.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.

- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

5.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

5.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

5.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limit (CSA 13.10.1):

$A_s \geq 0.002\ bh$ (CSA 7.8.1)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

5.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

5.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d*/2 from the face of the support (CSA 13.3.3.1 and CSA 13.3.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.3.3.3). Figure 5-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

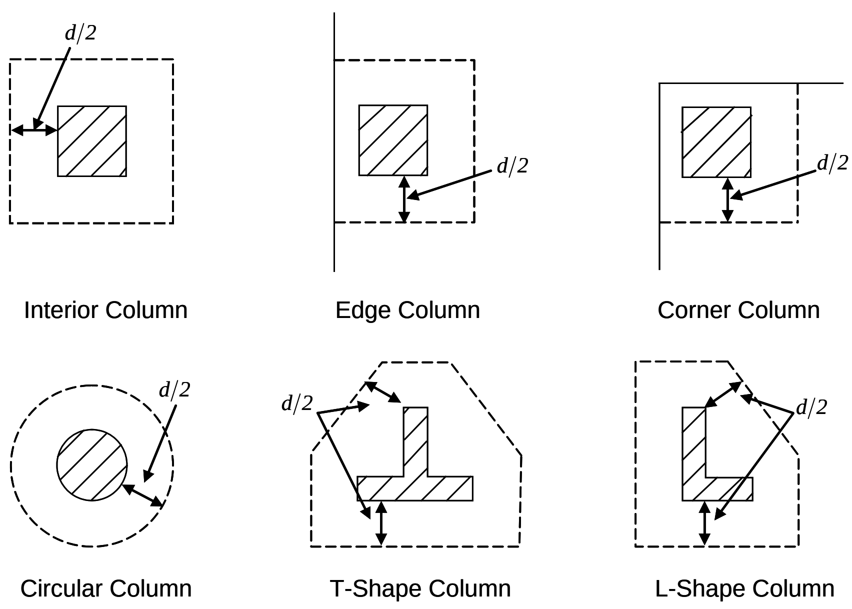


Figure 5-4 Punching Shear Perimeters

5.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(CSA 13.10.2)}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \qquad \text{(CSA 13.3.5.3)}$$

where b_1 is the width of the critical section measured in the direction of the span, and b_2 is the width of the critical section measured in the direction perpendicular to the span.

5.6.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_v = \min \begin{cases} \phi_c \left(1 + \dfrac{2}{\beta_c}\right) 0.19 \lambda \sqrt{f'_c} \\ \phi_c \left(0.19 + \dfrac{\alpha_s d}{b_0}\right) \lambda \sqrt{f'_c} \\ \phi_c 0.38 \lambda \sqrt{f'_c} \end{cases} \qquad \text{(CSA 13.3.4.1)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4, \text{ for interior columns} \\ 3, \text{ for edge colums, and} \\ 2, \text{ for corner columns.} \end{cases} \qquad \text{(CSA 13.3.4.1(b))}$$

The value of $\sqrt{f'_c}$ is limited to 8 MPa for the calculation of the concrete shear capacity (CSA 13.3.4.2).

If the effective depth, *d*, exceeds 300 mm, the value of v_c is reduced by a factor equal to 1300/(1000 + *d*) (CSA 13.3.4.3).

5.6.2.4 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

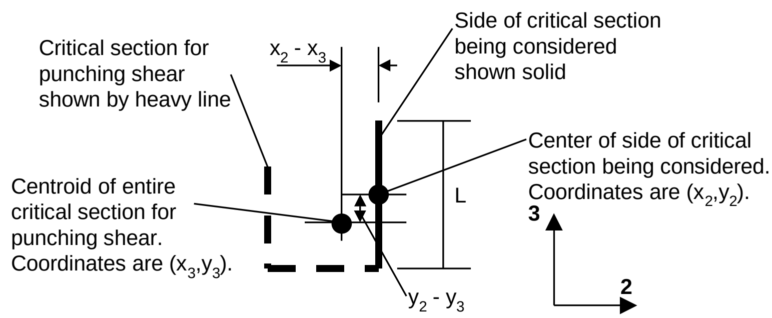
$$v_f = \frac{V_f}{b_0 d} + \frac{\gamma_{V2}[M_{f2} - V_f(y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^{\;2}} - \frac{\gamma_{V3}[M_{f3} - V_f(x_3 - x_1)][I_{22}(x_4 - x_3) - I_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}^{\;2}} \quad \text{Eq. 1}$$

$I_{22} = \sum_{sides=1}^{n} \overline{I}_{22}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 2

$I_{33} = \sum_{sides=1}^{n} \overline{I}_{33}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 3

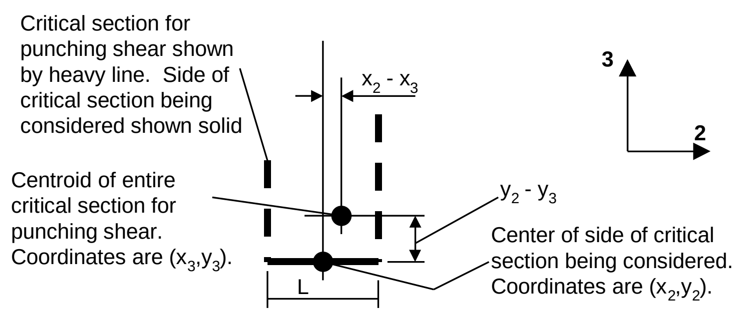
$I_{23} = \sum_{sides=1}^{n} \overline{I}_{23}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 4

The equations for $\overline{I}_{22}, \overline{I}_{33}$, and $\overline{I}_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 5-5.



Plan View For Side of Critical Section Parallel to 3-Axis

Work This Sketch With Equations 5b, 6b and 7



Plan View For Side of Critical Section Parallel to 2-Axis

Work This Sketch With Equations 5a, 6a and 7

Figure 5-5 Shear Stress Calculations at Critical Sections

$$\overline{I}_{22} = Ld(y_2 - y_3)^2, \text{ for side of critical section parallel to 2-axis} \qquad \text{Eq. 5a}$$

$$\overline{I}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2, \text{ for side of critical section parallel to 3-axis} \qquad \text{Eq. 5b}$$

$$\overline{I}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2, \text{ for side of critical section parallel to 2-axis} \qquad \text{Eq. 6a}$$

$$\overline{I}_{33} = Ld(x_2 - x_3)^2, \text{ for side of critical section parallel to 3-axis} \qquad \text{Eq. 6b}$$

$$\overline{I}_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \qquad \text{Eq. 7}$$

NOTE: $\overline{I}_{23}$ is explicitly set to zero for corner condition.

where,

b_0 = Perimeter of the critical section for punching shear

d = Effective depth at the critical section for punching shear based on the average of d for 2 direction and d for 3 direction

I_{22} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis

I_{33} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis

I_{23} = Product of inertia of the critical section for punching shear with respect to the 2 and 3 planes

L = Length of the side of the critical section for punching shear currently being considered

M_{f2} = Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)

M_{f3} = Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)

V_f = Punching shear stress

V_f = Shear at the center of the column (positive upward)

x_1, y_1 = Coordinates of the column centroid

x_2, y_2 = Coordinates of the center of one side of the critical section for punching shear

x_3, y_3 = Coordinates of the centroid of the critical section for punching shear

x_4, y_4 = Coordinates of the location where stress is being calculated

γ_{V2} = Percent of M_{f2} resisted by shear

γ_{V3} = Percent of M_{f3} resisted by shear

5.6.2.5 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

5.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CSA 13.2.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed, and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as explained in the subsections that follow.

5.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is taken as:

$$v_c = 0.28\lambda\phi_c\sqrt{f'_c} \quad \text{for shear studs} \qquad \text{(CSA 13.3.8.3)}$$

$$v_c = 0.19\lambda\phi_c\sqrt{f'_c} \quad \text{for shear stirrups} \qquad \text{(CSA 13.3.9.3)}$$

5.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of $v_{r,\max}$, where

$$v_{r,\max} = 0.75\lambda\phi_c\sqrt{f'_c} \text{ for shear studs} \quad \text{(CSA 13.3.8.2)}$$

$$v_{r,\max} = 0.55\lambda\phi_c\sqrt{f'_c} \text{ for shear stirrups} \quad \text{(CSA 13.3.9.2)}$$

Given v_f, v_c, and $v_{f,,\max}$, the required shear reinforcement is calculated as follows, where, ϕ_s, is the strength reduction factor.

- If $v_f > v_{r,\max}$,

$$\frac{A_v}{s} = \frac{\left(v_f - v_c\right)}{\phi_s f_{yv}} b_o \quad \text{(CSA 13.3.8.5, 13.3.9.4)}$$

- If $v_f > v_{r,\max}$, a failure condition is declared. (CSA 13.3.8.2)
- If v_f exceeds the maximum permitted value of $v_{r\max}$, the concrete section should be increased in size.

5.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 5-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

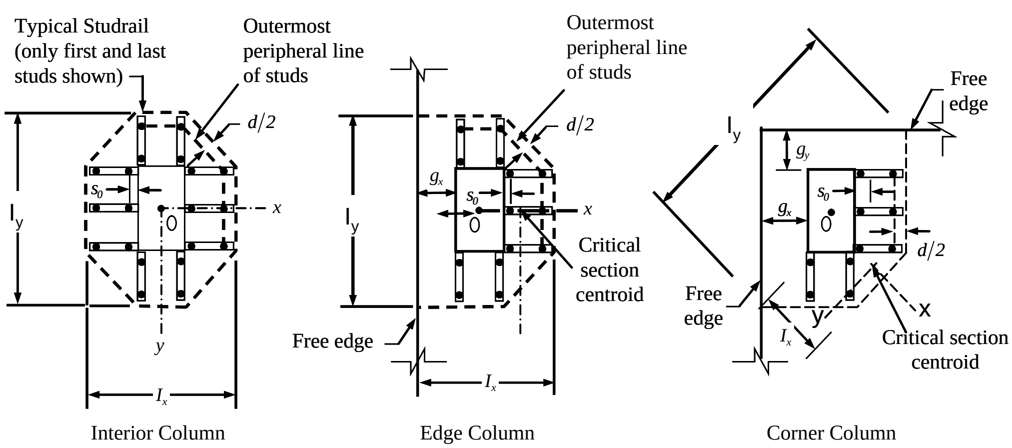


Figure 5-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed

0.4*d* for shear studs (CSA 13.3.8.6)

0.25*d* for shear stirrups (CSA 13.3.8.6)

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

5.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CSA 7.9 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.4*d*. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \le 0.4d \qquad \text{(CSA 13.3.8.6)}$$

$$s \le \begin{cases} 0.75d & v_f \le 0.56\lambda\phi_c\sqrt{f'_c} \\ 0.50d & v_f > 0.56\lambda\phi_c\sqrt{f'_c} \end{cases} \qquad \text{(CSA 13.3.8.6)}$$

For shear stirrups,

$$s_o \le 0.25d \qquad \text{(CSA 13.3.9.5)}$$

$$s \le 0.25d \qquad \text{(CSA 13.3.9.5)}$$

The minimum depth for reinforcement should be limited to 300 mm (CSA 13.3.9.1).

Chapter 6
Design for Eurocode 2-2004

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the European code Eurocode 2-2004 [EN 1992-1-1:2004] is selected. For the load combinations, reference is also made to Eurocode 0 [EN 1990], which is identified with the prefix “EC0.” Various notations used in this chapter are listed in Table 6-1. For referencing to the pertinent sections of the Eurocode in this chapter, a prefix “EC2” followed by the section number is used. It also should be noted that this section describes the implementation of the CEN Default version of Eurocode 2-2004, without a country specific National Annex. Where Nationally Determined Parameters [NDPs] are to be considered, this is highlighted in the respective section by the notation [*NDP*].

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

6.1 Notations

Table 6-1 List of Symbols Used in the Eurocode 2-2004

A_c	Area of concrete section, mm^2
A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sl}	Area of longitudinal reinforcement for torsion, mm^2
A_{sw}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sw}/s_v	Area of shear reinforcement per unit length, mm^2/mm
A_t/s	Area of transverse reinforcement per unit length for torsion, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement
f_{cd}	Design concrete strength = $\alpha_{cc} f_{ck} / \gamma_c$, MPa
f_{ck}	Characteristic compressive concrete cylinder strength at 28 days, MPa
f_{ctm}	Mean value of concrete axial tensile strength, MPa
f_{cwd}	Design concrete compressive strength for shear design = $\alpha_{cc} \, {f_{cwk}}/{\gamma_c}$, MPa
f_{cwk}	Characteristic compressive cylinder strength for shear design, MPa
f'_s	Compressive stress in compression reinforcement, MPa
f_{yd}	Design yield strength of reinforcement = f_{yk}/γ_s, MPa

Table 6-1 List of Symbols Used in the Eurocode 2-2004

f_{yk}	Characteristic strength of shear reinforcement, MPa
f_{ywd}	Design strength of shear reinforcement = f_{ywk} / γ_s, MPa
f_{ywk}	Characteristic strength of shear reinforcement, MPa
h	Overall depth of section, mm
h_f	Flange thickness, mm
M_{Ed}	Design moment at a section, N-mm
m	Normalized design moment, $M/bd^2\eta f_{cd}$
m_{lim}	Limiting normalized moment capacity as a singly reinforced beam
s_v	Spacing of the shear reinforcement, mm
T_{Ed}	Torsion at ultimate design load, N-mm
T_{Rdc}	Torsional cracking moment, N-mm
$T_{Rd,\text{max}}$	Design torsional resistance moment, N-mm
u	Perimeter of the punch critical section, mm
V_{Rdc}	Design shear resistance from concrete alone, N
$V_{Rd,\text{max}}$	Design limiting shear resistance of a cross-section, N
V_{Ed}	Shear force at ultimate design load, N
x	Depth of neutral axis, mm
x_{lim}	Limiting depth of neutral axis, mm
z	Lever arm, mm
α_{cc}	Coefficient accounting for long-term effects on the concrete compressive strength
α_{cw}	Coefficient accounting for the state of stress in the compression chord
δ	Redistribution factor
ε_c	Concrete strain
ε_s	Strain in tension reinforcement
ε'_s	Strain in compression steel

Table 6-1 List of Symbols Used in the Eurocode 2-2004

γ_c	Partial safety factor for concrete strength
γ_s	Partial safety factor for reinforcement strength
λ	Factor defining the effective depth of the compression zone
ν	Effectiveness factor for shear resistance without concrete crushing
η	Concrete strength reduction factor for sustained loading and stress block
ρ_l	Tension reinforcement ratio
σ_{cp}	Axial stress in the concrete, MPa
θ	Angle of the concrete compression strut
ω	Normalized tension reinforcement ratio
ω'	Normalized compression reinforcement ratio
ω_{lim}	Normalized limiting tension reinforcement ratio

6.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be checked. Eurocode 0-2002 allows load combinations to be defined based on EC0 Equation 6.10 or the less favorable of EC0 Equations 6.10a and 6.10b [*NDP*].

$$\sum_{j\geq 1}\gamma_{G,j}G_{k,j} + \gamma_P P + \gamma_{Q,1}Q_{k,1} + \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(EC0 Eq. 6.10)}$$

$$\sum_{j\geq 1}\gamma_{G,j}G_{k,j} + \gamma_P P + \gamma_{Q,1}\psi_{0,1}Q_{k,1} + \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(EC0 Eq. 6.10a)}$$

$$\sum_{j\geq 1}\xi_j\gamma_{G,j}G_{k,j} + \gamma_P P + \gamma_{Q,1}Q_{k,1} + \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(EC0 Eq. 6.10b)}$$

Load combinations considering seismic loading are automatically generated based on EC0 Equation 6.12b.

$$\sum_{j\geq1} G_{k,j} + P + A_{Ed} + \sum_{i>1} \psi_{2,i} Q_{k,i} \qquad \text{(EC0 Eq. 6.12b)}$$

For this code, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations need to be considered if equation 6.10 is specified for generation of the load combinations (EC0 6.4.3):

$\gamma_{Gj,sup}$ D (EC0 Eq. 6.10)

$\gamma_{Gj,sup}$D + $\gamma_{Q,1}$ L (EC0 Eq. 6.10)

$\gamma_{Gj,sup}$D + (0.75)$\gamma_{Q,1}$ PL (EC0 Eq. 6.10)

$\gamma_{Gj,inf}$ D ± $\gamma_{Q,1}$ W
$\gamma_{Gj,sup}$ D ± $\gamma_{Q,1}$ W (EC0 Eq. 6.10)

$\gamma_{Gj,sup}$ D + $\gamma_{Q,1}$ L ± $\gamma_{Q,i}$ $\psi_{0,i}$ W
$\gamma_{Gj,sup}$ D + $\gamma_{Q,1}$ L + $\gamma_{Q,i}$ $\psi_{0,i}$ S
$\gamma_{Gj,sup}$ D ± $\gamma_{Q,1}$ W + $\gamma_{Q,i}$ $\psi_{0,i}$ L
$\gamma_{Gj,sup}$ D ± $\gamma_{Q,1}$ W + $\gamma_{Q,i}$ $\psi_{0,i}$ S (EC0 Eq. 6.10)
$\gamma_{Gj,sup}$ D + $\gamma_{Q,1}$ S ± $\gamma_{Q,i}$ $\psi_{0,i}$ W
$\gamma_{Gj,sup}$ D + $\gamma_{Q,1}$ S + $\gamma_{Q,i}$ $\psi_{0,i}$ L

$\gamma_{Gj,sup}$ D + $\gamma_{Q,1}$ L + $\gamma_{Q,i}$ $\psi_{0,i}$ S ± $\gamma_{Q,i}$ $\psi_{0,i}$ W
$\gamma_{Gj,sup}$ D ± $\gamma_{Q,1}$ W + $\gamma_{Q,i}$ $\psi_{0,i}$ L + $\gamma_{Q,i}$ $\psi_{0,i}$ S (EC0 Eq. 6.10)
$\gamma_{Gj,sup}$ D + $\gamma_{Q,1}$ S ± $\gamma_{Q,i}$ $\psi_{0,i}$ W + $\gamma_{Q,i}$ $\psi_{0,i}$ L

D ± 1.0E
D ± 1.0E + $\psi_{2,i}$ L (EC0 Eq. 6.12b)
D ± 1.0E + $\psi_{2,i}$ L + $\psi_{2,i}$ S

If the load combinations are specified to be generated from the max of EC0 Equations 6.10a and 6.10b, the following load combinations from both equations are considered in the program.

$\gamma_{Gj,sup}$ D (EC0 Eq. 6.10a)
ξ $\gamma_{Gj,sup}$ D (EC0 Eq. 6.10b)

$\gamma_{Gj,sup}$D + $\gamma_{Q,1}$ $\psi_{0,1}$ L (EC0 Eq. 6.10a)

$\xi\, \gamma_{Gj,sup} D + \gamma_{Q,1} L$ (EC0 Eq. 6.10b)

$\gamma_{Gj,sup} D + (0.75)\gamma_{Q,1}\, \psi_{0,1}\, PL$ (EC0 Eq. 6.10a)
$\xi\, \gamma_{Gj,sup} D + (0.75)\gamma_{Q,1}\, PL$ (EC0 Eq. 6.10b)

$\gamma_{Gj,inf}\, D \pm \gamma_{Q,1}\, \psi_{0,1}\, W$ (EC0 Eq. 6.10a)
$\gamma_{Gj,sup}\, D \pm \gamma_{Q,1}\, \psi_{0,1}\, W$ (EC0 Eq. 6.10a)
$\gamma_{Gj,inf}\, D \pm \gamma_{Q,1}\, W$ (EC0 Eq. 6.10b)
$\xi\, \gamma_{Gj,sup}\, D \pm \gamma_{Q,1}\, W$ (EC0 Eq. 6.10b)

$\gamma_{Gj,sup}\, D + \gamma_{Q,1}\, \psi_{0,1}\, L \pm \gamma_{Q,i}\, \psi_{0,i}\, W$
$\gamma_{Gj,sup}\, D + \gamma_{Q,1}\, \psi_{0,1}\, L + \gamma_{Q,i}\, \psi_{0,i}\, S$
$\gamma_{Gj,sup}\, D \pm \gamma_{Q,1}\, \psi_{0,1}\, W + \gamma_{Q,i}\, \psi_{0,i}\, L$
$\gamma_{Gj,sup}\, D \pm \gamma_{Q,1}\, \psi_{0,1}\, W + \gamma_{Q,i}\, \psi_{0,i}\, S$
$\gamma_{Gj,sup}\, D + \gamma_{Q,1}\, \psi_{0,1}\, S + \gamma_{Q,i}\, \psi_{0,i}\, L$
$\gamma_{Gj,sup}\, D + \gamma_{Q,1}\, \psi_{0,1}\, S \pm \gamma_{Q,i}\, \psi_{0,i}\, W$
(EC0 Eq. 6.10a)

$\xi\, \gamma_{Gj,sup}\, D + \gamma_{Q,1}\, L \pm \gamma_{Q,i}\, \psi_{0,i}\, W$
$\xi\, \gamma_{Gj,sup}\, D + \gamma_{Q,1}\, L + \gamma_{Q,i}\, \psi_{0,i}\, S$
$\xi\, \gamma_{Gj,sup}\, D + \gamma_{Q,1}\, S \pm \gamma_{Q,i}\, \psi_{0,i}\, W$
$\xi\, \gamma_{Gj,sup}\, D + \gamma_{Q,1}\, S + \gamma_{Q,i}\, \psi_{0,i}\, L$
$\gamma_{Gj,inf}\, D \pm \gamma_{Q,1}\, W + \gamma_{Q,i}\, \psi_{0,i}\, L$
$\gamma_{Gj,inf}\, D \pm \gamma_{Q,1}\, W + \gamma_{Q,i}\, \psi_{0,i}\, S$
(EC0 Eq. 6.10b)

$D \pm 1.0E$
$D \pm 1.0E + \psi_{2,i}\, L$
$D \pm 1.0E + \psi_{2,i}\, L + \psi_{2,i}\, S$
(EC0 Eq. 6.12b)

For both sets of load combinations, the variable values for the CEN Default version of the load combinations are defined in the list that follows [*NDP*].

$\gamma_{Gj,sup} = 1.35$ (EC0 Table A1.2(B))

$\gamma_{Gj,inf} = 1.00$ (EC0 Table A1.2(B))

$\gamma_{Q,1} = 1.5$ (EC0 Table A1.2(B))

$\gamma_{Q,i} = 1.5$ (EC0 Table A1.2(B))

$\psi_{0,i} = 0.7$ (live load, assumed not to be storage) (EC0 Table A1.1)

$\psi_{0,i} = 0.6$ (wind load) (EC0 Table A1.1)

$\psi_{0,i}$ = 0.5 (snow load, assumed H ≤ 1,000 m) (EC0 Table A1.1)

ξ = 0.85 (EC0 Table A1.2(B))

$\psi_{2,i}$ = 0.3 (live, assumed office/residential space) (EC0 Table A1.1)

$\psi_{2,i}$ = 0 (snow, assumed H ≤ 1,000 m) (EC0 Table A1.1)

These are also the default design load combinations in SAFE whenever the Eurocode 2-2004 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

6.3 Limits on Material Strength

The concrete compressive strength, f_{ck}, should not be greater than 90 MPa (EC2 3.1.2(2)). The lower and upper limits of the reinforcement yield strength, f_{yk}, should be 400 and 600 MPa, respectively (EC2 3.2.2(3)).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. It is the user's responsibility to ensure that the minimum strength is satisfied.

6.4 Partial Safety Factors

The design strengths for concrete and steel are obtained by dividing the characteristic strengths of the materials by the partial safety factors, γ_s and γ_c as shown here [*NDP*].

$$f_{cd} = \alpha_{cc}\, f_{ck} / \gamma_c \qquad \text{(EC2 3.1.6(1))}$$

$$f_{yd} = f_{yk} / \gamma_s \qquad \text{(EC2 3.2.7(2))}$$

$$f_{ywd} = f_{ywk} / \gamma_s \qquad \text{(EC2 3.2.7(2))}$$

α_{cc} is the coefficient taking account of long term effects on the compressive strength. α_{cc} is taken as 1.0 by default and can be overwritten by the user (EC2 3.1.6(1)).

The partial safety factors for the materials and the design strengths of concrete and reinforcement are given in the text that follows (EC2 2.4.2.4(1), Table 2.1N):

Partial safety factor for reinforcement, $\gamma_s = 1.15$

Partial safety factor for concrete, $\gamma_c = 1.5$

These values are recommended by the code to give an acceptable level of safety for normal structures under regular design situations (EC2 2.4.2.4). For accidental and earthquake situations, the recommended values are less than the tabulated values. The user should consider those separately.

These values can be overwritten; however, caution is advised.

6.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

6.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

6.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

6.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 6-1 (EC2 3.1.7(3)). The area of the stress block and the depth of the compressive block is taken as:

$$F_c = \eta f_{cd} ab \qquad \text{(EC2 3.1.7(3), Fig 3.5)}$$

$$a = \lambda x \qquad \text{(EC2 3.1.7(3), Fig 3.5)}$$

where x is the depth of the neutral axis. The factor λ defining the effective height of the compression zone and the factor η defining the effective strength are given as:

$$\lambda = 0.8 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 3.1.7(3))}$$

$$\lambda = 0.8\left(\frac{f_{ck} - 50}{400}\right) \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa} \qquad \text{(EC2 3.1.7(3))}$$

$$\eta = 1.0 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 3.1.7(3))}$$

$$\eta = 1.0 - \left(\frac{f_{ck} - 50}{200}\right) \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa} \qquad \text{(EC2 3.1.7(3))}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed the code-specified limiting value. The code also places a limitation on the neutral axis depth, to safeguard against non-ductile failures (EC2 5.5(4)). When the applied moment exceeds the limiting moment capacity as a singly reinforced beam, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $\left(x/d\right)_{\text{lim}}$, is expressed as a function of the ratio of the redistributed moment to the moment before redistribution, δ, as follows:

$$\left(\frac{x}{d}\right)_{\text{lim}} = \frac{\delta - k_1}{k_2} \quad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 5.5(4))}$$

$$\left(\frac{x}{d}\right)_{\text{lim}} = \frac{\delta - k_3}{k_4} \quad \text{for } f_{ck} > 50 \text{ MPa} \qquad \text{(EC2 5.5(4))}$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$ [*NDP*] (EC 5.5(4))

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2})$ [*NDP*] (EC 5.5(4))

$k_3 = 0.54$ [*NDP*] (EC 5.5(4))

δ is assumed to be 1

where the ultimate strain, ε_{cu2} [*NDP*], is determined from EC2 Table 3.1 as:

$$\varepsilon_{cu2} = 0.0035 \text{ for } f_{ck} < 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

$$\varepsilon_{cu2} = 2.6 + 35\left[\left(90 - f_{ck}\right)/100\right]^4 \text{ for } f_{ck} \geq 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow.

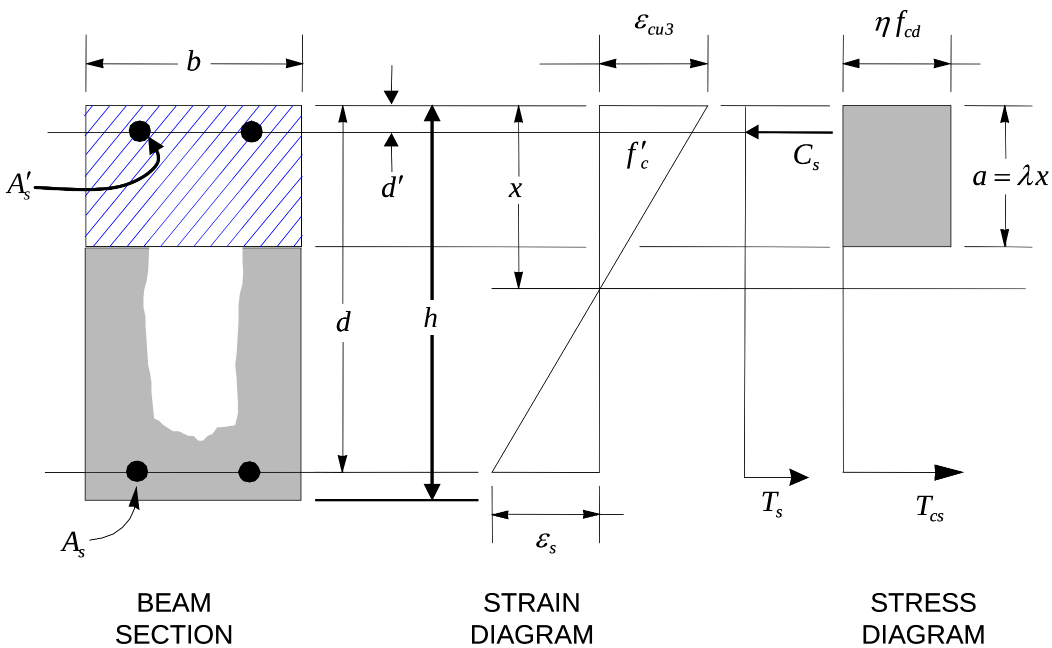


Figure 6-1 Rectangular Beam Design

6.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the normalized moment, m, and the normalized section capacity as a singly reinforce beam, m_{lim}, are obtained first. The reinforcement area is determined based on whether m is greater than, less than, or equal to m_{lim}.

- The normalized design moment, m, is calculated as:

$$m = \frac{M}{bd^2 \eta f_{cd}}$$

- The normalized concrete moment capacity as a singly reinforced beam, m_{lim}, is calculated as:

$$m_{\text{lim}} = \lambda \left(\frac{x}{d} \right)_{\text{lim}} \left[1 - \frac{\lambda}{2} \left(\frac{x}{d} \right)_{\text{lim}} \right]$$

- If $m \leq m_{\text{lim}}$, a singly reinforced beam is designed. The normalized reinforcement ratio is calculated as:

$$\omega = 1 - \sqrt{1 - 2m}$$

The area of tension reinforcement, A_s, is then given by:

$$A_s = \omega \left(\frac{\eta f_{cd} bd}{f_{yd}} \right)$$

This reinforcement is to be placed at the bottom if M_{Ed} is positive, or at the top if M_{Ed} is negative.

- If $m > m_{\text{lim}}$, both tension and compression reinforcement is designed as follows:

The normalized steel ratios ω', ω_{lim}, and ω are calculated as:

$$\omega_{\text{lim}} = \lambda \left(\frac{x}{d} \right)_{\text{lim}} = 1 - \sqrt{1 - 2m_{\text{lim}}}$$

$$\omega' = \frac{m - m_{\text{lim}}}{1 - d'/d}$$

$$\omega = \omega_{\text{lim}} + \omega'$$

where, d' is the depth to the compression reinforcement from the concrete compression face.

The area of compression and tension reinforcement, A'_s and A_s, are given by:

$$A'_s = \omega' \left[\frac{\eta f_{cd} bd}{f'_s - \eta f_{cd}} \right]$$

$$A_s = \omega \left[\frac{\eta f_{cd} bd}{f_{yd}} \right]$$

where, f'_s is the stress in the compression reinforcement, and is given by:

$$f'_s = E_s \varepsilon_{cu3} \left[1 - \frac{d'}{x_{\lim}} \right] \leq f_{yd} \qquad \text{(EC2 6.1, 3.2.7(4), Fig 3.8)}$$

6.5.1.2.2 Design of Flanged Beams

6.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_{Ed} (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

6.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located within the flange. Based on this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange, additional calculation is required. See Figure 6-2.

- The normalized design moment, m, is calculated as:

$$m = \frac{M}{bd^2 \eta f_{cd}} \qquad \text{(EC2 6.1, 3.1.7(3))}$$

- The limiting values are calculated as:

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] \quad \text{(EC2 5.5(4), 3.1.7(3))}$$

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}$$

$$a_{\max} = \omega_{\lim} d$$

- The values ω and a are calculated as:

$$\omega = 1 - \sqrt{1 - 2m}$$

$$a = \omega d$$

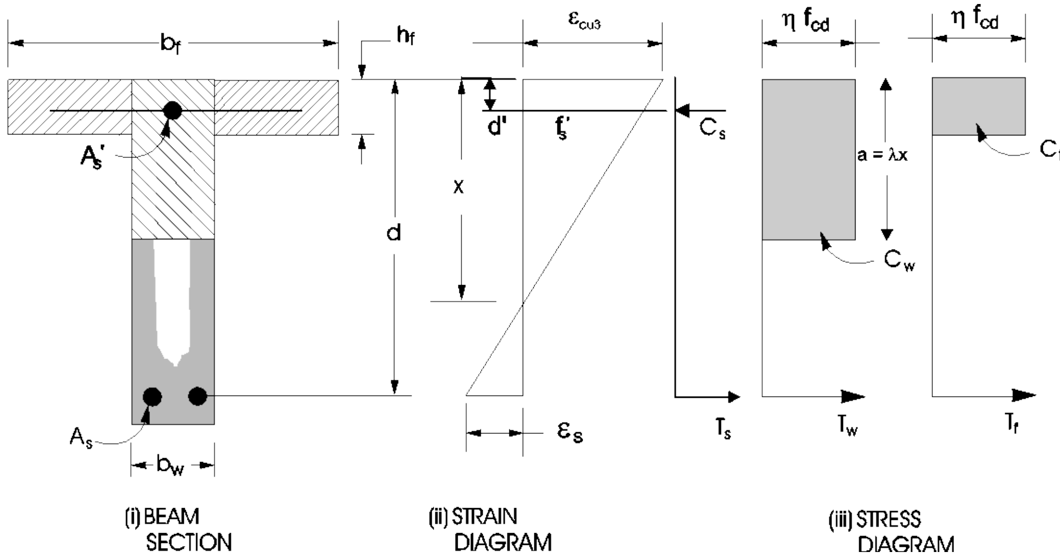


Figure 6-2 T-Beam Design

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as b_f. Compression reinforcement is required when $m > m_{\lim}$.

- If $a > h_f$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, and the second part is for balancing the compressive force from the web, as shown in Figure 6-2. The reinforcement area required for balancing the flange compression, A_{s2} is given as:

$$A_{s2} = \frac{\left(b_f - b_w\right) h_f \eta f_{cd}}{f_{yd}}$$

and the corresponding resistive moment is given by

$$M_2 = A_{s2} f_{yd} \left(d - \frac{h_f}{2} \right)$$

The reinforcement required for balancing the compressive force from the web, considering a rectangular section of width b_w to resist the moment, $M_1 = M - M_2$, is determined as follows:

$$m_1 = \frac{M_1}{b_w d^2 \eta f_{cd}}$$

- If $m_1 \leq m_{\text{lim}}$,

$$\omega_1 = 1 - \sqrt{1 - 2m_1}$$

$$A_{s1} = \omega_1 \left[\frac{\eta f_{cd} b_w d}{f_{yd}} \right]$$

- If $m_1 > m_{\text{lim}}$,

$$\omega' = \frac{m_1 - m_{\text{lim}}}{1 - d'/d}$$

$$\omega_{\text{lim}} = \lambda \left(\frac{x}{d} \right)_{\text{lim}}$$

$$\omega_1 = \omega_{\text{lim}} + \omega'$$

$$A'_s = \omega' \left[\frac{\eta f_{cd} b d}{f'_s - \eta f_{cd}} \right]$$

$$A_{s1} = \omega_1 \left[\frac{\eta f_{cd} b_w d}{f_{yd}} \right]$$

where, f'_s is given by:

$$f'_s = E_s \varepsilon_{cu3} \left[1 - \frac{d'}{x_{\lim}} \right] \leq f_{yd} \qquad \text{(EC2 6.1, 3.2.7(4), Fig 3.8)}$$

The total tension reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

6.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement, $A_{s,\min}$ [*NDP*], required in a beam section is given by the maximum of the following two limits:

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd \qquad \text{(EC2 9.2.1.1(1))}$$

$$A_{s,\min} = 0.0013bd \qquad \text{(EC2 9.2.1.1(1))}$$

where f_{ctm} is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \text{ for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm}/10\right) \text{ for } f_{ck} > 50 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement, $A_{s,\max}$ [*NDP*], has been imposed to be 0.04 times the gross cross-sectional area (EC 9.2.1.1(3)).

6.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (EC2 6.2):

- Determine the factored shear force, V_{Ed}.
- Determine the shear force, $V_{Rd,c}$, that can be resisted by the concrete.
- Determine the shear reinforcement required.

The following three sections describe in detail the algorithms associated with these steps.

6.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

6.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_{Rd,c}$, is calculated as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] b_w d \qquad \text{(EC2 6.2.2(1))}$$

with a minimum of:

$$V_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp} \right) b_w d \qquad \text{(EC2 6.2.2(1))}$$

where

f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.2.2(1))}$$

$$\rho_1 = \text{tension reinforcement ratio} = \frac{A_{s1}}{b_w d} \leq 0.02 \qquad \text{(EC2 6.2.2(1))}$$

A_{s1} = area of tension reinforcement (EC2 6.2.2(1))

$$\sigma_{cp} = N_{Ed} / A_c < 0.2\, f_{cd} \text{ MPa} \qquad \text{(EC2 6.2.2(1))}$$

The value of $C_{Rd,c}$, $v_{\min}$ and k_1 for use in a country may be found in its National Annex. The program default values for $C_{Rd,c}$ [*NDP*], $v_{\min}$ [*NDP*], and k_1 [*NDP*] are given as follows (EC2 6.2.2(1)):

$C_{Rd,c} = 0.18/\gamma_c$ (EC2 6.2.2(1))

$v_{\min} = 0.035\, k^{3/2} f_{ck}^{1/2}$ (EC2 6.2.2(1))

$k_1 = 0.15.$ (EC2 6.2.2(1))

For light-weight concrete:

$C_{Rd,c} = 0.18/\gamma_c$ (EC2 11.6.1(1))

$v_{\min} = 0.03\, k^{3/2} f_{ck}^{1/2}$ (EC2 11.6.1(1))

$k_1 = 0.15.$ (EC2 11.6.1(1))

6.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{Rd,\max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot\theta + \tan\theta}, \text{ where} \qquad \text{(EC2 6.2.3(3))}$$

α_{cw} [*NDP*] is conservatively taken as 1. (EC2 6.2.3(3))

The strength reduction factor for concrete cracked in shear, v_1 [*NDP*], is defined as:

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) \qquad \text{(EC2 6.2.2(6))}$$

$z = 0.9d$ (EC2 6.2.3(1))

θ is optimized by the program and is set to 45° for combinations including seismic loading (EC2 6.2.3(2)).

Given V_{Ed}, V_{Rdc}, and $V_{Rd,\max}$, the required shear reinforcement is calculated as follows:

- If $V_{Ed} \le V_{Rdc}$,

$$\frac{A_{sw}}{s_v} = \frac{A_{sw,\min}}{s}$$

- If $V_{R,dc} < V_{Ed} \le V_{Rd,\max}$

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{z f_{ywd} \cot\theta} \ge \frac{A_{sw,\min}}{s} \qquad \text{(EC2 6.2.3(3))}$$

- If $V_{Ed} > V_{Rd,\max}$, a failure condition is declared. (EC2 6.2.3(3))

The minimum shear reinforcement is defined as:

$$\frac{A_{sw,\min}}{s} = \frac{0.08\sqrt{f_{ck}}}{f_{yk}} b_w \qquad \text{(EC2 9.2.2(5))}$$

The maximum of all of the calculated A_{sw} / s_v values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

6.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_{Ed}.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

6.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_{Ed} is permitted to be reduced in accordance with the code (EC2 6.3.1(2)). However, the program does not automatically redistribute the internal forces and reduce T_{Ed}. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

6.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_k, t_{ef}, u, u_k, and z_i are calculated. These properties are described as follows (EC2 6.3.2).

A = Area enclosed by the outside perimeter of the cross-section

A_k = Area enclosed by centerlines of the connecting walls, where the centerline is located a distance of $t_{ef}/2$ from the outer surface

t_{ef} = Effective wall thickness, A/u. It is taken as at least twice the distance between the edge and center of the longitudinal rebar.

u = Outer perimeter of the cross-section

u_k = Perimeter of the area A_k

z_i = Side length of wall i, defined as the distance between the intersection points of the wall centerlines

In calculating the section properties involving reinforcement, such as A_k and u_k, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during calculation of torsion section properties. With this assumption, the special properties for a rectangular beam section are given as:

$$A = bh \quad \text{(EC2 6.3.2(1))}$$

$$A_k = (b - t_{ef})(h - t_{ef}) \quad \text{(EC2 6.3.2(1))}$$

$$u = 2b + 2h \quad \text{(EC2 6.3.2(1))}$$

$$u_k = 2(b - t_{ef}) + 2(h - t_{ef}) \quad \text{(EC2 6.3.2(3))}$$

$$t_{ef} = A/u \geq 2 \times \text{cover to center} \quad \text{(EC2 6.3.2(1))}$$

where, the section dimensions b, h, and c are shown in Figure 6-3. Similarly, the special section properties for a flanged beam section are given as:

$$A = b_w h \quad \text{(EC2 6.3.2(1))}$$

$$A_k = (b_w - t_{ef})(h - t_{ef}) \quad \text{(EC2 6.3.2(1))}$$

$$u = 2b_w + 2h \quad \text{(EC2 6.3.2(1))}$$

$$u_k = 2(h - t_{ef}) + 2(b_w - t_{ef}) \quad \text{(EC2 6.3.2(3))}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 6-3.

6.5.3.3 Determine Critical Torsion Capacity

The torsion in the section can be ignored, with only minimum shear reinforcement (EC2 9.2.1.1) required, if the following condition is satisfied:

$$\frac{T_{Ed}}{T_{Rd,c}} + \frac{V_{Ed}}{V_{Rd,c}} \leq 1.0 \quad \text{(EC2 6.3.2(5))}$$

where $V_{Rd,c}$ is as defined in the previous section and $T_{Rd,c}$ is the torsional cracking moment, calculated as:

$$T_{Rd,c} = f_{ctd} t_{ef} 2A_k \quad \text{(EC2 6.3.2(1), 6.3.2(5))}$$

where t_{ef}, and f_{ctd}, the design tensile strength, are defined as:

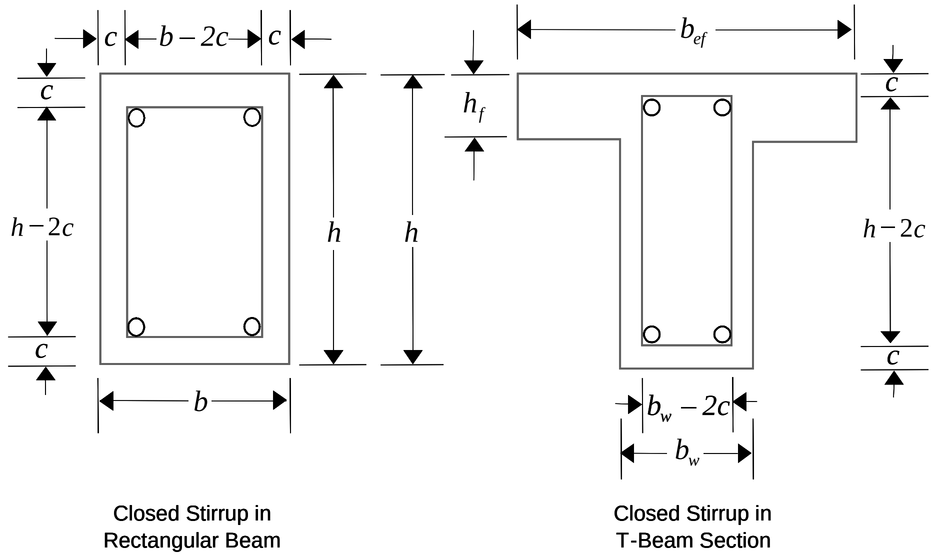


Figure 6-3 Closed stirrup and section dimensions for torsion design

$$t_{ef} = A/u \qquad \text{(EC2 6.3.2(1))}$$

$$f_{ctd} = \alpha_{ct} f_{ctk0.05}/\gamma_c \qquad \text{(EC2 Eq. 3.16)}$$

where A is the gross cross-section area, u is the outer circumference of the cross-section, α_{ct} [*NDP*] is a coefficient, taken as 1.0, taking account of long term effects on the tensile strength, and $f_{ctk0.05}$ is defined as:

$$f_{ctk0.05} = 0.7 f_{ctm} \qquad \text{(EC2 Table 3.1)}$$

6.5.3.4 Determine Torsion Reinforcement

If the expression in the previous subsection is satisfied, torsion can be safely ignored (EC2 6.3.2(5)), with only minimum shear reinforcement required. In that case, the program reports that no torsion reinforcement is required. However, if the equation is not satisfied, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals.

If torsion reinforcement in the form of closed stirrups is required, the shear due to this torsion, V_t, is first calculated, followed by the required stirrup area, as:

$$\frac{A_t}{s} = \frac{V_t}{z f_{ywd} \cot\theta} \qquad \text{(EC2 6.2.3(3))}$$

$$V_t = \left(h - t_{ef}\right)\frac{T_{Ed} - T_{con}}{2A_k} \qquad \text{(EC2 6.3.2(1))}$$

The required longitudinal reinforcement for torsion is defined as:

$$T_{con} = \left(1 - \frac{V_{Ed}}{V_{Rd,c}}\right)T_{Rd,c} \qquad \text{(EC2 6.3.2(5))}$$

$$A_{sl} = \frac{T_{Ed}}{2A_k}\cot\theta\frac{u_k}{f_{yd}} \qquad \text{(EC2 6.3.2(3))}$$

where θ is the angle of the compression struts, as previously defined for beam shear. In the preceding expressions, θ is taken as 45 degrees. The code allows any value between 21.8 and 45 degrees (EC2 6.2.3(2)), while the program assumes the conservative value of 45 degrees.

When torsional reinforcement is required, an upper limit on the combination of V_{Ed} and T_{Ed} that can be carried by the section without exceeding the capacity of the concrete struts also is checked using:

$$\frac{T_{Ed}}{T_{Rd,\max}} + \frac{V_{Ed}}{V_{Rd,\max}} \le 1.0 \qquad \text{(EC2 6.3.2(4))}$$

where $T_{Rd,\max}$, the design torsional resistance moment is defined as:

$$T_{Rd,\max} = 2\nu\alpha_{cw} f_{cd} A_k t_{ef} \sin\theta\cos\theta \qquad \text{(EC2 6.3.2(4))}$$

If this equation is not satisfied, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_{sl} and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

6.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (Eurocode 2-2004) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

6.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

6.6.1.1 Determine Factored Moments for Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

6.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

6.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (EC2 9.3.1.1) [*NDP*]:

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd \qquad \text{(EC2 9.2.1.1(1))}$$

$$A_{s,\min} = 0.0013 bd \qquad \text{(EC2 9.2.1.1(1))}$$

where f_{ctm} is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \text{ for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm}/10\right) \text{ for } f_{ck} > 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC 9.2.1.1(3)).

6.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

6.6.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (EC2 6.4.1(4)) and at a critical section at a distance of 2.0*d* from the face of the support (EC2 6.4.2(1)). The perimeter of the critical section should be constructed such that its length is minimized. Figure 6-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

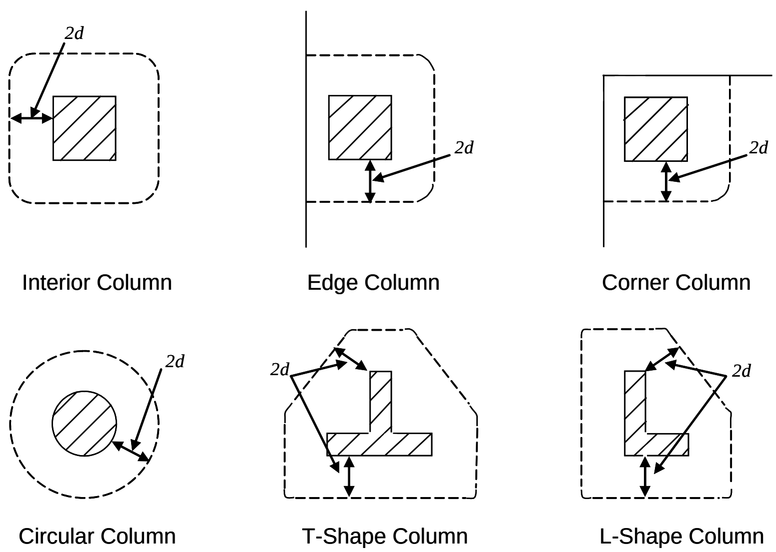


Figure 6-4 Punching Shear Perimeters

6.6.2.2 Determination of Concrete Capacity

The concrete punching shear stress capacity is taken as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp} \right] \qquad \text{(EC2 6.4.4(1))}$$

with a minimum of:

$$V_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp}\right) \qquad \text{(EC2 6.4.4(1))}$$

where f_{ck} is in MPa and

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.4.4(1))}$$

$$\rho_1 = \sqrt{\rho_{1x} \rho_{1y}} \leq 0.02 \qquad \text{(EC2 6.4.4(1))}$$

where ρ_{1x} and ρ_{1y} are the reinforcement ratios in the x and y directions respectively, which is taken as the average tension reinforcement ratios of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement ratio is taken as zero in the current implementation, and

$$\sigma_{cp} = (\sigma_{cx} + \sigma_{cy})/2 \qquad \text{(EC2 6.4.4(1))}$$

where σ_{cx} and σ_{cy} are the normal concrete stresses in the critical section in the x and y directions respectively, conservatively taken as zeros.

$$C_{Rd,c} = 0.18/\gamma_c \text{ [}NDP\text{]} \qquad \text{(EC2 6.4.4(1))}$$

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{\ 1/2} \text{ [}NDP\text{]} \qquad \text{(EC2 6.4.4(1))}$$

$$k_1 = 0.15 \text{ [}NDP\text{]}. \qquad \text{(EC2 6.4.4(1))}$$

6.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, v_{Ed}, is calculated as:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + k\frac{M_{Ed,2}u_1}{V_{Ed}W_{1,2}} + k\frac{M_{Ed,3}u_1}{V_{Ed}W_{1,3}}\right], \text{ where} \qquad \text{(EC2 6.4.4(2))}$$

k is the function of the aspect ratio of the loaded area in Table 6.1 of EN 1992-1-1

u_1 is the effective perimeter of the critical section

d is the mean effective depth of the slab

M_{Ed} is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3

V_{Ed} is the total punching shear force

W_1 accounts for the distribution of shear based on the control perimeter along bending axis 2 and 3.

6.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

6.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as described in the subsections that follow.

6.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

6.6.3.2 Determine Required Shear Reinforcement

The shear is limited to a maximum of $V_{Rd,\max}$ calculated in the same manner as explained previously for beams.

Given v_{Ed}, $v_{Rd,c}$, and $v_{Rd,\max}$, the required shear reinforcement is calculated as follows (EC2 6.4.5).

- If $v_{R,dc} < v_{Ed} \leq v_{Rd,\max}$

$$A_{sw} = \frac{\left(v_{Ed} - 0.75 v_{Rd,c}\right)}{1.5 f_{ywd,ef}} \left(u_1 d\right) s_r \quad \text{(EC2 6.4.5)}$$

- If $v_{Ed} > v_{Rd,\max,}$ a failure condition is declared. (EC2 6.2.3(3))
- If v_{Ed} exceeds the maximum permitted value of $V_{Rd,\max}$, the concrete section should be increased in size.

6.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 6-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

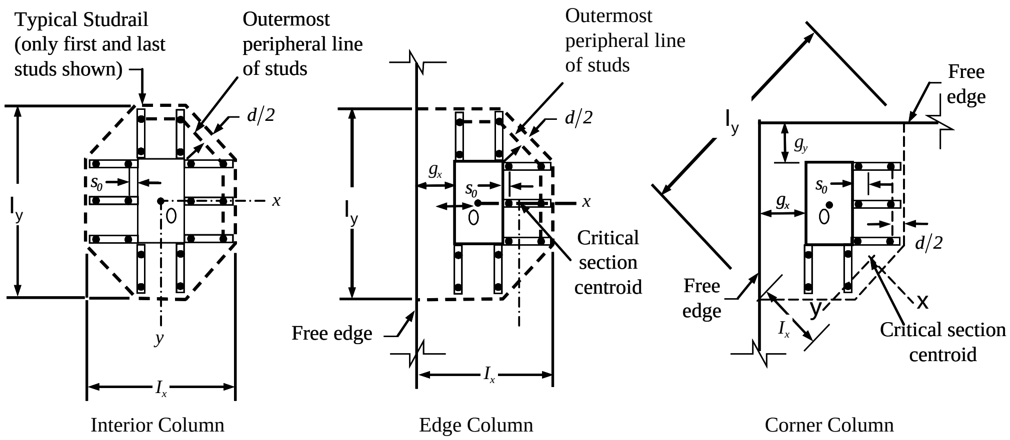


Figure 6-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed 2*d*. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5*d* measured in a direction parallel to the column face (EC2 9.4.3(1)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

6.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in EC2 4.4.1 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.3*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 1.5*d* and should not exceed 2*d* at additional perimeters. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$0.3d \leq s_o \leq 2d \qquad \text{(EC2 9.4.3(1))}$$

$$s \leq 0.75d \qquad \text{(EC2 9.4.3(1))}$$

$$g \leq 1.5d \text{ (first perimeter)} \qquad \text{(EC2 9.4.3(1))}$$

$$g \leq 2d \text{ (additional perimeters)} \qquad \text{(EC2 9.4.3(1))}$$

6.7 Nationally Determined Parameters (NDPs)

The Comité Européen de Normalisation (CEN) version of Eurocode 2-2004 specifies a set of clauses in the design code, for which Nationally Determined Parameters [NDPs] are permitted to be adjusted by each member country within their National Annex. Variations in these parameters between countries are considered in the program by choosing the desired country from the **Options menu > Preferences > Concrete Frame Design** command. This appendix lists the NDPs as adopted in the program for the CEN Default version of the design code. Additional tables are provided that list the NDPs that differ from the CEN Default values for each country supported in the program.

Table 6-2 CEN Default NDPs

NDP	Clause	Value
γ_c	2.4.2.4(1)	1.5
γ_s	2.4.2.4(1)	1.15
α_{cc}	3.1.6(1)	1.0
α_{ct}	3.1.6(2)	1.0
max f_{yk}	3.2.2(3)	600MPa
Load Combinations	5.1.3(1)	Combinations from Eq. 6.10

Table 6-2 CEN Default NDPs

NDP	Clause	Value
θ_0	5.2(5)	0.005
k_1	5.5(4)	0.44
k_2	5.5(4)	$1.25(0.6 + 0.0014/\varepsilon_{cu2})$
k_3	5.5(4)	0.54
k_4	5.5(4)	$1.25(0.6 + 0.0014/\varepsilon_{cu2})$
λ_{lim}	5.8.3.1(1)	$20 \cdot A \cdot B \cdot C / \sqrt{n}$
$C_{Rd,c}$	6.2.2(1)	$0.18/\gamma_c$
v_{min}	6.2.2(1)	$0.035k^{3/2}f_{ck}^{1/2}$
k_1	6.2.2(1)	0.15
θ	6.2.3(2)	45 degrees
v_1	6.2.3(3)	$0.6\left[1-\frac{f_{ck}}{250}\right]$
α_{cw}	6.2.3(3)	1.0
Beam $A_{s,\text{min}}$	9.2.1.1(1)	$0.26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0.0013 b_t d$
Beam $A_{s,\text{max}}$	9.2.1.1(3)	$0.04A_c$
Beam $\rho_{w,\text{min}}$	9.2.2(5)	$\left(0.08\sqrt{f_{ck}}\right)/ f_{yk}$
α_{lcc}	11.3.5(1)	0.85
α_{lct}	11.3.5(2)	0.85

Table 6-2 CEN Default NDPs

NDP	Clause	Value
$C_{lRd,c}$	11.6.1(1)	$0.15/\gamma_c$
$v_{l,\min}$	11.6.1(1)	$0.30k^{3/2}f_{lck}^{1/2}$
k_1	11.6.1(1)	0.15
ν_1	11.6.2(1)	$0.5\,\eta_1(1 - f_{lck}/250)$

Table 6-3 Denmark NDPs

NDP	Clause	Value
γ_c	2.4.2.4(1)	1.45
γ_s	2.4.2.4(1)	1.20
Max f_{yk}	3.2.2(3)	650MPa
Load Combinations	5.1.3(1)	Combinations from Eq. 6.10a/b
$\lambda_{\lim}$	5.8.3.1(1)	$20 \cdot \sqrt{\frac{A_c f_{cd}}{N_{Ed}}}$
Beam $\rho_{w,\min}$	9.2.2(5)	$\left(0.063\sqrt{f_{ck}}\right)/\ f_{yk}$
α_{lcc}	11.3.5(1)	1.0
α_{lct}	11.3.5(2)	1.0
$v_{l,\min}$	11.6.1(1)	$0.03k^{2/3}f_{lck}^{1/2}$

Table 6-4 Finland NDPs

NDP	Clause	Value
α_{cc}	3.1.6(1)	0.85
Max f_{yk}	3.2.2(3)	700MPa
Load Combinations	5.1.3(1)	Combinations from Eq. 6.10a/b
k_2	5.5(4)	1.10
Beam $A_{s,max}$	9.2.1.1(3)	Unlimited

Table 6-5 Norway NDPs

NDP	Clause	Value
α_{cc}	3.1.6(1)	0.85
α_{ct}	3.1.6(2)	0.85
λ_{lim}	5.8.3.1(1)	$13(2 - r_m)A_f$
k_1	6.2.2(1)	0.15 for compression 0.3 for tension
v_{min}	6.2.2(1)	$0.035k^{3/2}f_{ck}^{1/2}$
Beam $\rho_{w,\text{min}}$	9.2.2(5)	$\left(0.1\sqrt{f_{ck}}\right)/\,f_{yk}$
$v_{l,\text{min}}$	11.6.1(1)	$0.03k^{2/3}f_{lck}^{1/2}$
k_1	11.6.1(1)	0.15 for compression 0.3 for tension
ν_1	11.6.2(1)	$0.5(1 - f_{lck}/250)$

Table 6-6 Singapore NDPs

NDP	Clause	Value
α_{cc}	3.1.6(1)	0.85
k_1	5.5(4)	0.4
k_2	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
k_3	5.5(4)	0.54
k_4	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
$v_{\lim}$	5.8.3.1(1)	$0.30k^{3/2}f_{lck}^{1/2}$

Table 6-7 Slovenia NDPs

NDP	Clause	Value
Same As CEN Default		

Table 6-8 Sweden NDPs

NDP	Clause	Value
Beam $A_{s,max}$	9.2.1.1(3)	Unlimited
α_{lcc}	11.3.5(1)	1.0
α_{lct}	11.3.5(2)	1.0

Table 6-9 United Kingdom NDPs

NDP	Clause	Value
$\psi_{0,i}$ (wind load)	EC0 Combos	0.5

Table 6-9 United Kingdom NDPs

NDP	Clause	Value
α_{cc}	3.1.6(1)	0.85
k_1	5.5(4)	0.4
k_2	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
k_3	5.5(4)	0.4
k_4	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
$v_{l,\min}$	11.6.1(1)	$0.30k^{3/2}f_{lck}^{1/2}$

Chapter 7
Design for Hong Kong CP-04

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Hong Kong limit state code CP-04 [CP 04], which also incorporates Amendment 1 published in June 2007, is selected. The various notations used in this chapter are listed in Table 7-1. For referencing to the pertinent sections of the Hong Kong code in this chapter, a prefix "CP" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

7.1 Notations

Table 7-1 List of Symbols Used in the CP-04 Code

A_g	Gross area of cross-section, mm^2
A_l	Area of longitudinal reinforcement for torsion, mm^2

Table 7-1 List of Symbols Used in the CP-04 Code

A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
$A_{sv,t}$	Total cross-sectional area of closed links for torsion, mm^2
A_{sv}/s_v	Area of shear reinforcement per unit length, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
C	Torsional constant, mm^4
d	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, N/mm^2
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 N/mm^2
f	Punching shear factor considering column location
f_{cu}	Characteristic cube strength, N/mm^2
f'_s	Stress in the compression reinforcement, N/mm^2
f_y	Characteristic strength of reinforcement, N/mm^2
f_{yv}	Characteristic strength of shear reinforcement, N/mm^2
h	Overall depth of a section in the plane of bending, mm
h_f	Flange thickness, mm
$h_{\min}$	Smaller dimension of a rectangular section, mm
$h_{\max}$	Larger dimension of a rectangular section, mm
K	Normalized design moment, $\frac{M_u}{bd^2 f_{cu}}$
K'	Maximum $\frac{M_u}{bd^2 f_{cu}}$ for a singly reinforced concrete section

Table 7-1 List of Symbols Used in the CP-04 Code

k_1	Shear strength enhancement factor for support compression
k_2	Concrete shear strength factor, $\left[f_{cu}/25\right]^{1/3}$
M	Design moment at a section, N-mm
M_{single}	Limiting moment capacity as singly reinforced beam, N-mm
s_v	Spacing of the links along the length of the beam, mm
T	Design torsion at ultimate design load, N-mm
u	Perimeter of the punch critical section, mm
V	Design shear force at ultimate design load, N
v	Design shear stress at a beam cross-section or at a punching critical section, N/mm^2
v_c	Design concrete shear stress capacity, N/mm^2
$v_{\max}$	Maximum permitted design factored shear stress, N/mm^2
v_t	Torsional shear stress, N/mm^2
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm
β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain
ε_s	Strain in tension reinforcement
ε'_s	Strain in compression reinforcement

7.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (CP 2.3.1.3). For CP-04, if a structure is subjected to dead (D), live (L), pattern live (PL), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered. (CP 2.3.2.1, Table 2.1).

1.4D
1.4D + 1.6L (CP 2.3.2)

1.4D + 1.6(0.75PL) (CP 2.3.2)

1.0D ± 1.4W
1.4D ± 1.4W (CP 2.3.2)
1.2D + 1.2L ± 1.2W

These are also the default design load combinations in SAFE whenever the CP-04 code is used. If roof live load is separately treated or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

7.3 Limits on Material Strength

The concrete compressive strength, f_{cu}, should not be less than 20 N/mm^2 (CP 3.1.3). The program does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsible to use the proper strength values while defining the materials.

7.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the following table, as taken from CP Table 2.2 (CP 2.4.3.2):

Values of γ_m for the Ultimate Limit State	
Reinforcement	1.15
Concrete in flexure and axial load	1.50
Concrete shear strength without shear reinforcement	1.25

These factors are incorporated into the design equations and tables in the code, but can be overwritten.

7.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the sections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

7.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the length of the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

7.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

7.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 7-1 (CP 6.1.2.4(a)), where $\varepsilon_{c,\max}$ is defined as:

$$\varepsilon_{c,\max} = \begin{cases} 0.0035 & \text{if} \quad f_{cu} \le 60\text{N/mm}^2 \\ 0.0035 - 0.00006\left(f_{cu} - 60\right)^{1/2} & \text{if} \quad f_{cu} > 60\text{N/mm}^2 \end{cases}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \ge 0.9$; CP 6.1.2.4(b)). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \le \begin{cases} 0.5 & \text{for} \quad f_{cu} \le 45\text{ N/mm}^2 \\ 0.4 & \text{for} \quad 45 < f_{cu} \le 70\text{ N/mm}^2 \\ 0.33 & \text{for} \quad 70 < f_{cu} \le 100\text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(b))}$$

to safeguard against non-ductile failures (CP 6.1.2.4(b)). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a), Fig 6.1)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed ($0.1f_{cu}A_g$) (CP 6.1.2.4(a)), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $0.1f_{cu}A_g$ and axial tensions are always included in flexural and shear design.

7.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam, M_{single}, is obtained first for a section. The reinforcing is determined based on whether M is greater than, less than, or equal to M_{single}. See Figure 7-1

Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

$$M_{\text{single}} = K' f_{cu}\, bd^2, \text{ where} \qquad \text{(CP 6.1.2.4(c))}$$

$$K' = \begin{cases} 0.156 \text{ for } & f_{cu} \le 45 \text{ N/mm}^2 \\ 0.120 \text{ for } & 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.094 \text{ for } & 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases}$$

- If $M \le M_{\text{single}}$, the area of tension reinforcement, A_s, is obtained from:

$$A_s = \frac{M}{0.87 f_y z}, \text{ where} \qquad \text{(CP 6.1.2.4(c))}$$

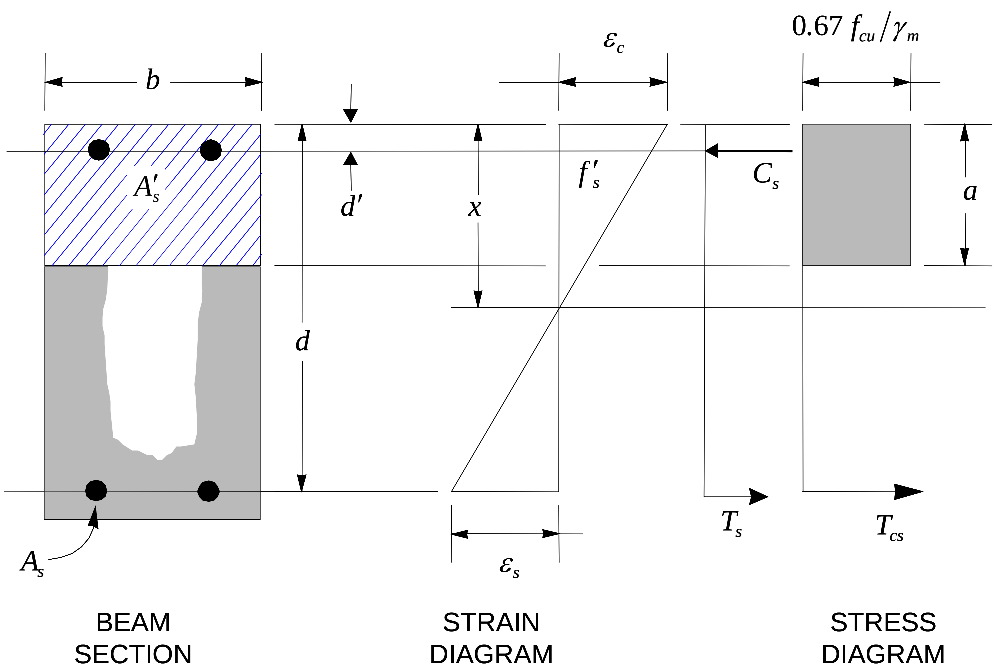


Figure 7-1 Rectangular Beam Design

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \leq 0.95d \qquad \text{(CP 6.1.2.4(c))}$$

$$K = \frac{M}{f_{cu}bd^2} \qquad \text{(CP 6.1.2.4(c))}$$

This reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

- If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$A'_s = \frac{M - M_{single}}{\left(f'_s - \dfrac{0.67 f_{cu}}{\gamma_c}\right)(d - d')} \qquad \text{(CP 6.1.2.4(c))}$$

where d' is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = E_s \varepsilon_c \left(1 - \frac{d'}{x}\right) \leq 0.87 f_y, \qquad \text{(CP 6.1.2.4(c), 3.2.6, Fig. 3.9)}$$

$$x = \begin{cases} \dfrac{d-z}{0.45}, & \text{for} \quad f_{cu} \leq 45\,\text{N/mm}^2 \\ \dfrac{d-z}{0.40}, & \text{for} \quad 45 < f_{cu} \leq 70\ \text{N/mm}^2 \\ \dfrac{d-z}{0.36}, & \text{for} \quad 70 < f_{cu} \leq 100\ \text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a), Fig 6.1)}$$

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right\} \leq 0.95d \qquad \text{(CP 6.1.2.4(c))}$$

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$A_s = \frac{M_{\text{single}}}{0.87 f_y z} + \frac{M - M_{\text{single}}}{0.87 f_y \left(d - d'\right)} \qquad \text{(CP 6.1.2.4(c))}$$

7.5.1.2.2 Design of Flanged Beams

7.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

7.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located in the flange. On the basis of this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange depth, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 7-2.

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

$$K = \frac{M}{f_{cu}\, b_f\, d^2}. \qquad \text{(CP 6.1.2.4(c))}$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \qquad \text{(CP 6.1.2.4(c))}$$

the depth of the neutral axis is computed as:

$$x = \begin{cases} \dfrac{d-z}{0.45}, & \text{for} \quad f_{cu} \le 45\,\text{N/mm}^2 \\ \dfrac{d-z}{0.40}, & \text{for} \quad 45 < f_{cu} \le 70\,\text{N/mm}^2 \\ \dfrac{d-z}{0.36}, & \text{for} \quad 70 < f_{cu} \le 100\,\text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(c), Fig 6.1)}$$

and the depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 45\ \text{N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \le 70\ \text{N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \le 100\ \text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a), Fig 6.1)}$$

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as b_f. Compression reinforcement is required when $K > K'$.

- If $a > h_f$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 7-2.

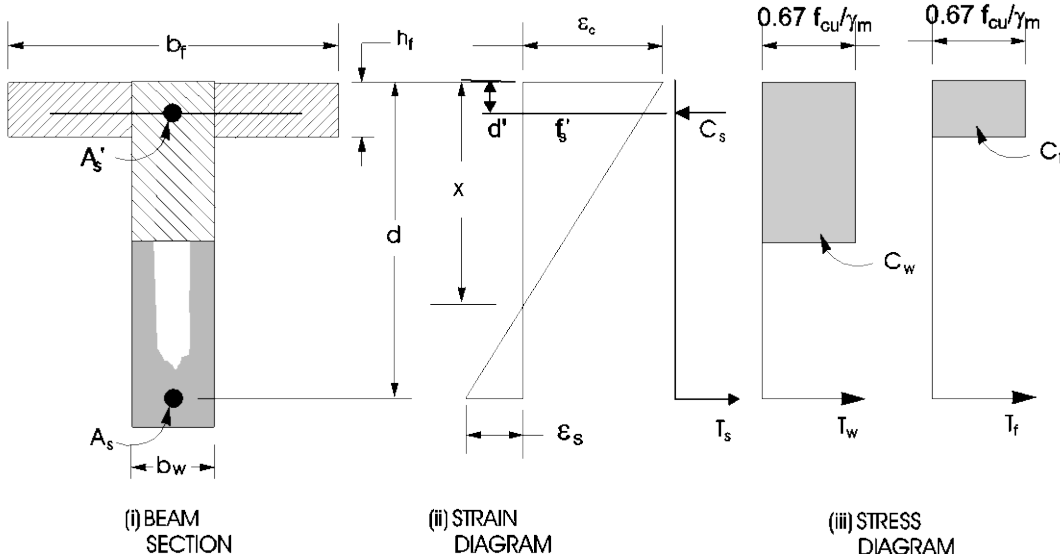


Figure 7-2 Design of a T-Beam Section

In that case, the ultimate resistance moment of the flange is given by:

$$M_f = \frac{0.67}{\gamma_c} f_{cu}\left(b_f - b_w\right)h_f\left(d - 0.5h_f\right)$$

The moment taken by the web is computed as:

$$M_w = M - M_f$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2}$$

- If $K_w \leq K'$ (CP 6.1.2.4(c)), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{0.87 f_y \left(d - 0.5 h_f\right)} + \frac{M_w}{0.87 f_y z}, \text{ where}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \le 0.95d$$

- If $K_w > K'$, compression reinforcement is required and is calculated as follows:

  The ultimate moment of resistance of the web only is given by:

$$M_{uw} = K f_{cu} b_w d^2$$

  The compression reinforcement is required to resist a moment of magnitude $M_w - M_{uw}$. The compression reinforcement is computed as:

$$A'_s = \frac{M_w - M_{uw}}{\left(f'_s - \dfrac{0.67 f_{cu}}{\gamma_c}\right)\left(d - d'\right)}$$

  where, d' is the depth of the compression reinforcement from the concrete compression face, and

$$f_s = E_s \varepsilon_c \left(1 - \frac{d}{x}\right) \le 0.87 f_y \qquad \text{(CP 6.1.2.4(c), 3.2.6, Fig 3.9)}$$

The area of tension reinforcement is obtained from equilibrium as:

$$A_s = \frac{1}{0.87 f_y}\left[\frac{M_f}{d - 0.5 h_f} + \frac{M_{uw}}{z} + \frac{M_w - M_{uw}}{d - d'}\right]$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right) \le 0.95d$$

7.5.1.2.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from CP Table 9.1(CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage f_y = 250 MPa	Minimum percentage f_y = 460 MPa
Rectangular	—	$100\ \frac{A_s}{bh}$	0.24	0.13
T or L-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100\ \frac{A_s}{b_w h}$	0.32	0.18
T or L-Beam with web in tension	$\frac{b_w}{b_f} \geq 0.4$	$100\ \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.48	0.26
L-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from CP Table 9.1 (CP 9.2.1.1).

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100\ \frac{A'_s}{bh}$	0.20
T or L-Beam	Web in tension	$100\ \frac{A'_s}{b_f h_f}$	0.40
T or L-Beam	Web in compression	$100\ \frac{A'_s}{b_w h}$	0.20

An upper limit of 0.04 times the gross cross-sectional area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows (CP 9.2.1.3):

$$A_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$ (CP 9.2.1.3)

7.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (CP 6.1.2.5):

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

7.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors. The shear stress is then calculated as:

$$v = \frac{V}{bd}$$ (CP 6.1.2.5(a))

The maximum allowable shear stress, v_{max} is defined as:

$$v_{max} = \min(0.8\sqrt{f_{cu}}\text{, 7 MPa})$$ (CP 6.1.2.5(a))

7.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, v_c, is calculated as:

$$v'_c = v_c + 0.6\frac{NVh}{A_c M} \le v_c\sqrt{1+\frac{N}{A_c v_c}}$$ (CP 6.1.2.5(k))

$$v_c = \frac{0.79k_1k_2}{\gamma_m}\left(\frac{100A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}$$ (CP 6.1.2.5(c), Table 6.3)

k_1 is the enhancement factor for support compression,
and is conservatively taken as 1 (CP 6.1.2.5(g))

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3}, \qquad 1 \le k_2 \le \left(\frac{80}{25}\right)^{1/3}$$ (CP 6.1.2.5(c), Table 6.3)

$$\gamma_m = 1.25$$ (CP 6.1.2.5(c), Table 6.3)

However, the following limitations also apply:

$$0.15 \le \frac{100A_s}{bd} \le 3,$$ (CP 6.1.2.5(c), Table 6.3)

$$\left(\frac{400}{d}\right)^{1/4} \ge \begin{matrix} 0.67, & \text{Members without shear reinforcement} \\ 1.00, & \text{Members with shear reinforcement} \end{matrix}$$

(CP 6.1.2.5(c), Table 6.3)

$$\frac{Vh}{M} \le 1$$ (CP 6.1.2.5(k))

7.5.2.3 Determine Required Shear Reinforcement

Given v, v_c, and v_{max}, the required shear reinforcement is calculated as follows (CP Table 6.2, CP 6.1.2.5(b)):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, v_r, as:

$$v_r = \begin{cases} 0.4 & \text{if} \quad f_{cu} \le 40 \text{ N/mm}^2 \\ 0.4\left(\frac{f_{cu}}{40} \right)^{2/3} & \text{if } 40 < f_{cu} \le 80 \text{ N/mm}^2 \\ 0.4\left(\frac{80}{40} \right)^{2/3} & \text{if} \quad f_{cu} > 80 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.5(b), Table 6.2)}$$

- If $v \le v'_c + v_r$, minimum reinforcement is required:

$$\frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}}, \qquad \text{(CP 6.1.2.5(b))}$$

- If $v > v'_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{\left(v - v'_c\right) b}{0.87 f_{yv}} \qquad \text{(CP 6.1.2.5(b))}$$

- If $v > v_{max}$, a failure condition is declared. (CP 6.1.2.5(b))

The maximum of all the calculated A_{sv}/s_v values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

7.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t.

- Determine special section properties.

- Determine critical torsion stress.
- Determine the torsion reinforcement required.

7.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given, as follows (CP 6.3.1).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2\left(h_{\max} - h_{\min}/3\right)} \qquad \text{(CP 6.3.3(a))}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{seg} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(CP 6.3.3(b))}$$

$h_{\max}$ = Larger dimension of a rectangular section

$h_{\min}$ = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

$$v_t \le \min(0.8\sqrt{f_{cu}}, 7\,\text{N/mm}^2) \times \frac{y_1}{550} \qquad \text{(CP 6.3.4, Table 6.17)}$$

7.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.6\ \text{N/mm}^2\right) \qquad \text{(CP 6.3.4, Table 6.17)}$$

where f_{cu} is the specified concrete compressive strength.

7.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (CP 6.3.5). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP 6.3.5).

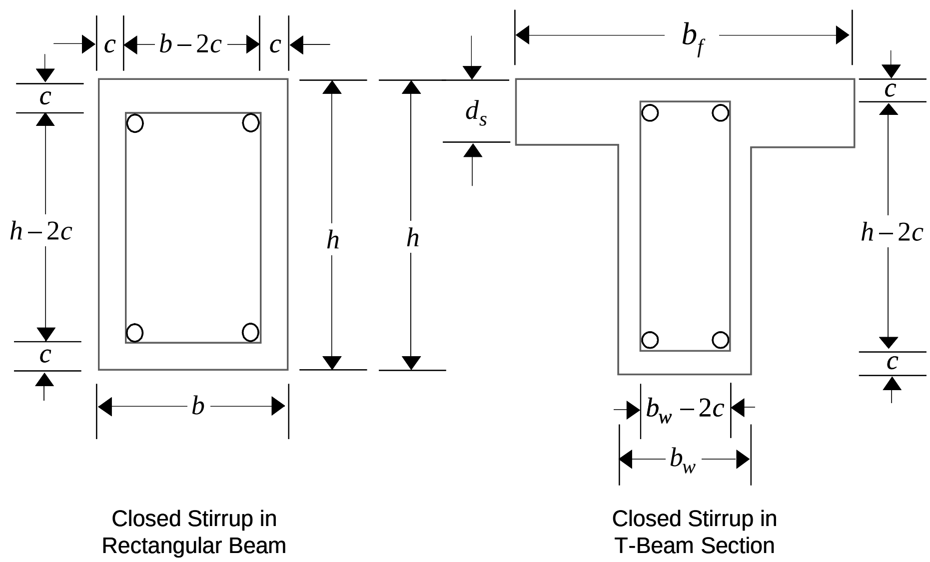


Figure 7-3 Closed stirrup and section dimensions for torsion design

- If $v_t > v_{t,\min}$, the required closed stirrup area per unit spacing, $A_{sv,t} / s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8 x_1 y_1 \left(0.87 f_{yv}\right)} \qquad \text{(CP 6.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t}\, f_{yv} \left(x_1 + y_1\right)}{s_v\, f_y} \qquad \text{(CP 6.3.6)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link, and y_1 is the larger center-to-center dimension of the closed link.

An upper limit of the combination of v and v_t that can be carried by the section also is checked using the equation:

$$v + v_t \le \min\left(0.8\sqrt{f_{cu}}, 7\ \text{N/mm}^2\right) \qquad \text{(CP 6.3.4)}$$

If the combination of v and v_t exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

7.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is performed based on the ultimate strength design method (CP-04) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

7.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

7.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

7.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

7.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required in each direction of a slab is given by the following limits (CP 9.3.1.1), with interpolation for reinforcement of intermediate strength:

$$A_s \geq \begin{cases} 0.0024bh & \text{if} \quad f_y \leq 250 \text{ MPa} \\ 0.0013bh & \text{if} \quad f_y \geq 460 \text{ MPa} \end{cases} \qquad \text{(CP 9.3.1.1(a))}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 9.2.1.3).

7.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

7.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5d from the face of the support (CP 6.1.5.7(d)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CP 6.1.5.7). Figure 7-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

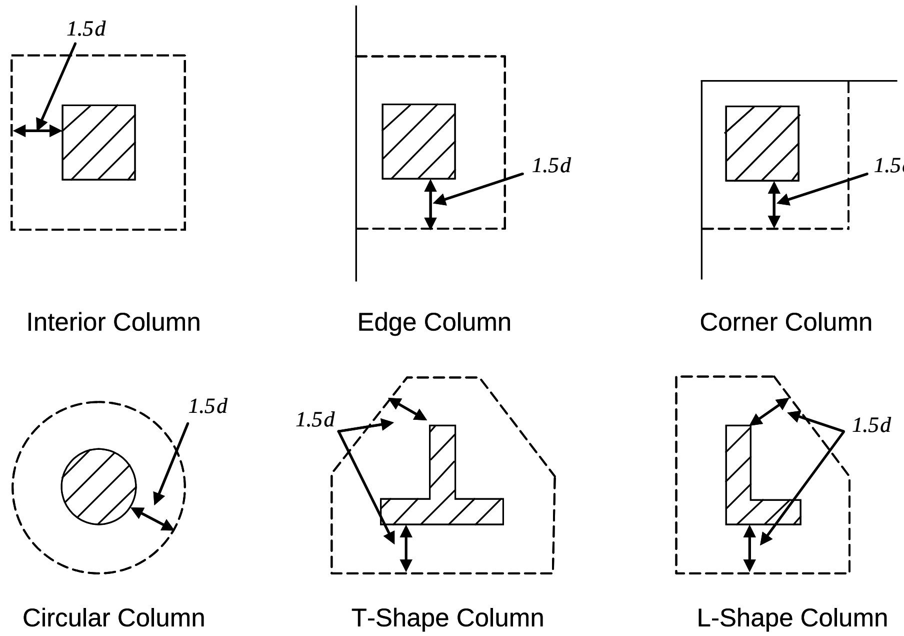


Figure 7-4 Punching Shear Perimeters

7.6.2.2 Determine Concrete Capacity

The concrete punching shear factored strength is taken as (CP 6.1.5.7(d), Table 6.3):

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} \qquad \text{(CP 6.1.2.5(d), Table 6.3)}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 (CP 6.1.2.5(g), 6.1.5.7(d))

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} \quad 1 \le k_2 \le \left(\frac{80}{25}\right)^{1/3} \qquad \text{(CP 6.1.2.5(c), Table 6.3)}$$

$$\gamma_m = 1.25 \qquad \text{(CP 2.4.3.2, Table 2.2)}$$

However, the following limitations also apply:

$$0.15 \le \frac{100A_s}{bd} \le 3, \qquad \text{(CP 6.1.2.5(c), Table 6.3)}$$

$$\left(\frac{400}{d}\right)^{1/4} \ge \begin{cases} 0.67, & \text{Members without shear reinforcement} \\ 1.00, & \text{Members with shear reinforcement} \end{cases}$$

(CP 6.1.2.5(c), Table 6.3)

A_s = area of tension reinforcement, which is taken as the average tension reinforcement of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement is taken as zero in the current implementation.

$v \le \min(0.8\sqrt{f_{cu}}, 7 \text{ MPa})$ (CP 6.1.5.7(b))

$f_{cu} \le 80$ MPa (for calculation purpose only) (CP Table 6.3)

7.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, $v_{\max}$, is calculated as:

$$V_{eff,x} = V\left(f + \frac{1.5M_x}{Vy}\right) \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

$$V_{eff,y} = V\left(f + \frac{1.5M_y}{Vx}\right) \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

$$v_{\max} = \max\begin{cases} \dfrac{V_{eff,x}}{u\,d} \\ \dfrac{V_{eff,y}}{u\,d} \end{cases} \qquad \text{(CP 6.1.5.7)}$$

where,

u is the perimeter of the critical section,

x and y are the lengths of the sides of the critical section parallel to the axis of bending,

M_x and M_y are the design moments transmitted from the slab to the column at the connection,

V is the total punching shear force, and

f is a factor to consider the eccentricity of punching shear force and is taken as

$$f = \begin{cases} 1.00 & \text{for interior columns} \\ 1.25 & \text{for edge columns} \\ 1.25 & \text{for corner columns} \end{cases} \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

7.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

7.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 6.1.5.7(e)). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

7.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

7.6.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

$$v_{\max} = 2v_c \qquad \text{(CP 6.1.5.7(e))}$$

Given v, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows (CP 6.1.5.7(e)).

- If $v \le 1.6v_c$,

$$\frac{A_v}{s} = \frac{\left(v - v_c\right)ud}{0.87\, f_{yv}} \ge \frac{v_r ud}{0.87\, f_{yv}}, \qquad \text{(CP 6.1.5.7(e))}$$

- If $1.6v_c \le v < 2.0v_c$,

$$\frac{A_v}{s} = \frac{5\left(0.7v - v_c\right)ud}{0.87\, f_{yv}} \ge \frac{v_r ud}{0.87\, f_{yv}}, \qquad \text{(CP 6.1.5.7(e))}$$

$$v_r = \begin{cases} 0.4 \\ 0.4\left(\dfrac{f_{cu}}{40}\right)^{2/3} \end{cases} \text{N/mm}^2 \qquad \text{(CP 6.1.5.7, Table 6.2)}$$

- If $v > 2.0v_c$, a failure condition is declared. (CP 6.1.5.7(e))

If v exceeds the maximum permitted value of $v_{\max}$, the concrete section should be increased in size.

7.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 7-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5d measured in a direction parallel to the column face (CP 6.1.5.7(f)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

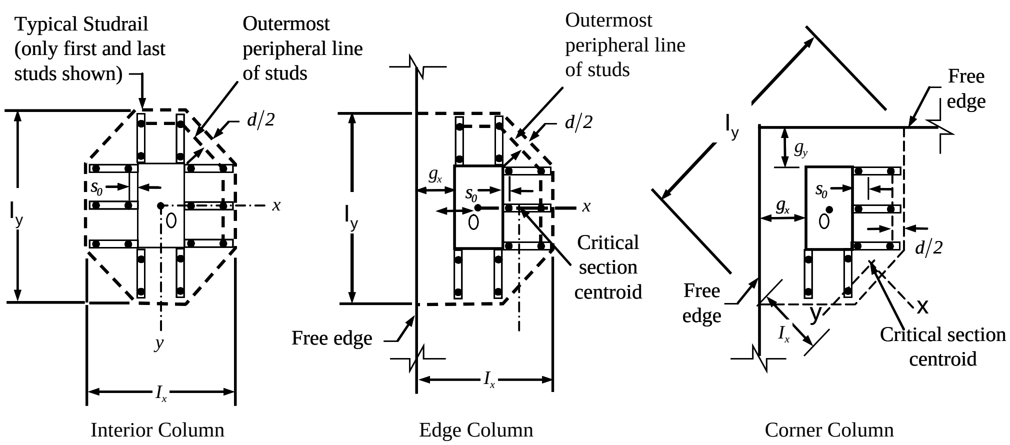


Figure 7-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

7.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 4.2.4 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 1.5d. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(CP 6.1.5.7(f))}$$

$$s \leq 0.75d \qquad \text{(CP 6.1.5.7(f))}$$

$$g \leq 1.5d \qquad \text{(CP 6.1.5.7(f))}$$

Stirrups are only permitted when slab thickness is greater than 200 mm (CP 6.1.5.7(e)).

Chapter 8
Design for IS 456-2000

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Indian Code IS 456-2000 [IS 2000] is selected. Various notations used in this chapter are listed in Table 8-1. For referencing to the pertinent sections of the Indian code in this chapter, a prefix "IS" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

8.1 Notations

Table 8-1 List of Symbols Used in the IS 456-2000 Code

A_c	Area of concrete, mm^2
A_{cv}	Area of section for shear resistance, mm^2
A_g	Gross cross-sectional area of a frame member, mm^2

Table 8-1 List of Symbols Used in the IS 456-2000 Code

A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv}/s_v	Area of shear reinforcement per unit length, mm^2/mm
a	Depth to the center of the compression block, mm
a_1	Width of the punching critical section in the direction of bending, mm
a_2	Width of the punching critical section perpendicular to the direction of bending, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm
D	Overall depth of a beam or slab, mm
D_f	Flange thickness in a flanged beam, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{cd}	Design concrete strength = f_{ck} / γ_c, MPa
f_{ck}	Characteristic compressive strength of concrete, MPa
f_{sc}	Compressive stress in beam compression steel, MPa
f_{yd}	Design yield strength of reinforcement = f_y / γ_s, MPa
f_y	Characteristic strength of reinforcement, MPa
f_{ys}	Characteristic strength of shear reinforcement, MPa
k	Enhancement factor of shear strength for depth of the beam
M_{single}	Design moment resistance of a section as a singly reinforced section, N-mm
M_u	Ultimate factored design moment at a section, N-mm

Table 8-1 List of Symbols Used in the IS 456-2000 Code

M_t	Equivalent factored bending moment due to torsion at a section, N-mm
M_{e1}	Equivalent factored moment including moment and torsion effects ($M_{e1} = M_u + M_t$) at a section, N-mm
M_{e2}	Residual factored moment when $M_t > M_u$ at a section applied in the opposite sense of M_{e1} at a section, N-mm
m	Normalized design moment, $M/bd^2\alpha f_{ck}$
s_v	Spacing of the shear reinforcement along the length of the beam, mm
T_u	Factored torsional moment at a section, N-mm
V_u	Factored shear force at a section, N
V_e	Equivalent factored shear force including torsion effects, N
v_c	Allowable shear stress in punching shear mode, N
x_u	Depth of neutral axis, mm
$x_{u,\max}$	Maximum permitted depth of neutral axis, mm
z	Lever arm, mm
α	Concrete strength reduction factor for sustained loading, as well as reinforcement over strength factor for computing capacity moment at a section
β	Factor for the depth of compressive force resultant of the concrete stress block
β_c	Ratio of the minimum to maximum dimensions of the punching critical section
γ_c	Partial safety factor for concrete strength
γ_f	Partial safety factor for load, and fraction of unbalanced moment transferred by flexure
γ_m	Partial safety factor for material strength
γ_s	Partial safety factor for reinforcement strength
δ	Enhancement factor of shear strength for compression

Table 8-1 List of Symbols Used in the IS 456-2000 Code

$\varepsilon_{c,\max}$	Maximum concrete strain in the beam and slab (= 0.0035)
ε_s	Strain in tension steel
ε_s'	Strain in compression steel
τ_v	Average design shear stress resisted by concrete, MPa
τ_c	Basic design shear stress resisted by concrete, MPa
$\tau_{c,\max}$	Maximum possible design shear stress permitted at a section, MPa
τ_{cd}	Design shear stress resisted by concrete, MPa

8.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For IS 456-2000, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (IS 36.4, Table 18):

1.5D	(IS 36.4.1)
1.5D + 1.5L 1.5D + 1.5S	(IS 36.4.1)
1.5D + 1.5(0.75 PL)	(IS 31.5.2.3)
1.5D ± 1.5W 0.9D ± 1.5W 1.2D + 1.2L ± 1.2W 1.5D + 1.5L ± 1.0W	(IS 36.4.1)
1.5D ± 1.5E 0.9D ± 1.5E 1.2D + 1.2L ± 1.2E 1.5D + 1.5L ± 1.0E	(IS 36.4.1)

1.5D + 1.5L + 1.5S
1.2D + 1.2S ± 1.2W
1.2D + 1.2L + 1.2S ± 1.2W (IS 36.4.1)
1.2D + 1.2S ± 1.2E
1.2D + 1.2L + 1.2S ± 1.2E

These are also the default design load combinations in SAFE whenever the IS 456-2000 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

8.3 Partial Safety Factors

The design strength for concrete and reinforcement is obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are as follows:

Partial safety factor for reinforcement, $\gamma_s = 1.15$ (IS 36.4.2.1)

Partial safety factor for concrete, $\gamma_c = 1.5$ (IS 36.4.2.1)

These factors are already incorporated into the design equations and tables in the code. These values can be overwritten; however, caution is advised.

8.4 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

8.4.1 Effects of Torsion

IS 456, 14.1 states that wherever torsion is required to maintain equilibrium, beams must be designed for torsion. However, torsion can be ignored for indeterminate structures where torsion develops primarily because of compatibility of deformations. However, the program does not automatically redistribute the internal forces and reduce torsion. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

Note that the torsion design can be turned off by choosing not to consider torsion in the Design Preferences.

The beam design procedure involves the following steps:

- Determine design bending moments and shears
- Design flexural reinforcement
- Design shear reinforcement

8.4.1.1 Determine Design Bending Moments and Shears

IS 456 uses a simplified approach and does not require the calculation of shear stresses produced by torsion separately. Rather, torsion and bending shear are combined as an equivalent shear V_e, and bending moment and torsion are combined as an equivalent bending moment M_e. The beam is checked for adequacy and then designed for the equivalent moment and shear. If the shear stress caused by equivalent shear is less than the concrete shear capacity, torsion is ignored completely and only required minimum shear links are computed. If the shear stress caused by equivalent shear is more than the concrete shear capacity, additional longitudinal reinforcement and shear links are computed, as detailed in the subsections that follow.

8.4.1.2 Determine Factored Moments when Torsion is Excluded

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

8.4.1.3 Determine Factored Moments when Torsion is Included

In the design of flexural reinforcement of concrete beams, the factored moments and torsion for each load combination at a particular beam station are obtained by factoring the corresponding moments and torsion for different load cases with the corresponding load factors.

The equivalent moment at a particular station is computed as described in the text that follows. The beam is then designed for the maximum positive and maximum negative factored moments obtained from all the of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

The equivalent moment is calculated from the following equation:

$$M_{e1} = M_u + M_t \text{, where} \qquad \text{(IS 41.4.2)}$$

$$M_t = T_u \left(\frac{1 + D/b}{1.7} \right) \qquad \text{(IS 41.4.2)}$$

and D and b are the overall depth and width of the beam, respectively.

If M_t exceeds M_u, additional reinforcement will be computed for the moment M_{e2} applied in the opposite sense of M_u. Effectively, this will result in additional longitudinal reinforcement on the compression face of the beam because the moment sign is reversed. The additional moment M_{e2} is computed as:

$$M_{e2} = M_t - M_u \qquad \text{(IS 41.4.2.1)}$$

8.4.1.4 Determine Factored Shears when Torsion is Excluded

In the design of the beam shear reinforcement, the factored shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

8.4.1.5 Determine Factored Shears when Torsion is Included

In the design of beam shear reinforcement, the factored shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

The equivalent shear at a particular station is computed as described in the text that follows. The beam is then designed for the equivalent shear at the station.

When a torsional moment is to be included, the equivalent shear V_e is calculated from the following equation:

$$V_e = V_u + 1.6\left(\frac{T_u}{b}\right) \qquad \text{(IS 41.3.1)}$$

where b is the width of the beam web.

8.4.2 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam.

8.4.2.1 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified parabolic stress block shown in Figure 8-1 (IS 38.1). The area of the stress block, c, and the depth of the center of the compressive force from the extreme compression fiber, a, are taken as

$$c = \alpha f_{\text{ck}} x_u \qquad \text{(IS 38.1)}$$

$$a = \beta x_{\text{u}} \qquad \text{(IS 38.1)}$$

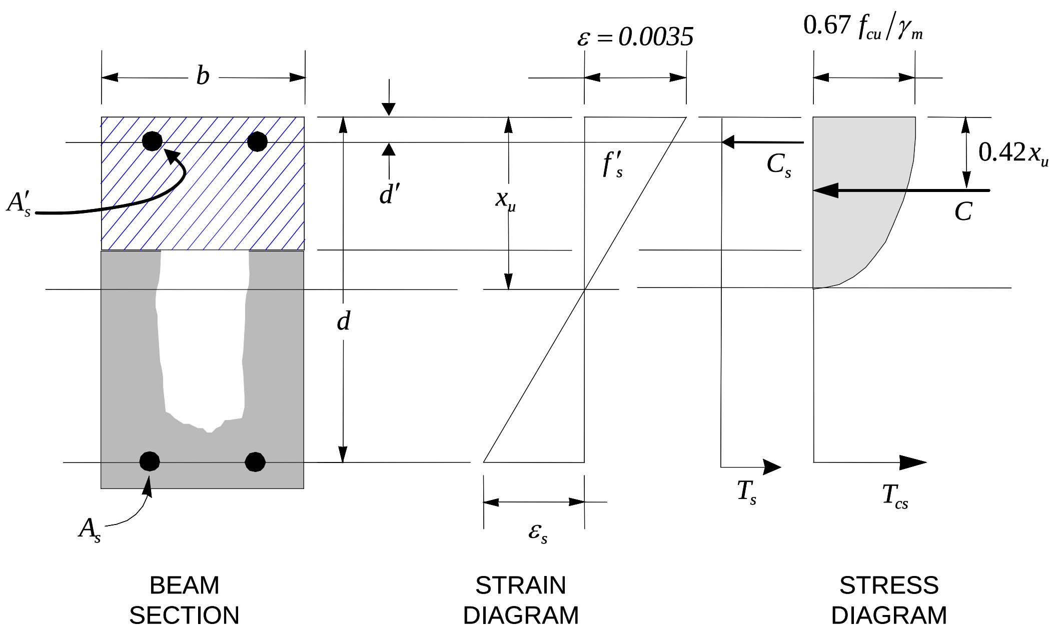


Figure 8-1 Rectangular Beam Design

where x_u is the depth of the neutral axis, and α and β are taken as:

$\alpha = 0.36$ (IS 38.1)

$\beta = 0.42$ (IS 38.1)

where α is the reduction factor to account for sustained compression and the partial safety factor for concrete and is generally taken to be 0.36 for the assumed parabolic stress block (IS 38.1). The β factor considers the depth to the center of the compressive force.

Furthermore, it is assumed that moment redistribution in the member does not exceed the code-specified limiting value. The code also places a limitation on the neutral axis depth, as shown in the following table, to safeguard against non-ductile failures (IS 38.1). SAFE uses interpolation between these three values.

f_y (MPa)	$x_{u,\max}/d$
250	0.53
415	0.48

500	0.46

When the applied moment exceeds the moment capacity of the beam as a singly reinforced beam, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value. The maximum fiber compression is taken as:

$$\varepsilon_{c,\max} = 0.0035 \qquad \text{(IS 38.1)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force can be neglected; hence all beams are designed for major direction flexure, shear, and torsion only.

8.4.2.2 Design of Rectangular Beams

For rectangular beams, the limiting depth of the neutral axis, $x_{u,\max}$, and the moment capacity as a singly reinforced beam, M_{single}, are obtained first. The reinforcement area is determined based on whether M_u is greater than, less than, or equal to M_{single}.

- Calculate the limiting depth of the neutral axis.

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \text{ MPa} \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \text{ MPa} \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \text{ MPa} \\ 0.46 & \text{if} \quad f_y \ge 500 \text{ MPa} \end{cases} \qquad \text{(IS 38.1)}$$

- Calculate the limiting ultimate moment of resistance as a singly reinforced beam.

$$M_{\text{single}} = \alpha \frac{x_{u,\max}}{d}\left(1 - \beta \frac{x_{u,\max}}{d}\right) bd^2 f_{ck} \qquad \text{(IS G-1.1)}$$

- Calculate the depth of the neutral axis as:

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta}$$

where the normalized design moment, m, is given by

$$m = \frac{M_u}{bd^2 \alpha f_{ck}}$$

– If $M_u \le M_{\text{single}}$ the area of tension reinforcement, A_s, is obtained from

$$A_s = \frac{M_u}{\left(f_y / \gamma_s\right) z}, \text{ where} \qquad \text{(IS G-1.1)}$$

$$z = d\left\{1 - \beta \frac{x_u}{d}\right\}. \qquad \text{(IS 38.1)}$$

This reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

– If $M_u > M_{\text{single}}$, the area of compression reinforcement, A'_s, is given by:

$$A'_s = \frac{M_u - M_{\text{single}}}{\left(f_{sc} - \frac{0.67\, f_{ck}}{\gamma_m}\right)\left(d - d'\right)} \qquad \text{(IS G-1.2)}$$

where d' is the depth of the compression reinforcement from the concrete compression face, and

$$f_{sc} = \varepsilon_{c,\max} E_s \left[1 - \frac{d'}{x_{u,\max}}\right] \le \frac{f_y}{\gamma_s} \qquad \text{(IS G-1.2)}$$

The required tension reinforcement is calculated as:

$$A_s = \frac{M_{\text{single}}}{\left(f_y / \gamma_s\right) z} + \frac{M_u - M_{\text{single}}}{\left(f_y / \gamma_s\right)\left(d - d'\right)}, \text{ where} \qquad \text{(IS G-1.2)}$$

$$z = d\left\{1 - \beta \frac{x_{u,\max}}{d}\right\} \qquad \text{(IS 38.1)}$$

A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

8.4.2.3 Design of Flanged Beams

8.4.2.3.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

8.4.2.3.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located within the flange. On the basis of this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange depth, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 8-2.

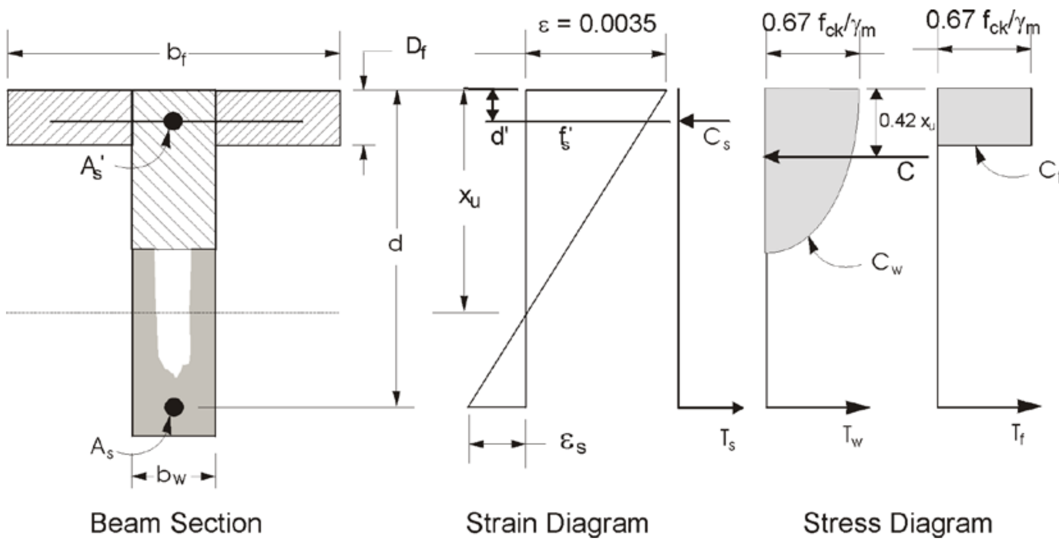


Figure 8-2 Design of a T-Beam Section

Assuming the neutral axis lies in the flange, the depth of the neutral axis is calculated as:

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta}$$

where the normalized design moment, m, is given by

$$m = \frac{M_u}{b_f d^2 \alpha f_{ck}}$$

- If $\left(\frac{x_u}{d}\right) \leq \left(\frac{D_f}{d}\right)$, the neutral axis lies within the flange and the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design (IS G-2.1). However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required when $M_u > M_{\text{single}}$.

- If $\left(\frac{x_u}{d}\right) > \left(\frac{D_f}{d}\right)$, the neutral axis lies below the flange and the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 8-2.

  – Calculate the ultimate resistance moment of the flange as:

$$M_f = 0.45 f_{ck} \left(b_f - b_w\right) \gamma_f \left(d - \frac{\gamma_f}{2}\right) \quad \text{(IS G-2.2)}$$

  where γ_f is taken as:

$$\gamma_f = \begin{cases} D_f & \text{if } D_f \leq 0.2d \\ 0.15x_u + 0.65D_f & \text{if } D_f > 0.2d \end{cases} \quad \text{(IS G-2.2)}$$

  – Calculate the moment taken by the web as

$$M_w = M_u - M_f.$$

  – Calculate the limiting ultimate moment of resistance of the web for tension reinforcement as:

$$M_{w,\text{single}} = \alpha f_{ck} b_w d^2 \, \frac{x_{u,\max}}{d}\left[1-\beta\frac{x_{u,\max}}{d}\right] \text{ where} \qquad \text{(IS G-1.1)}$$

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \text{ MPa} \\ 0.53-0.05\dfrac{f_y-250}{165} & \text{if} \quad 250 < f_y \le 415 \text{ MPa} \\ 0.48-0.02\dfrac{f_y-415}{85} & \text{if} \quad 415 < f_y \le 500 \text{ MPa} \\ 0.46 & \text{if} \quad f_y \ge 500 \text{ MPa} \end{cases} \qquad \text{(IS 38.1)}$$

- If $M_w \le M_{w,\text{single}}$, the beam is designed as a singly reinforced concrete beam. The area of reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{\left(f_y/\gamma_s\right)\left(d-0.5y_f\right)} + \frac{M_w}{\left(f_y/\gamma_s\right)z}, \text{ where}$$

$$z = d\left\{1-\beta\frac{x_u}{d}\right\}$$

$$\frac{x_u}{d} = \frac{1-\sqrt{1-4\beta m}}{2\beta}$$

$$m = \frac{M_w}{b_w d^2 \alpha f_{ck}}$$

- If $M_w > M_{w,\text{single}}$, the area of compression reinforcement, A'_s, is given by:

$$A'_s = \frac{M_w - M_{w,\text{single}}}{\left(f'_s - \dfrac{0.67 f_{ck}}{\gamma_m}\right)\left(d-d'\right)}$$

where d' is the depth of the compression reinforcement from the concrete compression face, and

$$f_{sc} = \varepsilon_{c,\max} E_s \left[1 - \frac{d'}{x_{u,\max}} \right] \le \frac{f_y}{\gamma_s} \qquad \text{(IS G-1.2)}$$

The required tension reinforcement is calculated as:

$$A_s = \frac{M_f}{\left(f_y/\gamma_s\right)\left(d - 0.5\gamma_f\right)} + \frac{M_{w,\text{single}}}{\left(f_y/\gamma_s\right)z} + \frac{M_w - M_{w,\text{single}}}{\left(f_y/\gamma_s\right)\left(d - d'\right)} \quad \text{where}$$

$$z = d\left\{1 - \beta \frac{x_{u,\max}}{d}\right\}$$

8.4.2.4 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given as (IS 26.5.1.1):

$$A_s \ge \frac{0.85}{f_y} bd \qquad \text{(IS 26.5.1.1)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement (IS 26.5.1.1) and the compression reinforcement (IS 26.5.1.2) is imposed upon request as follows:

$$A_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases} \qquad \text{(IS 26.5.1.1)}$$

$$A_s' \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases} \qquad \text{(IS 26.5.1.2)}$$

8.4.3 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station, the following steps are involved (IS 40.1):

- Determine the design shear stress
- Determine the shear stress that can be resisted by the concrete

- Determine the shear reinforcement required to carry the balance

8.4.3.1 Design for Shear when Torsion is Excluded

Determine the design nominal shear stress as follows.

- For prismatic sections

$$\tau_v = \frac{V_u}{bd} \qquad \text{(IS 40.1)}$$

- For non-prismatic sections (beams with varying depth)

$$\tau_v = \frac{V_u \pm \frac{M_u}{d} \tan\beta}{bd}, \text{ where} \qquad \text{(IS 40.1.1)}$$

β = angle between the top and bottom edges of the beam

M_u is the moment at the section, and the negative sign is considered when the numerical value of the moment increases in the same direction as the depth, d, and the positive sign is considered when the numerical value of the moment decreases in the same direction as the depth increases.

$$\tau_v \leq \tau_{c,\max} \qquad \text{(IS 40.2.3, Table 20)}$$

The maximum nominal shear stress, $\tau_{c,\max}$ is given in IS Table 20 as follows:

Maximum Shear Stress, $\tau_{c,\max}$ (MPa) (IS 40.2.3, IS Table 20)						
Concrete Grade	M15	M20	M25	M30	M35	M40
$\tau_{c,\max}$ (MPa)	2.5	2.8	3.1	3.5	3.7	4.0

The maximum nominal shear stress, $\tau_{c,\max}$, is computed using linear interpolation for concrete grades between those indicated in IS Table 20.

Determine the design shear stress that can be carried by the concrete, as:

$$\tau_{cd} = k\delta\lambda\tau_c, \qquad \text{(IS 40.2)}$$

where k is the enhancement factor for the depth of the section, taken as 1.0 for beams, and is computed as follows for other slabs:

$$k = 1 \qquad \text{(IS 40.2.1.1)}$$

δ is the enhancement factor for compression and is given as:

$$\delta = \begin{cases} 1 + 3\dfrac{P_u}{A_g f_{ck}} \le 1.5 & \text{if } P_u > 0, \text{ Under Compression} \\ 1 & \text{if } P_u \le 0, \text{ Under Tension} \end{cases} \qquad \text{(IS 40.2.2)}$$

δ is always taken as 1

λ is the factor for light-weight concrete, and

τ_c is the basic design shear strength for concrete, which is given by:

$$\tau_c = 0.64\left(\frac{100A_s}{bd}\right)^{1/3}\left(\frac{f_{ck}}{25}\right)^{1/4} \qquad \text{(IS 40.2.1)}$$

The preceding expression approximates IS Table 19. It should be noted that the value of γ_c has already been incorporated in IS Table 19 (see note in IS 36.4.2.1). The following limitations are enforced in the determination of the design shear strength as is the case in the Table.

$$0.15 \le \frac{100\,A_s}{bd} \le 3 \qquad \text{(IS 40.2.1, Table 19)}$$

$$f_{ck} \le 40 \text{ MPa (for calculation purpose only)} \qquad \text{(IS 40.2.1, Table 19)}$$

Determine required shear reinforcement:

- If $\tau_v \le \tau_{cd} + 0.4$,

$$\frac{A_{sv}}{s_v} = \frac{0.4\,b}{0.87\,f_y} \qquad \text{(IS 40.3, 26.5.1.6)}$$

- If $\tau_{cd} + 0.4 < \tau_v \le \tau_{c,\max}$,

$$\frac{A_{sv}}{s_v} = \frac{\left(\tau_v - \tau_{cd}\right) b}{0.87\ f_y}$$ (IS 40.4(a))

$$\frac{A_{sv}}{s_v} \geq \frac{0.4b}{0.87\ f_y}$$ (IS 40.4(a))

- If $\tau_v > \tau_{c,\max}$, a failure condition is declared. (IS 40.2.3)

In calculating the shear reinforcement, a limit is imposed on the f_y as:

$$f_y \leq 415 \text{ MPa}$$ (IS 40.4)

8.4.3.2 Design for Shear when Torsion is Included

Determine the design nominal shear stress as:

$$\tau_{ve} = \frac{V_e}{bd}$$ (IS 40.1)

$$\tau_{ve} \leq \tau_{c,\max}$$ (IS 40.2.3)

The maximum nominal shear stress, $\tau_{c,\max}$ is determined as defined in the last section.

Determine required shear reinforcement:

- If $\tau_{ve} \leq \tau_{cd}$,

$$\frac{A_{sv}}{s_v} = \frac{0.4\, b}{0.87\, f_y}$$ (IS 41.3, 26.5.1.6)

- If $\tau_{ve} \geq \tau_{cd}$, provide 2-legged closed stirrups, taken as the maximum of:

$$\frac{A_{sv}}{s_v} = \frac{T_u}{b_1 d_1 (0.87\, f_y)} + \frac{V_u}{2.5 d_1 (0.87\, f_y)} \text{ and}$$ (IS 41.4.3)

$$\frac{A_{sv}}{s_v} = \frac{(\tau_{ve} - \tau_c) b}{0.87\, f_y}$$ (IS 41.4.3)

The maximum of all of the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

8.5 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the limit state of collapse (IS 456-2000) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

8.5.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.

- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

8.5.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

8.5.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

8.5.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (IS 26.5.2):

$$A_s \leq \begin{cases} 0.0015bD & \text{if} \quad f_y < 415\,\text{MPa} \\ 0.0012bD & \text{if} \quad f_y \geq 415\,\text{MPa} \end{cases} \qquad \text{(IS 26.5.2.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (IS 26.5.1.1).

8.5.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following sections.

8.5.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (IS 31.6.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (IS 31.6.1). Figure 8-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner), and the punching perimeter may be overwritten using the Punching Check Overwrites.

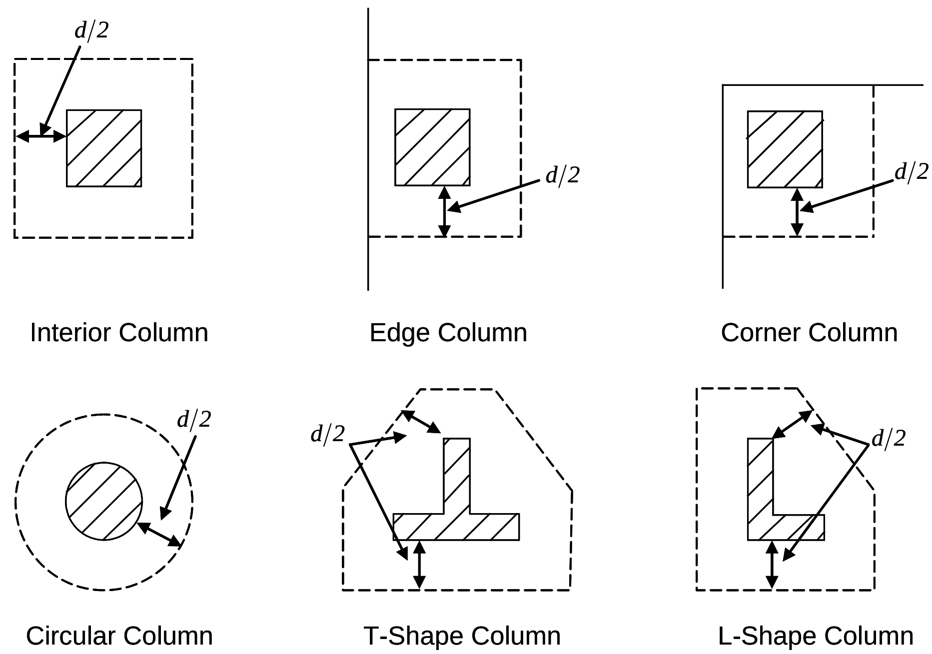


Figure 8-4 Punching Shear Perimeters

8.5.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be αM_u and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $(1 - \alpha)\, M_u$ (IS 31.6.2.2), where:

$$\alpha = \frac{1}{1+(2/3)\sqrt{a_1/a_2}} \qquad \text{(IS 31.3.3)}$$

and a_1 is the width of the critical section measured in the direction of the span and a_2 is the width of the critical section measured in the direction perpendicular to the span.

8.5.2.3 Determine Concrete Capacity

The concrete punching shear factored strength is taken as:

$$v_c = k_s \tau_c \quad \text{(IS 31.6.3.1)}$$

$$k_s = 0.5 + \beta_c \leq 1.0 \quad \text{(IS 31.6.3.1)}$$

$$\tau_c = 0.25\sqrt{f_{ck}} \quad \text{(IS 31.6.3.1)}$$

β_c = ratio of the minimum to the maximum dimensions of the support section.

8.5.2.4 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

$$v_U = \frac{V_U}{b_0 d} + \frac{\gamma_{V2}[M_{U2} - V_U(y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^{\;2}} - \frac{\gamma_{V3}[M_{U3} - V_U(x_3 - x_1)][I_{22}(x_4 - x_3) - I_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}^{\;2}} \quad \text{Eq. 1}$$

$$I_{22} = \sum_{sides=1}^{n} \overline{I}_{22}\,, \quad \text{where "}\textit{sides}\text{" refers to the sides of the critical section for punching shear} \quad \text{Eq. 2}$$

$$I_{33} = \sum_{sides=1}^{n} \overline{I}_{33}\,, \quad \text{where "}\textit{sides}\text{" refers to the sides of the critical section for punching shear} \quad \text{Eq. 3}$$

$$I_{23} = \sum_{sides=1}^{n} \overline{I}_{23}\,, \quad \text{where "}\textit{sides}\text{" refers to the sides of the critical section for punching shear} \quad \text{Eq. 4}$$

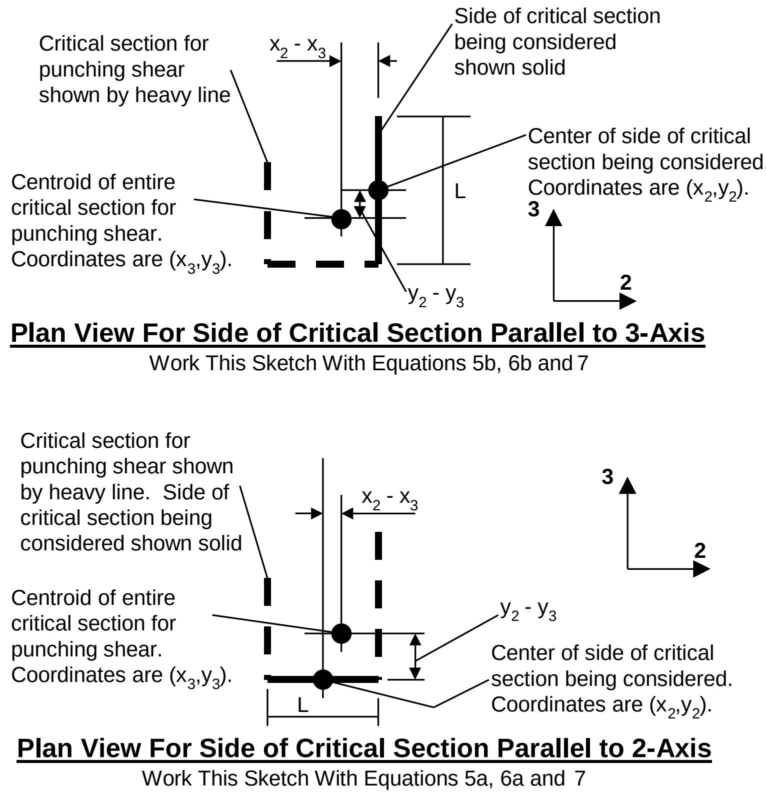


Figure 8-5 Shear Stress Calculations at Critical Sections

The equations for $\overline{I}_{22}, \overline{I}_{33}$ and $\overline{I}_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 8-5.

$\overline{I}_{22} = Ld(y_2 - y_3)^2$, for side of critical section parallel to 2-axis Eq. 5a

$\overline{I}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2$, for side of critical section parallel to 3-axis Eq. 5b

$\overline{I}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2$, for side of critical section parallel to 2-axis Eq. 6a

$\bar{I}_{33} = Ld(x_2 - x_3)^2$, for side of critical section parallel to 3-axis Eq. 6b

$\bar{I}_{23} = Ld(x_2 - x_3)(y_2 - y_3)$, for side of critical section parallel to 2-axis or 3-axis Eq. 7

NOTE: $\bar{I}_{23}$ is explicitly set to zero for corner condition.

where,

b_0 = Perimeter of critical section for punching shear

d = Effective depth at critical section for punching shear based on the average of d for the 2 direction and d for the 3 direction

I_{22} = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 2-axis

I_{33} = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 3-axis

I_{23} = Product of inertia of critical section for punching shear with respect to the 2 and 3 planes

L = Length of side of critical section for punching shear currently being considered

M_{U2} = Moment about line parallel to 2-axis at center of column (positive in accordance with the right-hand rule)

M_{U3} = Moment about line parallel to 3-axis at center of column (positive in accordance with the right-hand rule)

v_U = Punching shear stress

V_U = Shear at center of column (positive upward)

x_1, y_1 = Coordinates of column centroid

x_2, y_2 = Coordinates of center of one side of critical section for punching shear

x_3, y_3 = Coordinates of centroid of critical section for punching shear

x_4, y_4 = Coordinates of location where you are calculating stress

γ_{V2} = Percent of M_{U2} resisted by shear

γ_{V3} = Percent of M_{U3} resisted by shear

8.5.2.5 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

8.5.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is completed as described in the subsections that follow.

8.5.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined, but limited to:

$$v_c \leq 1.5\tau_c \qquad \text{(IS 31.6.3.2)}$$

8.5.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{max} = 0.5\,\tau_c\,b_o d \qquad \text{(IS 31.6.3.2)}$$

Given V_u, V_c, and V_{max}, the required shear reinforcement is calculated as follows (IS 31.6.3.2).

$$A_v = \frac{\left(V_u - 0.5V_c\right)S}{0.87\,f_y d} \qquad \text{(IS 31.6.3.2, 40.4(a))}$$

- If $V_u > V_{\max}$, a failure condition is declared. (IS 31.6.3.2)
- If V_u exceeds the maximum permitted value of $V_{\max}$, the concrete section should be increased in size.

8.5.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 8-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

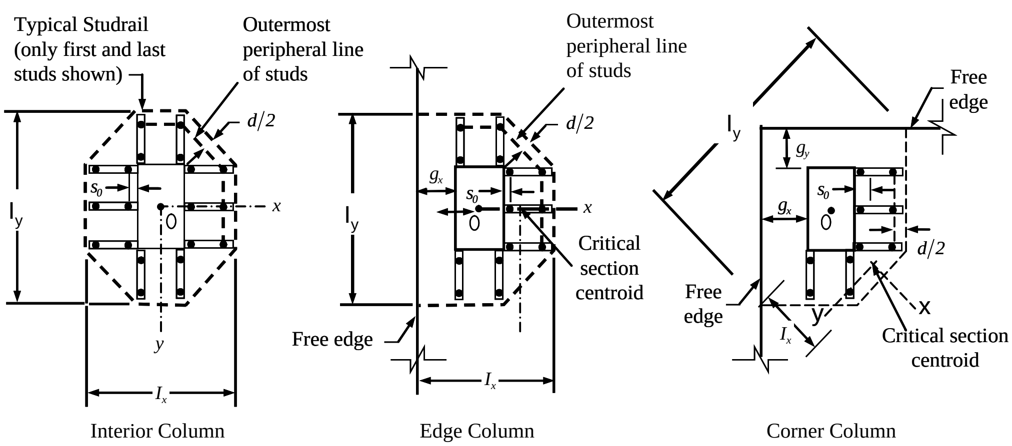


Figure 8-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed *d*/2. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face.

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

8.5.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in IS 26.4 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 2d. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d$$

$$s \leq 0.5d$$

$$g \leq 2d$$

Chapter 9
Design for NZS 3101-06

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the New Zealand code NZS 3101-06 [NZS 06] is selected. Various notations used in this chapter are listed in Table 9-1. For referencing to the pertinent sections of the New Zealand code in this chapter, a prefix "NZS" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

9.1 Notations

Table 9-1 List of Symbols Used in the NZS 3101-06 Code

A_{co}	Area enclosed by perimeter of the section, sq-mm
A_{cv}	Area of concrete used to determine shear stress, sq-mm
A_g	Gross area of concrete, sq-mm

Table 9-1 List of Symbols Used in the NZS 3101-06 Code

A_l	Area of longitudinal reinforcement for torsion, sq-mm
A_o	Gross area enclosed by shear flow path, sq-mm
A_s	Area of tension reinforcement, sq-mm
A'_s	Area of compression reinforcement, sq-mm
$A_{s(required)}$	Area of steel required for tension reinforcement, sq-mm
A_t/s	Area of closed shear reinforcement per unit length for torsion, sq-mm/mm
A_v	Area of shear reinforcement, sq-mm
A_v/s	Area of shear reinforcement per unit length, sq-mm/mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
a_{max}	Maximum allowed depth of compression block, mm
b	Width of member, mm
b_f	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Distance from extreme compression fiber to the neutral axis, mm
c_b	Distance from extreme compression fiber to neutral axis at balanced condition, mm
d	Distance from extreme compression fiber to tension reinforcement, mm
d'	Distance from extreme compression fiber to compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa

Table 9-1 List of Symbols Used in the NZS 3101-06 Code

f'_c	Specified compressive strength of concrete, MPa
f'_s	Stress in the compression reinforcement, psi
f_y	Specified yield strength of flexural reinforcement, MPa
f_{yt}	Specified yield strength of shear reinforcement, MPa
h	Overall depth of sections, mm
h_f	Thickness of slab or flange, mm
k_a	Factor accounting for influence of aggregate size on shear strength
k_d	Factor accounting for influence of member depth on shear strength
M^*	Factored design moment at a section, N-mm
p_c	Outside perimeter of concrete section, mm
p_o	Perimeter of area A_o, mm
s	Spacing of shear reinforcement along the length, mm
T^*	Factored design torsion at a section, N-mm
t_c	Assumed wall thickness of an equivalent tube for the gross section, mm
t_o	Assumed wall thickness of an equivalent tube for the area enclosed by the shear flow path, mm
V_c	Shear force resisted by concrete, N
V^*	Factored shear force at a section, N
v^*	Average design shear stress at a section, MPa
v_c	Design shear stress resisted by concrete, MPa
$v_{\max}$	Maximum design shear stress permitted at a section, MPa
v_{tn}	Shear stress due to torsion, MPa
α_s	Punching shear factor accounting for column location
α_1	Concrete strength factor to account for sustained loading and equivalent stress block
β_1	Factor for obtaining depth of compression block in concrete

Table 9-1 List of Symbols Used in the NZS 3101-06 Code

β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)
ε_s	Strain in reinforcement
ϕ_b	Strength reduction factor for bending
ϕ_s	Strength reduction factor for shear and torsion
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

9.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For NZS 3101-06, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (AS/NZS 1170.0, 4.2.2):

1.35D	(AS/NZS 1170.0, 4.2.2(a))
1.2D + 1.5L	(AS/NZS 1170.0, 4.2.2(b))
1.2D + 1.5(0.75 PL)	(AS/NZS 1170.0, 4.2.2(b))
1.2D + 0.4L + 1.0S	(AS/NZS 1170.0, 4.2.2(g))
1.2D ± 1.0W	(AS/NZS 1170.0, 4.2.2(d))
0.9D ± 1.0W	(AS/NZS 1170.0, 4.2.2(e))
1.2D + 0.4L ± 1.0W	(AS/NZS 1170.0, 4.2.2(d))
1.0D ± 1.0E	(AS/NZS 1170.0, 4.2.2(f))
1.0D + 0.4L ± 1.0E	(AS/NZS 1170.0, 4.2.2(f))

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in SAFE whenever the NZS 3101-06 code is used. If roof live load is treated separately or if other types of loads are present, other appropriate load combinations should be used.

9.3 Limits on Material Strength

The upper and lower limits of f'_c shall be as follows:

$$25 \leq f'_c \leq 100 \text{ MPa} \qquad \text{(NZS 5.2.1)}$$

The lower characteristic yield strength of longitudinal reinforcement, f_y, should be equal to or less than 500 MPa for all frames (NZS 5.3.3). The lower characteristic yield strength of transverse (stirrup) reinforcement, f_{yt}, should not be greater than 500 MPa for shear or 800 MPa for confinement (NZS 5.3.3).

The code allows use of f'_c and f_y beyond the given limits, provided special study is conducted (NZS 5.2.1).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

9.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied to the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

ϕ_b = 0.85 for flexure (NZS 2.3.2.2)

ϕ_s = 0.75 for shear and torsion (NZS 2.3.2.2)

These values can be overwritten; however, caution is advised.

9.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

9.5.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

9.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

9.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 9-1 (NZS 7.4.2.7). Furthermore, it is assumed that the compression carried by the concrete is 0.75 times that which can be carried at the balanced condition (NZS 9.3.8.1). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

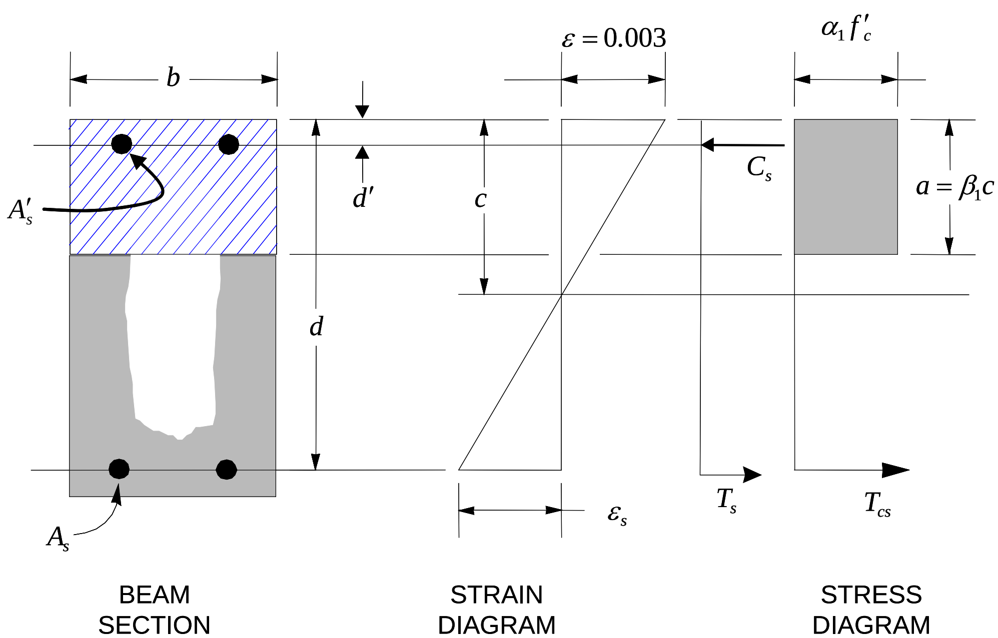


Figure 9-1 Rectangular Beam Design

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed (0.1 $f'_c A_g$), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than 0.1 $f'_c A_g$ and axial tensions are always included in flexural and shear design.

9.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive M^* (i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 9-1), where,

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \phi_b b}} \qquad \text{(NZS 7.4.2)}$$

where the default value of ϕ_b is 0.85 (NZS 2.3.2.2) in the preceding and following equations. The factor α_1 is calculated as follows (NZS 7.4.2.7):

$$\alpha_1 = 0.85 \quad \text{for} \quad f'_c \leq 55\,\text{MPa}$$

$$\alpha_1 = 0.85 - 0.004(f'_c - 55) \quad \text{for} \quad f'_c \geq 55\text{MPa}, \quad 0.75 \leq \alpha_1 \leq 0.85$$

The value β_1 and c_b are calculated as follows:

$$\beta_1 = 0.85 \quad \text{for} \quad f'_c \leq 30, \qquad \text{(NZS 7.4.2.7)}$$

$$\beta_1 = 0.85 - 0.008(f'_c - 30), \quad 0.65 \leq \beta_1 \leq 0.85 \qquad \text{(NZS 7.4.2.7)}$$

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + f_y/E_s} d \qquad \text{(NZS 7.4.2.8)}$$

The maximum allowed depth of the rectangular compression bloack, $a_{\max}$, is given by:

$$a_{\max} = 0.75\beta_1 c_b \qquad \text{(NZS 7.4.2.7, 9.3.8.1)}$$

- If $a \leq a_{\max}$ (NZS 9.3.8.1), the area of tension reinforcement is give by:

$$A_s = \frac{M^*}{\phi_b f_y \left(d - \frac{a}{2} \right)}$$

The reinforcement is to be placed at the bottom if M^* is positive, or at the top if M^* is negative.

- If $a > a_{\max}$ (NZS 9.3.8.1), compression reinforcement is required (NZS 7.4.2.9) and is calculated as follows:

  The compressive force developed in the concrete alone is given by:

  $$C = \alpha_1 f'_c b a_{\max} \qquad \text{(NZS 7.4.2.7)}$$

  and the moment resisted by concrete compression and tension reinforcement is:

  $$M^*_c = C\left(d - \frac{a_{max}}{2} \right)\phi_b$$

  Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

  $$M^*_s = M^* - M^*_c$$

  The required compression reinforcement is given by:

  $$A'_s = \frac{M^*_s}{\left(f'_s - \alpha_1 f'_c \right)\left(d - d' \right)\phi_b}, \text{ where}$$

  $$f'_s = \varepsilon_{c,max} E_s \left[\frac{c - d'}{c} \right] \le f_y \qquad \text{(NZS 7.4.2.2, 7.4.2.4)}$$

The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M^*_c}{f_y \left(d - \frac{a_{max}}{2} \right)\phi_b}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_s^*}{f_y\left(d - d'\right)\phi_b}$$

Therefore, the total tension reinforcement, $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M^* is positive, and vice versa if M^* is negative.

9.5.1.2.2 Design of Flanged Beams

9.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M^* (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

9.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M^* > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \phi_b b_f}} \qquad \text{(NZS 7.4.2)}$$

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = 0.75\beta_1 c_b \qquad \text{(NZS 7.4.2.7, 9.3.8.1)}$$

- If $a \leq h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{\max}$.

- If $a > h_f$, calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 9-2.

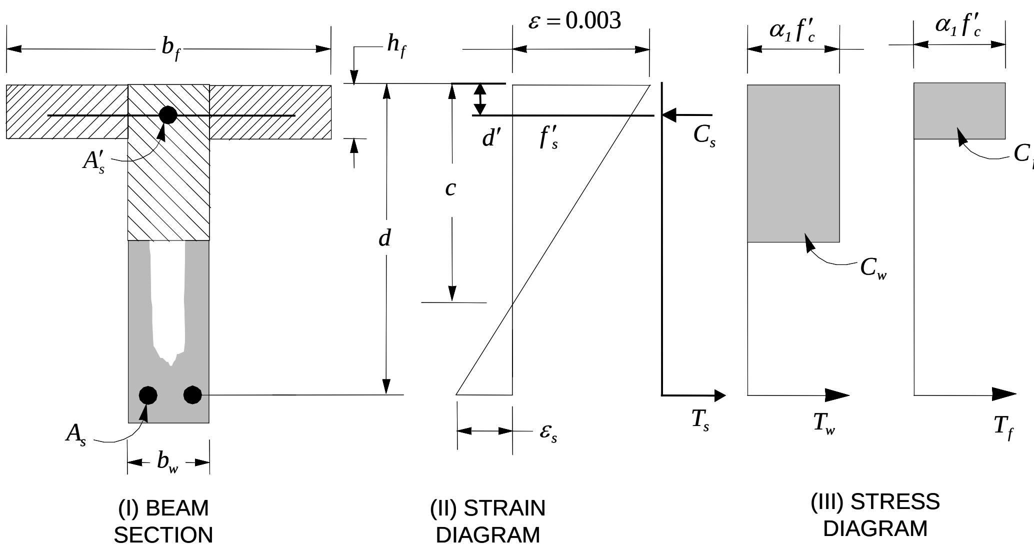


Figure 9-2 Design of a T-Beam Section

C_f is given by:

$$C_f = \alpha_1 f'_c \left(b_f - b_w\right) h_f \qquad \text{(NZS 7.4.2.7)}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of M^* that is resisted by the flange is given by:

$$M^*_f = C_f \left(d - \frac{d_s}{2}\right)\phi_b$$

Therefore, the balance of the moment, M^*, to be carried by the web is:

$$M^*_w = M^* - M^*_f$$

The web is a rectangular section with dimensions b_w and d, for which the depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M^*_w}{\alpha_1 f'_c \phi_b b_w}} \qquad \text{(NZS 7.4.2)}$$

- If $a_1 \le a_{\max}$ (NZS 9.3.8.1), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_w^*}{\phi_b f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_{\max}$ (NZS 9.3.8.1), compression reinforcement is required and is calculated as follows:

The compressive force in the web concrete alone is given by:

$$C_w = \alpha_1 f'_c b_w a_{\max} \qquad \text{(NZS 7.4.2.7)}$$

and the moment resisted by the concrete web and tension reinforcement is:

$$M_c^* = C_w \left(d - \frac{a_{\max}}{2} \right) \phi_b$$

The moment resisted by compression and tension reinforcement is:

$$M^*_s = M^*_w - M^*_c$$

Therefore, the compression reinforcement is computed as:

$$A'_s = \frac{M_s^*}{\left(f'_s - \alpha_1 f'_c \right)\left(d - d' \right) \phi_b}, \text{ where}$$

$$f'_s = \varepsilon_{c,\max} E_s \left[\frac{c - d'}{c} \right] \le f_y \qquad \text{(NZS 7.4.2.2, 7.4.2.4)}$$

The tension reinforcement for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_c^*}{f_y \left(d - \frac{a_{\max}}{2} \right) \phi_b}$$

and the tension reinforcement for balancing the compression reinforcement is:

$$A_{s3} = \frac{M_s^*}{f_y(d - d')\phi_b}$$

Total tension reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top.

9.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the maximum of the two limits:

$$A_s \geq \frac{\sqrt{f'_c}}{4 f_y} b_w d \qquad \text{(NZS 9.3.8.2.1)}$$

$$A_s \geq 1.4 \frac{b_w d}{f_y} \qquad \text{(NZS 9.3.8.2.1)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

9.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V^*.

- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

9.5.2.1 Determine Shear Force and Moment

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam section are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

9.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = v_c A_{cv} \qquad \text{(NZS 9.3.9.3.4)}$$

The allowable shear stress capacity is given by:

$$v_c = k_d k_a k_n v_b \qquad \text{(NZS 9.3.9.3.4)}$$

The basic shear strength for a rectangular section is computed as,

$$v_b = \left[0.07 + 10\frac{A_s}{b_w d}\right] \lambda\sqrt{f'_c}\text{, where} \qquad \text{(NZS 9.3.9.3.4)}$$

$$f'_c \leq 50 \text{ MPa, and} \qquad \text{(NZS 9.3.9.3.4)}$$

$$0.08\lambda\sqrt{f'_c} \leq v_b \leq 0.2\lambda\sqrt{f'_c} \qquad \text{(NZS 9.3.9.3.4)}$$

where

$$\lambda = \begin{cases} 1.0, & \text{normal concrete} \\ 0.85, & \text{sand light-weight concrete} \\ 0.75, & \text{all light-weight concrete} \end{cases} \qquad \text{(NZS 9.3.9.3.5)}$$

The factor k_a allows for the influence of maximum aggregate size on shear strength. For concrete with a maximum aggregate size of 20 mm or more, k_a shall be taken as 1.0. For concrete where the maximum aggregate size is 10 mm or less, the value of k_a shall be taken as 0.85. Interpolation is used between these limits. The program default for k_a is 1.0.

$$k_a = \begin{cases} 0.85, & a_g \le 10\,\text{mm} \\ 0.85 + 0.15\left(\dfrac{a_g - 10}{20}\right), & a_g < 20\,\text{mm} \\ 1.00, & a_g \ge 20\,\text{mm} \end{cases} \qquad \text{(NZS 9.3.9.3.4)}$$

The factor k_d allows for the influence of member depth on strength and it shall be calculated from the following conditions:

- For members with shear reinforcement equal to or greater than the nominal shear reinforcement given in NZS 9.3.9.4.15, k_d = 1.0
- For members with an effective depth equal to or smaller than 400 mm, k_d = 1.0 (NZS 9.3.9.3.4)
- For members with an effective depth greater than 400, $k_d = \left(400/d\right)^{0.25}$ where d is in mm (NZS 9.3.9.3.4)

The factor k_n allows for the influence of axial loading (NZS 10.3.10.3.1).

$$k_n = \begin{cases} 1, & \text{N}^* = 0 \\ 1 + 3\left(\dfrac{N^*}{A_g f_c'}\right), & \text{N}^* > 0 \\ 1 + 12\left(\dfrac{N^*}{A_g f_c'}\right), & \text{N}^* < 0 \end{cases} \qquad \text{(NZS 10.3.10.3.1)}$$

9.5.2.3 Determine Required Shear Reinforcement

The average shear stress is computed for rectangular and flanged sections as:

$$v^* = \frac{V^*}{b_w d} \qquad \text{(NZS 7.5.1)}$$

The average shear stress is limited to a maximum of,

$$v_{\max} = \min\left\{0.2 f'_c, 8 \text{ MPa}\right\} \qquad \text{(NZS 7.5.2, 9.3.9.3.3)}$$

The shear reinforcement is computed as follows:

- If $v^* \le \phi_s (v_c/2)$ or $h \le \max(300 \text{ mm}, 0.5b_w)$,

$$\frac{A_v}{s} = 0 \qquad \text{(NZS 9.3.9.4.13)}$$

- If $\phi_s (v_c/2) < v^* \le \phi_s v_c$,

$$\frac{A_v}{s} = \frac{1}{16}\sqrt{f'_c}\frac{b_w}{f_{yt}} \qquad \text{(NZS 7.5.10, 9.3.9.4.15)}$$

- If $\phi_s v_c < v^* \le \phi_s v_{\max}$, (NZS 9.3.9.4.2)

$$\frac{A_v}{s} = \frac{\left(v^* - \phi_s v_c\right)}{\phi_s f_{yt} d}$$

- If $v^* > v_{\max}$, a failure condition is declared. (NZS 7.5.2, 9.3.9.3.3)

If the beam depth h is less than the minimum of 300 mm and $0.5b_w$, no shear reinforcement is required (NZS 9.3.9.4.13).

The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

9.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T^*.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

Note that the torsion design can be turned off by choosing not to consider torsion in the Design Preferences.

9.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T^* is permitted to be reduced in accordance with the code (NZS 7.6.1.3). However, the program does not automatically redistribute the internal forces and reduce T^*. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

9.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{co}, A_o, p_c, p_o, t_c, and t_o are calculated. These properties are described in the following (NZS 7.1).

A_{co} = Area enclosed by outside perimeter of concrete cross-section

A_o = Gross area enclosed by shear flow path

p_c = Outside perimeter of concrete cross-section

p_o = Perimeter of area A_o

t_c = Assumed wall thickness of an equivalent tube for the gross section

t_o = Assumed wall thickness of an equivalent tube for the area enclosed by the shear flow path

In calculating the section properties involving reinforcement, such as A_o, p_o, and t_o, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to a 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{co} = bh \quad \text{(NZS 7.1)}$$

$$A_o = (b - 2c)(h - 2c) \quad \text{(NZS 7.1)}$$

$$p_c = 2b + 2h \quad \text{(NZS 7.1)}$$

$$p_o = 2(b - 2c) + 2(h - 2c) \quad \text{(NZS 7.1)}$$

$$t_c = 0.75\, A_o/p_o \quad \text{(NZS 7.1)}$$

$$t_o = 0.75\, A_{co}/p_c \quad \text{(NZS 7.1)}$$

where, the section dimensions b, h, and c are shown in Figure 9-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_{co} = b_w h + (b_f - b_w)h_f \quad \text{(NZS 7.1)}$$

$$A_o = (b_w - 2c)(h - 2c) \quad \text{(NZS 7.1)}$$

$$p_c = 2b_f + 2h \quad \text{(NZS 7.1)}$$

$$p_o = 2(h - 2c) + 2(b_w - 2c) \quad \text{(NZS 7.1)}$$

$$t_c = 0.75\, A_o/p_o \quad \text{(NZS 7.1)}$$

$$t_o = 0.75\, A_{co}/p_c \quad \text{(NZS 7.1)}$$

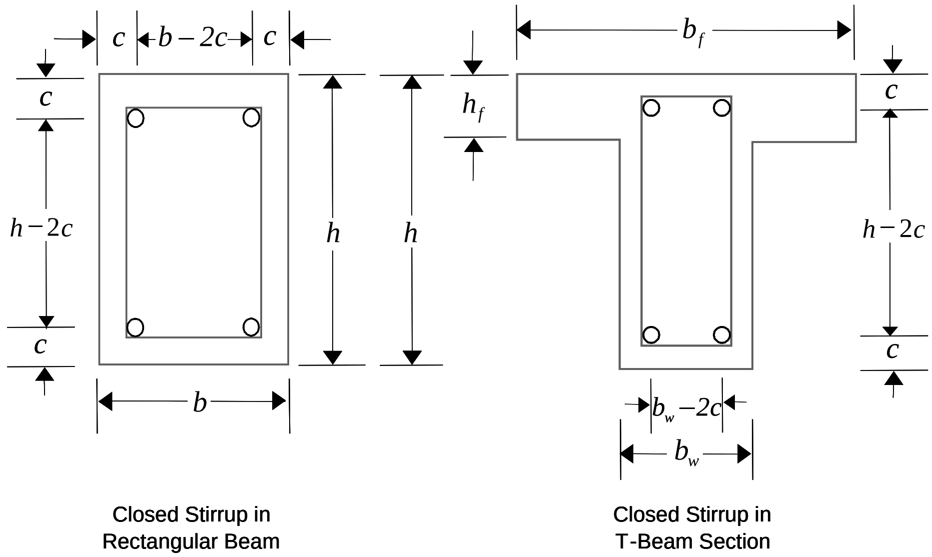


Figure 9-3 Closed stirrup and section dimensions for torsion design

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 9-3. Note that the flange width on either side of the beam web is limited to the smaller of $3h_f$ (NZS 7.6.1.7).

9.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \phi 0.1 A_{co} t_c \sqrt{f'_c} \qquad \text{(NZS 7.6.1.2)}$$

where A_{co} and t_c are as described in the previous section, and f'_c is the specified concrete compressive strength. The stress caused by torsion should also be limited in order to ignore torsion, defined as:

$$\frac{T^*}{\phi 2 A_o t_o} \leq 0.08 \sqrt{f'_c} \qquad \text{(NZS 7.6.1.3)}$$

9.5.3.4 Determine Torsion Reinforcement

If the factored torsion, T^*, is less than the threshold limit, T_{cr}, and meets the torsion stress limit, torsion can be safely ignored (NZS 7.6.1). In that case, the program reports that no torsion reinforcement is required. However, if T^* exceeds the threshold limit, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (NZS 7.6.4.1).

- If $T^* > T_{cr}$ and/or the torsion stress limit is not met, the required closed stirrup area per unit spacing, A_t/s, is calculated as:

$$\frac{A_t}{s} = \frac{v_{tn} t_o}{f_{yt}} \qquad \text{(NZS 7.6.4.2)}$$

  and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{v_{tn} t_o p_o}{f_y} \qquad \text{(NZS 7.6.4.3)}$$

  where the torsional shear stress v_{tn} is defined as:

$$v_{tn} = \frac{T^*}{\phi 2 A_o t_o} \qquad \text{(NZS 7.6.1.6)}$$

The minimum closed stirrups and longitudinal reinforcement shall be such that the following is satisfied, where A_t/s can be from any closed stirrups for shear and A_l can include flexure reinforcement, provided it is fully developed.

$$\sqrt{\frac{A_t A_l}{s p_o}} = \frac{1.5 A_o t_c}{f_y A_o} \qquad \text{(NZS 7.6.2)}$$

An upper limit of the combination of V^* and T^* that can be carried by the section is also checked using the equation:

$$v_n + v_{tn} < \min(0.2 f'_c, 8\,\text{MPa}) \qquad \text{(NZS 7.6.1.8, 7.5.2)}$$

For rectangular sections, b_w is replaced with b. If the combination of V^* and T^* exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

9.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method (NZS 3101-06) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

9.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

9.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

9.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

9.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limit (NZS 12.5.6.2, 8.8, 2.4.4):

$$A_s \geq \begin{cases} \dfrac{0.7}{f_y} bh & f_y < 500 \text{ MPa} \\ 0.0014bh & f_y \geq 500 \text{ MPa} \end{cases} \qquad \text{(NZS 12.5.6.2, 8.8.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

The slab reinforcement requirements reported by the program do not consider crack control. Any minimum requirements to satisfy crack limitations must be investigated independently of the program by the user.

9.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following.

9.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (NZS 12.7.1(b)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (NZS 12.7.1(b)). Figure 9-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

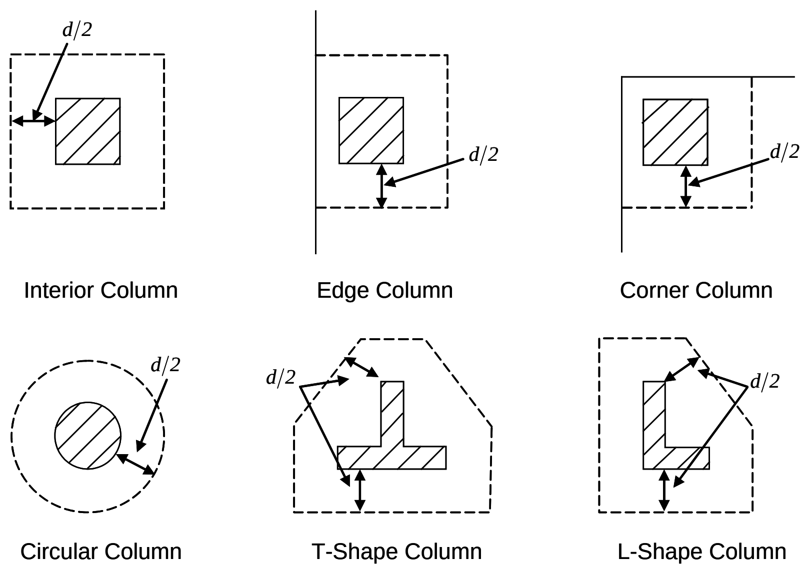


Figure 9-4 Punching Shear Perimeters

9.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M^*$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M^*$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(NZS 12.7.7.2)}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(NZS 12.7.7.1)}$$

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span.

9.6.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = \min\begin{cases} \dfrac{1}{6}\left(1+\dfrac{2}{\beta_c}\right)\sqrt{f'_c} \\ \dfrac{1}{6}\left(1+\dfrac{\alpha_s d}{b_0}\right)\sqrt{f'_c} \\ \dfrac{1}{3}\sqrt{f'_c} \end{cases} \qquad \text{(NZS 12.7.3.2)}$$

where, β_c is the ratio of the maximum to the minimum dimension of the critical section (NZS 12.1, 12.7.3.2(a)), b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 20 & \text{for interior columns,} \\ 15 & \text{for edge columns,} \\ 10 & \text{for corner columns.} \end{cases} \qquad \text{(NZS 12.7.3.2(b))}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as follows:

$$\lambda\sqrt{f'_c} \leq \sqrt{100} \qquad \text{(NZS 9.3.9.3.5(6))}$$

9.6.2.4 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

$$v^* = \frac{V^*}{b_0 d} + \frac{\gamma_{V2}[M_2^* - V^*(y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^{\ 2}} - \frac{\gamma_{V3}[M_3^* - V^*(x_3 - x_1)][I_{22}(x_4 - x_3) - I_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}^{\ 2}} \qquad \text{Eq. 1}$$

$I_{22} = \sum_{sides=1}^{n} \overline{I}_{22}$, where "*sides*" refers to the sides of the critical section for punching shear — Eq. 2

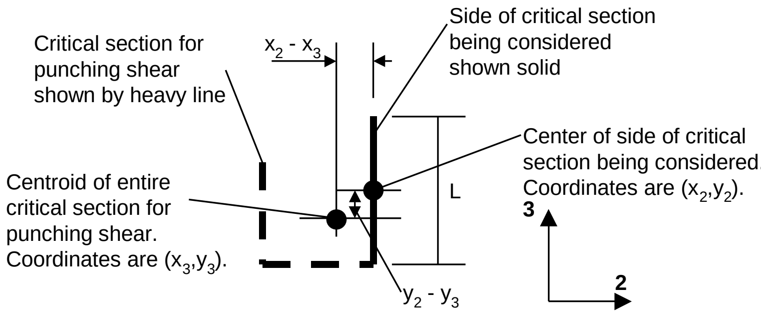
$I_{33} = \sum_{sides=1}^{n} \overline{I}_{33}$, where "*sides*" refers to the sides of the critical section for punching shear — Eq. 3

$I_{23} = \sum_{sides=1}^{n} \overline{I}_{23}$, where "*sides*" refers to the sides of the critical section for punching shear — Eq. 4

The equations for $\overline{I}_{22}, \overline{I}_{33,}$ and $\overline{I}_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 9-5.

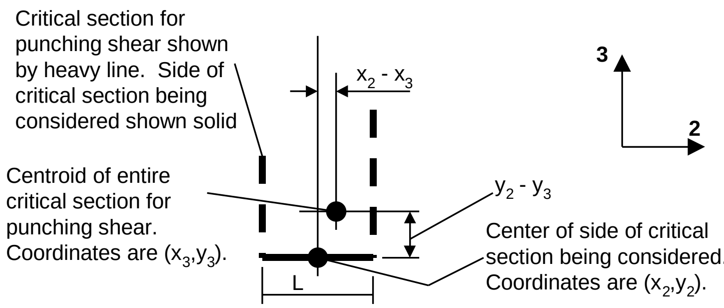
$\overline{I}_{22} = Ld(y_2 - y_3)^2$, for side of critical section parallel to 2-axis — Eq. 5a

$\overline{I}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2$, for the side of the critical section parallel to the 3-axis — Eq. 5b



Plan View For Side of Critical Section Parallel to 3-Axis

Work This Sketch With Equations 5b, 6b and 7



Plan View For Side of Critical Section Parallel to 2-Axis

Work This Sketch With Equations 5a, 6a and 7

Figure 9-5 Shear Stress Calculations at Critical Sections

$$\bar{I}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - y_3)^2$$, for the side of the critical section parallel to the 2-axis Eq. 6

$$\bar{I}_{33} = Ld(x_2 - x_3)^2$$, for the side of the critical section parallel to the 3-axis Eq. 6b

$$\bar{I}_{23} = Ld(x_2 - x_3)(y_2 - y_3)$$, for the side of the critical section parallel to the 2-axis or 3-axis Eq. 7

NOTE: $\bar{I}_{23}$ is explicitly set to zero for corner condition.

where,

b_0 = Perimeter of critical section for punching shear

d = Effective depth at critical section for punching shear based on average of d for 2 direction and d for 3 direction

I_{22} = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 2-axis

I_{33} = Moment of inertia of critical section for punching shear about an axis that is parallel to the local 3-axis

I_{23} = Product of inertia of critical section for punching shear with respect to the 2 and 3 planes

L = Length of the side of the critical section for punching shear currently being considered

M_2^* = Moment about line parallel to 2-axis at center of column (positive per right-hand rule)

M_3^* = Moment about line parallel to 3-axis at center of column (positive per right-hand rule)

V^* = Punching shear stress

V^* = Shear at center of column (positive upward)

x_1, y_1 = Coordinates of column centroid

x_2, y_2 = Coordinates of center of one side of critical section for punching shear

x_3, y_3 = Coordinates of centroid of critical section for punching shear

x_4, y_4 = Coordinates of location where you are calculating stress

γ_{V2} = Percent of M_{U2} resisted by shear

γ_{V3} = Percent of M_{U3} resisted by shear

9.6.2.5 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

9.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as described in the subsections that follow.

9.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is determined as:

$$v_c = \frac{1}{6}\sqrt{f'_c} \qquad \text{(NZS 12.7.3.5)}$$

9.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$v_{\max} = 0.5\sqrt{f'_c} \qquad \text{(NZS 12.7.3.4)}$$

Given v^*, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows, where, ϕ, is the strength reduction factor.

$$\frac{A_v}{s} = \frac{(v_n - v_c)}{\phi f_{yv} d} \qquad \text{(NZS 12.7.4.2(a))}$$

Minimum punching shear reinforcement should be provided such that:

$$V_s \geq \frac{1}{16}\sqrt{f'_c}\, b_o d \qquad \text{(NZS 12.7.4.3)}$$

- If $v_n > \phi v_{\max}$, a failure condition is declared. (NZS 12.7.3.4)
- If v_n exceeds the maximum permitted value of $\phi v_{\max}$, the concrete section should be increased in size.

9.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 9-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

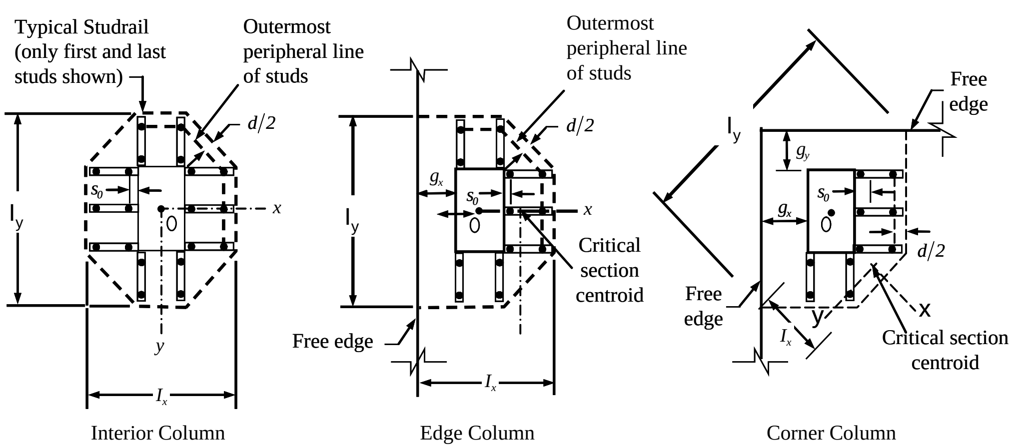


Figure 9-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (NZS 12.7.4.4).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

9.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 2*d* and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(NZS 12.7.4.4)}$$

$$s \leq 0.5d \qquad \text{(NZS 12.7.4.4)}$$

$$g \leq 2d \qquad \text{(NZS 12.7.4.4)}$$

Chapter 10
Design for Singapore CP 65-99

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Singapore standard, Structural Use of Concrete code CP 65-99 [CP 99], is selected. The program also includes the recomendations of BC 2:2008 Design Guide of High Strength Concrete to Singapore Standard CP65 [BC 2008]. Various notations used in this chapter are listed in Table 10-1. For referencing to the pertinent sections of the Singapore code in this chapter, a prefix "CP" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

10.1 Notations

Table 10-1 List of Symbols Used in the CP 65-99 Code

A_g	Gross area of cross-section, mm^2

Table 10-1 List of Symbols Used in the CP 65-99 Code

A_l	Area of longitudinal reinforcement for torsion, mm^2
A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
$A_{sv,t}$	Total cross-sectional area of closed links for torsion, mm^2
A_{sv}/s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
C	Torsional constant, mm^4
d	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f	Punching shear factor considering column location
f_{cu}	Characteristic cube strength, MPa
f'_s	Stress in the compression reinforcement, MPa
f_y	Characteristic strength of reinforcement, MPa
f_{yv}	Characteristic strength of shear reinforcement, MPa (< 460 MPa)
h	Overall depth of a section in the plane of bending, mm
h_f	Flange thickness, mm
$h_{\min}$	Smaller dimension of a rectangular section, mm
$h_{\max}$	Larger dimension of a rectangular section, mm
K	Normalized design moment, M_u/bd^2f_{cu}

Table 10-1 List of Symbols Used in the CP 65-99 Code

K'	Maximum $\frac{M_u}{bd^2 f_{cu}}$ for a singly reinforced concrete section
k_1	Shear strength enhancement factor for support compression
k_2	Concrete shear strength factor, $\left[f_{cu}/30 \right]^{1/3}$
M	Design moment at a section, N-mm
M_{single}	Limiting moment capacity as singly reinforced beam, N-mm
s_v	Spacing of the links along the length of the beam, mm
T	Design torsion at ultimate design load, N-mm
u	Perimeter of the punching critical section, mm
V	Design shear force at ultimate design load, N
v	Design shear stress at a beam cross-section or at a punching critical section, MPa
v_c	Design concrete shear stress capacity, MPa
$v_{\max}$	Maximum permitted design factored shear stress, MPa
v_t	Torsional shear stress, MPa
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm
β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain
ε_s	Strain in tension reinforcement
ε'_s	Strain in compression reinforcement

10.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (CP 2.4.1.3). If a structures is subjected to dead (D), live (L), pattern live (PL), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered (CP 2.4.3).

1.4D
1.4D + 1.6L (CP 2.4.3)

1.4D + 1.6(0.75PL) (CP 2.4.3)

1.0D ± 1.4W
1.4D ± 1.4W (CP 2.4.3)
1.2D + 1.2L ± 1.2W

These are also the default design load combinations in SAFE whenever the CP 65-99 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

10.3 Limits on Material Strength

The concrete compressive strength, f_{cu}, should not be less than 30 MPa (CP 3.1.7.2).

The program does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsible to use the proper strength values while defining the materials.

10.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the table that follows and are taken from CP Table 2.2 (CP 2.4.4.1):

Values of γ_m for the Ultimate Limit State	
Reinforcement	1.15
Concrete in flexure and axial load	1.50
Concrete shear strength without shear reinforcement	1.25

These factors are incorporated into the design equations and tables in the code, but can be overwritten.

10.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

10.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

10.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

10.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 10-1 (CP 3.4.4.4), where ε_c is defined as:

$$\varepsilon_c = \begin{cases} 0.0035 & \text{if} \quad f_{cu} \le 60\text{MPa} \\ 0.0035 - \dfrac{\left(f_{cu} - 60\right)}{50000} & \text{if} \quad f_{cu} > 60\text{MPa} \end{cases} \qquad \text{(CP 2.5.3, BC 2.2)}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$; CP 3.4.4.4). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \leq \begin{cases} 0.5 & \text{for} \quad f_{cu} \leq 60\,\text{N/mm}^2 \\ 0.4 & \text{for} \quad 60 < f_{cu} \leq 75\,\text{N/mm}^2 \\ 0.33 & \text{for} \quad 75 < f_{cu} \leq 105\,\text{N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.4, BC 2.2)}$$

to safeguard against non-ductile failures (CP 3.4.4.4). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \leq 60\ \text{N/mm}^2 \\ 0.8x & \text{for} \quad 60 < f_{cu} \leq 75\ \text{N/mm}^2 \\ 0.72x & \text{for} \quad 75 < f_{cu} \leq 105\ \text{N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.4, BC 2.2)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed ($0.1f_{cu}A_g$) (CP 3.4.4.1), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $0.1f_{cu}A_{g\ g}$ and axial tensions are always included in flexural and shear design.

10.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam, M_{single}, is first calculated for a section. The reinforcement is determined based on whether M is greater than, less than, or equal to M_{single}. See Figure 10-1.

Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

$$M_{\text{single}} = K' f_{cu}\, bd^2, \text{ where} \qquad \text{(CP 3.4.4.4)}$$

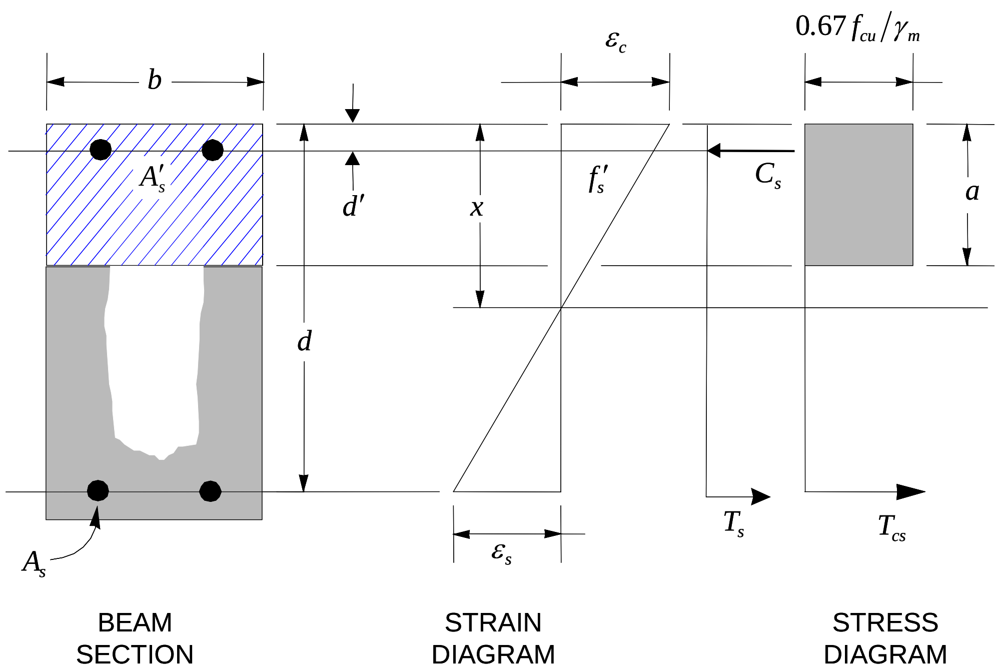


Figure 10-1 Rectangular Beam Design

$$K' = \begin{cases} 0.156 \text{ for } \quad f_{cu} \leq 60\,\text{N/mm}^2 \\ 0.120 \text{ for } 60 < f_{cu} \leq 75\text{N/mm}^2 \\ 0.094 \text{ for } 75 < f_{cu} \leq 105\text{N/mm}^2 \text{ and no moment redistribution.} \end{cases}$$

- If $M \leq M_{\text{single}}$, the area of tension reinforcement, A_s, is given by:

$$A_s = \frac{M}{0.87 f_y z}, \text{ where} \qquad \text{(CP 3.4.4.4)}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \leq 0.95d \qquad \text{(CP 3.4.4.4)}$$

$$K = \frac{M}{f_{cu} b d^2} \qquad \text{(CP 3.4.4.4)}$$

This reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

- If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$A'_s = \frac{M - M_{\text{single}}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_m}\right)(d - d')} \qquad \text{(CP 3.4.4.4)}$$

where d' is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = 0.87 f_y \qquad \text{if } d'/d \le \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(CP 3.4.4.1, 2.5.3, Fig 2.2)}$$

$$f'_s = E_s \varepsilon_c \left[1 - \frac{2d'}{d}\right] \text{ if } d'/d > \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(CP 3.4.4.4, 2.5.3, Fig 2.2)}$$

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$A_s = \frac{M_{\text{single}}}{0.87 f_y z} + \frac{M - M_{\text{single}}}{0.87 f_y (d - d')}, \text{ where} \qquad \text{(CP 3.4.4.4)}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right) \le 0.95d \qquad \text{(CP 3.4.4.4)}$$

10.5.1.2.2 Design of Flanged Beams

10.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

10.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located in the flange. On the basis of this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange width, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 10-2.

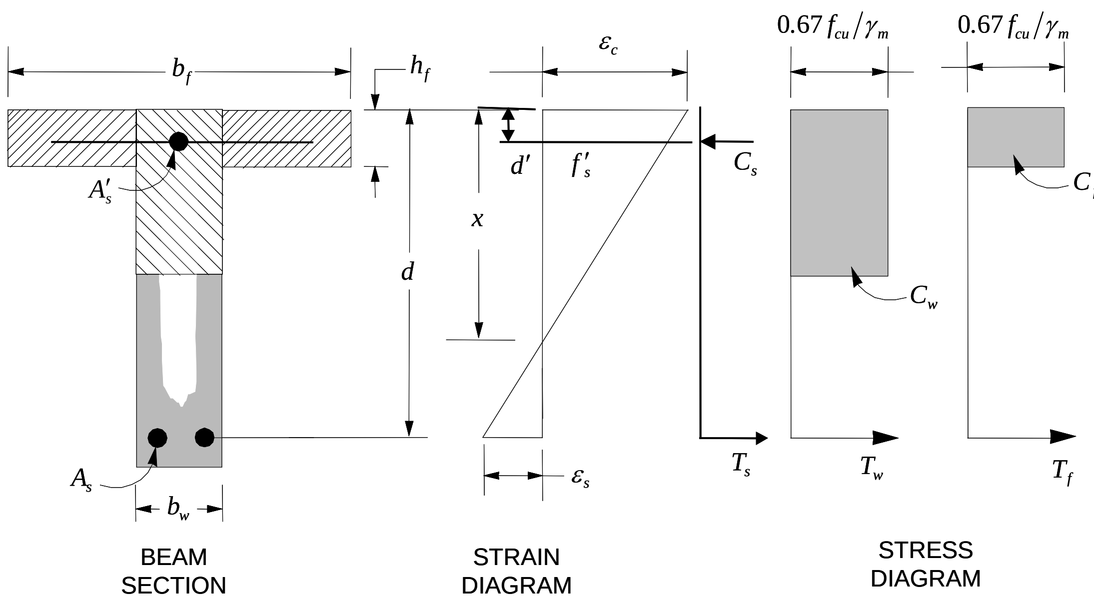


Figure 10-2 Design of a T-Beam Section

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

$$K = \frac{M}{f_{cu}\, b_f\, d^2} \qquad \text{(CP 3.4.4.4)}$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d \qquad \text{(CP 3.4.4.4)}$$

the depth of neutral axis is computed as:

$$x = \begin{cases} \dfrac{d-z}{0.45}, & \text{for} \quad f_{cu} \le 60\,\text{N/mm}^2 \\ \dfrac{d-z}{0.40}, & \text{for} \quad 60 < f_{cu} \le 75\,\text{N/mm}^2 \\ \dfrac{d-z}{0.36}, & \text{for} \quad 75 < f_{cu} \le 105\,\text{N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.4, BC 2.2, Fig 2.3)}$$

and the depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 60\ \text{N/mm}^2 \\ 0.8x & \text{for} \quad 60 < f_{cu} \le 75\ \text{N/mm}^2 \\ 0.72x & \text{for} \quad 75 < f_{cu} \le 105\ \text{N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.4, BC 2.2, Fig 2.3)}$$

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $K > K'$.

- If $a > h_f$,

If $M \le \beta_f f_{cu} b d^2$ and $h_f \le 0.45d$ then,

$$A_s = \frac{M + 0.1 f_{cu} bd\left(0.45d - h_f\right)}{0.87 f_y\left(d - 0.5h_f\right)}, \text{ where} \qquad \text{(BS 3.4.4.5)}$$

$$\beta_f = 0.45\frac{h_f}{d}\left(1 - \frac{b_w}{b}\right)\left(1 - \frac{h_f}{2d}\right) + 0.15\frac{b_w}{b} \qquad \text{(BS 3.4.4.5)}$$

Otherwise the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 10-2.

In that case, the ultimate resistance moment of the flange is given by:

$$M_f = 0.45 f_{cu} \left(b_f - b_w\right) h_f \left(d - 0.5 h_f\right) \qquad \text{(CP 3.4.4.5)}$$

The moment taken by the web is computed as:

$$M_w = M - M_f$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2} \qquad \text{(CP 3.4.4.4)}$$

- If $K_w \leq 0.156$ (CP 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{0.87 f_y \left(d - 0.5 h_f\right)} + \frac{M_w}{0.87 f_y z}, \text{ where}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \leq 0.95d$$

- If $K_w > K'$ (CP 3.4.4.4), compression reinforcement is required and is calculated as follows:

The ultimate moment of resistance of the web only is given by:

$$M_{uw} = K' f_{cu} b_w d^2 \qquad \text{(CP 3.4.4.4)}$$

The compression reinforcement is required to resist a moment of magnitude $M_w - M_{uw}$. The compression reinforcement is computed as:

$$A'_s = \frac{M_w - M_{uw}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_m}\right)\left(d - d'\right)}$$

where, d' is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = 0.87 f_y \qquad \text{if } d'/d \le \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(CP 3.4.4.4, 2.5.3, Fig 2.2)}$$

$$f'_s = E_s \varepsilon_c \left[1 - \frac{2d'}{d}\right] \quad \text{if } d'/d > \frac{1}{2}\left[1 - \frac{f_y}{800}\right] \qquad \text{(CP 3.4.4.4, 2.5.3, Fig 2.2)}$$

The area of tension reinforcement is obtained from equilibrium as:

$$A_s = \frac{1}{0.87 f_y}\left[\frac{M_f}{d - 0.5h_f} + \frac{M_{uw}}{z} + \frac{M_w - M_{uw}}{d - d'}\right]$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right) \le 0.95d$$

10.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from CP Table 3.27 (CP 3.12.5.3) with interpolation for reinforcement of intermediate strength.

Section	Situation	Definition of percentage	Minimum percentage	
			f_y = 250 MPa	f_y = 460 MPa
Rectangular	—	$100\ \frac{A_s}{bh}$	0.24	0.13
T- or L-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100\ \frac{A_s}{b_w h}$	0.32	0.18
	$\frac{b_w}{b_f} \ge 0.4$	$100\ \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.48	0.26

L-Beam with web in compression	—	$100\,\dfrac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from CP Table 3.27 (CP 3.12.5.3).

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100\,\dfrac{A'_s}{bh}$	0.20
T- or L-Beam	Web in tension	$100\,\dfrac{A'_s}{b_f h_f}$	0.40
	Web in compression	$100\,\dfrac{A'_s}{b_w h}$	0.20

For f_{cu} > 40 MPa, the minimum percentage shown in CP Table 3.27 shall be multiplied by a factor of $\left(f_{cu}/40\right)^{2/3}$ (CP 3.12.5.3, BC 2.2).

An upper limit of 0.04 times the gross cross-sectional area on both the tension reinforcement and the compression reinforcement is imposed upon request (CP 3.12.6.1).

10.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (CP 3.4.5):

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

10.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors. The shear stress is then calculated as:

$$v = \frac{V}{b_w d} \qquad \text{(CP 3.4.5.2)}$$

The maximum allowable shear stress, v_{max} is defined as:

$$v_{max} = \min\left(0.8\sqrt{f_{cu}}, 7 \text{ MPa}\right). \qquad \text{(CP 3.4.5.2)}$$

For light-weight concrete, v_{max} is defined as:

$$v_{max} = \min\left(0.63\sqrt{f_{cu}}, 4 \text{ MPa}\right) \qquad \text{(CP Part 2 5.4)}$$

10.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, v_c, is calculated as:

$$v'_c = v_c + 0.6\frac{NVh}{A_c M} \le v_c\sqrt{1+\frac{N}{A_c v_c}} \qquad \text{(CP 3.4.5.12)}$$

$$v_c = \frac{0.84 k_1 k_2}{\gamma_m}\left(\frac{100A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} \qquad \text{(CP 3.4.5.4, Table 3.9)}$$

k_1 is the enhancement factor for support compression,
and is conservatively taken as 1 (CP 3.4.5.8)

$$k_2 = \left(\frac{f_{cu}}{30}\right)^{1/3}, 1 \le k_2 \le \left(\frac{80}{30}\right)^{1/3} \qquad \text{(CP 3.4.5.4)}$$

$\gamma_m = 1.25$ (CP 2.4.4.1)

However, the following limitations also apply:

$$0.15 \le \frac{100\,A_s}{bd} \le 3$$ (CP 3.4.5.4, Table 3.9)

$$\left(\frac{400}{d}\right)^{1/4} \ge 0.67 \text{ (unreinforced) or} \ge 1 \text{ (reinforced)}$$ (CP 3.4.5.4, Table 3.9)

$f_{cu} \le 80$ MPa (for calculation purpose only) (CP 3.4.5.4, Table 3.9)

$$\frac{Vh}{M} \le 1$$ (CP 3.4.5.12)

A_s is the area of tension reinforcement

10.5.2.3 Determine Required Shear Reinforcement

Given v, v'_c, and v_{max}, the required shear reinforcement is calculated as follows (CP Table 3.8, CP 3.4.5.3):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, v_r, as:

- $$v_r = \begin{cases} 0.4 & \text{if} \quad f_{cu} \le 40\text{N/mm}^2 \\ 0.4\left(f_{cu}/40\right)^{2/3} & \text{if } 40 < f_{cu} \le 80\text{N/mm}^2 \end{cases}$$ (CP 3.4.5.3, Table 3.8)

  $f_{cu} \le 80\text{N/mm}^2$ (for calculation purpose only) (CP 3.4.5.3, Table 3.8)

- If $v \le v'_c + v_r$,

  $$\frac{A_s}{s_v} = \frac{v_r b}{0.87\,f_{yv}},$$ (CP 3.4.5.3, Table 3.8)

- If $v > v'_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v'_c)b}{0.87 f_{yv}}$$ (CP 3.4.5.3, Table 3.8)

- If $v > v_{max}$, a failure condition is declared. (CP 3.4.5.2)

In the preceding expressions, a limit is imposed on the f_{yv} as

$$f_{yv} \le 460 \text{ MPa}$$ (CP 3.4.5.1)

The maximum of all of the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

10.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t.
- Determine special section properties.
- Determine critical torsion stress.
- Determine the torsion reinforcement required.

Note that references in this section refer to CP 65:Part 2.

10.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following sections (CP Part 2 3.4.5.15).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2 \left(h_{\max} - h_{\min}/3\right)} \qquad \text{(CP Part 2 2.4.4.1)}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{seg} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(CP Part 2 2.4.4.2)}$$

$h_{\max}$ = Larger dimension of a rectangular section

$h_{\min}$ = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, does not satisfy the following limit for sections with a larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

$$v_t \le \min\left(0.8\sqrt{f_{cu}}, 7\text{N/mm}^2\right) \times \frac{y_1}{550} \qquad \text{(CP Part 2 2.4.5)}$$

10.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.6\text{N/mm}^2\right) \qquad \text{(CP Part 2 2.4.6)}$$

where f_{cu} is the specified concrete compressive strength.

For light-weight concrete, $v_{t,\min}$ is defined as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.6\text{N/mm}^2\right) \times 0.8 \qquad \text{(CP Part 2 5.5)}$$

10.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t, is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (CP Part 2 2.4.6). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP Part 2 2.4.6).

- If $v_t > v_{t,\min}$, the required closed stirrup area per unit spacing, $A_{sv,t}/s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8x_1y_1\left(0.87 f_{yv}\right)} \qquad \text{(CP Part 2 2.4.7)}$$

  and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t} f_{yv}\left(x_1 + y_1\right)}{s_v f_y} \qquad \text{(CP Part 2 2.4.7)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link and y_1 is the larger center-to-center dimension of the closed link.

An upper limit of the combination of v and v_t that can be carried by the section also is checked using the equation:

$$v + v_t \le \min\left(0.8\sqrt{f_{cu}}, 7\text{N/mm}^2\right) \qquad \text{(CP Part 2 2.4.5)}$$

If the combination of v and v_t exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

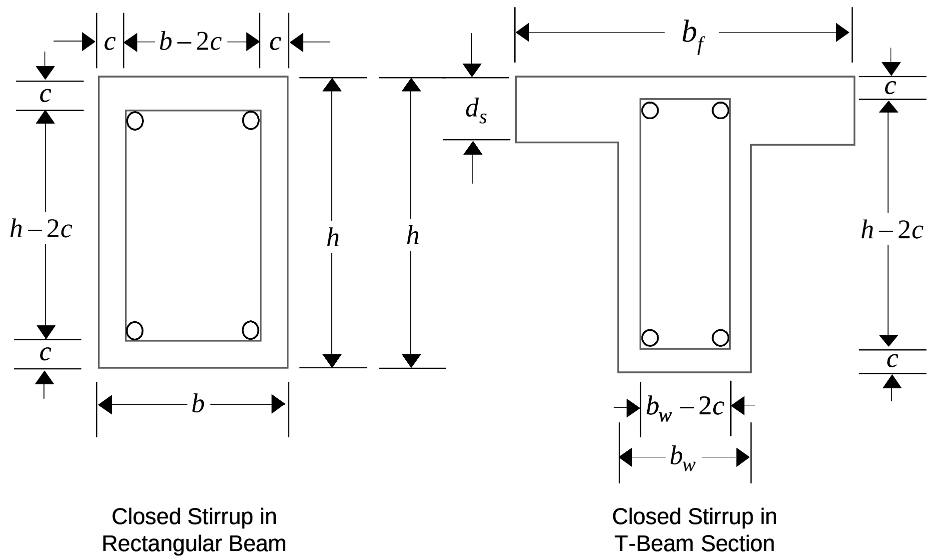


Figure 10-3 Closed stirrup and section dimensions for torsion design

10.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method (CP 65-99) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

10.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is performed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps described in the subsections that follow are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

10.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

10.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

10.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limit (CP 3.12.5.3, CP Table 3.25) with interpolation for reinforcement of intermediate strength:

$$A_s \geq \begin{cases} 0.0024bh & \text{if} \quad f_y = 250 \text{ MPa} \\ 0.0013bh & \text{if} \quad f_y = 460 \text{ MPa} \end{cases} \qquad \text{(CP 3.12.5.3)}$$

For f_{cu} > 40 N/mm^2, the preceding minimum reinforcement shall be multiplied by $(f_{cu}/40)^{2/3}$.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 3.12.6.1).

10.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

10.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5*d* from the face of the support (CP 3.7.7.4, 3.7.7.6). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CP 3.7.7.1). Figure 10-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

10.6.2.2 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as (CP 3.7.7.4, 3.7.7.6):

$$v_c = \frac{0.84 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}$$ (CP 3.4.5.4, Table 3.9)

k_1 is the enhancement factor for support compression,
and is conservatively taken as 1 (CP 3.4.5.8)

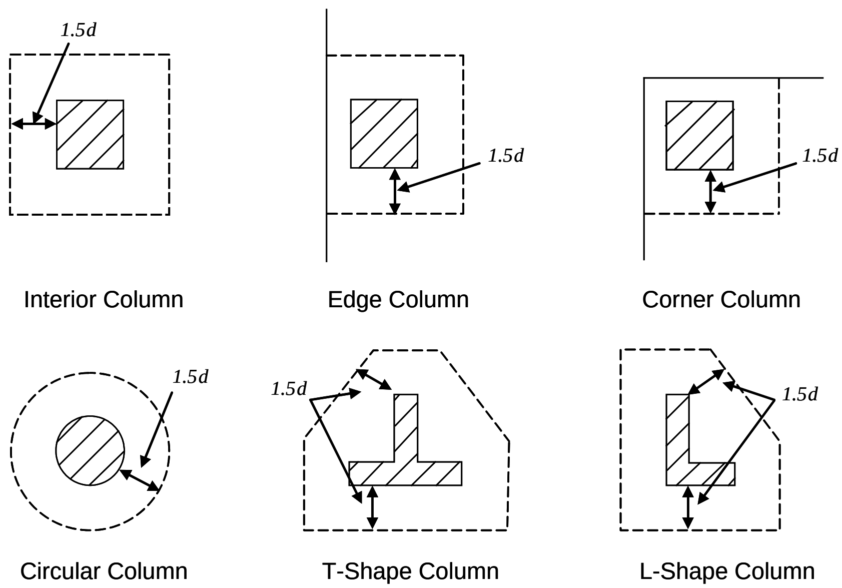


Figure 10-4 Punching Shear Perimeters

$$k_2 = \left(\frac{f_{cu}}{30}\right)^{1/3} \quad 1 \le k_2 \le \left(\frac{80}{30}\right)^{1/3}$$ (CP 3.4.5.4, Table 3.9)

$\gamma_m = 1.25$ (CP 3.4.5.2)

However, the following limitations also apply:

$$0.15 \le \frac{100\ A_s}{bd} \le 3$$ (CP 3.4.5.4, Table 3.9)

$$\left(\frac{400}{d}\right)^{1/4} \geq 0.67 \text{ (unreinforced) or} \geq 1 \text{ (reinforced)} \quad \text{(CP 3.4.5.4, Table 3.9)}$$

For light-weight concrete, v_{max} is defined as:

$$v \leq \min(0.63\sqrt{f_{cu}},\ 4 \text{ MPa}) \quad \text{(CP Part 2 5.4)}$$

$$v \leq \min\ (0.8\sqrt{f_{cu}},\ 7 \text{ MPa}) \quad \text{(CP 3.4.5.2, Table 3.9)}$$

$f_{cu} \leq 80$ MPa (for calculation purpose only) (CP 3.4.5.4, Table 3.9)

A_s = area of tension reinforcement, which is taken as the average tension reinforcement of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement is taken as zero in the current implementation.

10.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the nominal design shear stress, v, is calculated as:

$$V_{eff,x} = V\left(f + \frac{1.5M_x}{V_y}\right) \quad \text{(CP 3.7.6.2, 3.7.6.3)}$$

$$V_{eff,y} = V\left(f + \frac{1.5M_y}{V_x}\right) \quad \text{(CP 3.7.6.2, 3.7.6.3)}$$

$$v_{\max} = \max\begin{cases} \dfrac{V_{eff,x}}{u\,d} \\ \dfrac{V_{eff,y}}{u\,d} \end{cases} \quad \text{(CP 3.7.7.3)}$$

where,

u is the perimeter of the critical section,

x and y are the lengths of the sides of the critical section parallel to the axis of bending

M_x and M_y are the design moments transmitted from the slab to the column at the connection

V is the total punching shear force

f is a factor to consider the eccentricity of punching shear force and is taken as:

$$f = \begin{cases} 1.00 & \text{for interior columns,} \\ 1.25 & \text{for edge columns, and} \\ 1.25 & \text{for corner columns.} \end{cases} \qquad \text{(CP 3.7.6.2, 3.7.6.3)}$$

10.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

10.6.3 Design Punching Shear Reinforcement

The use of shear links as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 3.7.7.5). If the slab thickness does not meet this requirement, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is completed as described in the following subsections.

10.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

10.6.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

$$v_{max} = 2v_c \qquad \text{(CP 3.7.7.5)}$$

Given v, v_c, and v_{max}, the required shear reinforcement is calculated as follows (CP 3.7.7.5).

- If $v \leq 1.6v_c$,

$$\frac{A_v}{s} = \frac{(v - v_c)ud}{0.87\, f_{yv}} \geq \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(CP 3.7.7.5)}$$

- If $1.6v_c \leq v < 2.0v_c$,

$$\frac{A_v}{s} = \frac{5(0.7v - v_c)ud}{0.87\, f_{yv}} \geq \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(CP 3.7.7.5)}$$

- If $v > v_{max}$, a failure condition is declared. (CP 3.7.7.5)

If v exceeds the maximum permitted value of v_{max}, the concrete section should be increased in size.

10.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 10-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

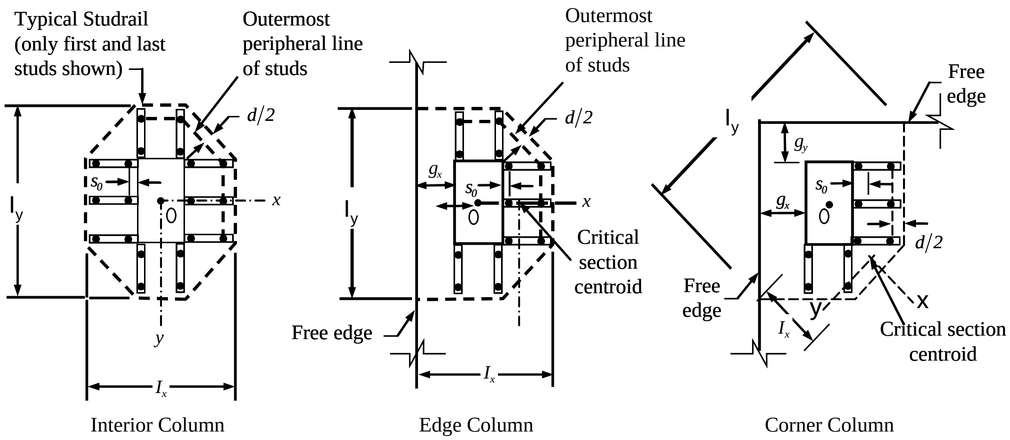


Figure 10-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed *d*/2. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5*d* measured in a direction parallel to the column face (CP 3.7.7.6).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

10.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 3.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 1.5*d*. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(CP 3.7.7.6)}$$

$$s \leq 0.75d \qquad \text{(CP 3.7.7.6)}$$

$$g \leq 1.5d \qquad \text{(CP 3.7.7.6)}$$

Chapter 11
Design for AS 3600-09

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Australian code AS 3600-2009 [AS 2009] is selected. Various notations used in this chapter are listed in Table 11-1. For referencing to the pertinent sections of the AS code in this chapter, a prefix "AS" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

11.1 Notations

Table 11-1 List of Symbols Used in the AS 3600-2009 Code

A_g	Gross area of concrete, mm^2
A_l	Area of longitudinal reinforcement for torsion, mm^2
A_{sc}	Area of compression reinforcement, mm^2

Table 11-1 List of Symbols Used in the AS 3600-2009 Code

A_{st}	Area of tension reinforcement, mm^2
$A_{st(\text{required})}$	Area of required tension reinforcement, mm^2
A_{sv}	Area of shear reinforcement, mm^2
$A_{sv,\min}$	Minimum area of shear reinforcement, mm^2
A_{sv}/s	Area of shear reinforcement per unit length, mm^2 /mm
A_{sw}/s	Area of shear reinforcement per unit length consisting of closed ties, mm^2/mm
A_t	Area of a polygon with vertices at the center of longitudinal bars at the corners of a section, mm^2
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
$a_{\max}$	Maximum allowed depth of compression block, mm
b	Width of member, mm
b_{ef}	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
c	Depth to neutral axis, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to compression reinforcement, mm
d_o	Distance from the extreme compression fiber to the centroid of the outermost tension reinforcement, mm
d_{om}	Mean value of d_o, averaged around the critical shear perimeter, mm
D	Overall depth of a section, mm
D_s	Thickness of slab (flanged section), mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, MPa
f'_c	Specified compressive strength of concrete, MPa
f'_{cf}	Characteristic flexural tensile strength of concrete, MPa

Table 11-1 List of Symbols Used in the AS 3600-2009 Code

f_{cv}	Concrete shear strength, MPa
f_{sy}	Specified yield strength of flexural reinforcement, MPa
$f_{sy,f}$	Specified yield strength of shear reinforcement, MPa
f'_s	Stress in the compression reinforcement, MPa
J_t	Torsional modulus, mm^3
k_u	Ratio of the depth to the neutral axis from the compression face, to the effective depth, d
M_{ud}	Reduced ultimate strength in bending without axial force, N-mm
M^*	Factored moment at section, N-mm
N^*	Factored axial load at section, N
s	Spacing of shear reinforcement along the beam, mm
T_{uc}	Torsional strength of section without torsional reinforcement, N-mm
$T_{u.\max}$	Maximum permitted total factored torsion at a section, N-mm
T_{us}	Torsion strength of section with torsion reinforcement, N-mm
T^*	Factored torsional moment at a section, N-mm
u_t	Perimeter of the polygon defined by A_t, mm
V^*	Factored shear force at a section, N
$V_{u.\max}$	Maximum permitted total factored shear force at a section, N
$V_{u.\min}$	Shear strength provided by minimum shear reinforcement, N
V_{uc}	Shear force resisted by concrete, N
V_{us}	Shear force resisted by reinforcement, N
γ_1	Factor for obtaining depth of compression block in concrete
β_h	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber, (0.003 mm/mm)

Table 11-1 List of Symbols Used in the AS 3600-2009 Code

ε_s	Strain in reinforcement
ϕ	Strength reduction factor
θ_t	Angle of compression strut for torsion, degrees
θ_v	Angle of compression strut for shear, degrees

11.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For AS 3600-09, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be defined (AS 2.4.2):

1.35D	(AS/NZS 1170.0-02, 4.2.2(a))
1.2D + 1.5L	(AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 1.5(0.75 PL)	(AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 0.4L + 1.0S	(AS/NZS 1170.0-02, 4.2.2(g))
0.9D ± 1.0W	(AS/NZS 1170.0-02, 4.2.2(e))
1.2D ± 1.0W	(AS/NZS 1170.0-02, 4.2.2(d))
1.2D + 0.4L ± 1.0W	(AS/NZS 1170.0-02, 4.2.2(d))
1.0D ± 1.0E	(AS/NZS 1170.0-02, 4.2.2(f))
1.0D + 0.4L ± 1.0E	(AS/NZS 1170.0-02, 4.2.2(f))

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in SAFE whenever the AS 3600-2009 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

11.3 Limits on Material Strength

The upper and lower limits of f'_c are 100 MPa and 20 MPa, respectively, for all framing types (AS 3.1.1.1(b)).

$$f'_c \leq 100\,\text{MPa} \qquad \text{(AS 3.1.1.1)}$$

$$f'_c \geq 20\,\text{MPa} \qquad \text{(AS 3.1.1.1)}$$

The upper limit of f_{sy} is 500 MPa for all frames (AS 3.2.1, Table 3.2.1).

The code allows use of f'_c and f_{sy} beyond the given limits, provided special care is taken regarding the detailing and ductility (AS 3.1.1, 3.2.1, 17.2.1.1).

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

11.4 Strength Reduction Factors

The strength reduction factor, ϕ, is defined as given in AS 2.2.2(ii), Table 2.2.2:

For members with Class N reinforcement only

ϕ = 0.80 for flexure (tension controlled) (Table 2.2.2(b))

ϕ = 0.60 for flexure (compression controlled) (Table 2.2.2(b))

For members with Class L reinforcement

ϕ = 0.64 for flexure (tension controlled) (Table 2.2.2(b))

ϕ = 0.60 for flexure (compression controlled) (Table 2.2.2(b))

ϕ = 0.70 for shear and torsion (Table 2.2.2(b))

These values can be overwritten; however, caution is advised.

11.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, axial, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement for bending and axial loading
- Design shear reinforcement
- Design torsion reinforcement

11.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments and factored axial forces
- Determine required flexural reinforcement for bending and axial forces

11.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments and axial forces for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases the beam may be designed as a rectangular or inverted flanged beam.

11.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, f_y, is limited to 500MPa (AS 3.2.1), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 11-1 (AS 8.1.2).

The following assumptions apply to the stress block used to compute the flexural bending capacity of rectangular sections (AS 8.1.2).

- The maximum strain in the extreme compression fiber is taken as 0.003 (AS 8.1.3(a)).

- A uniform compressive stress of $\alpha_2 f'_c$ acts on an area (AS 8.1.3(b)) bounded by:

  – The edges of the cross-sections.

  – A line parallel to the neutral axis at the strength limit under the loading concerned, and located at a distance $\gamma k_u d$ from the extreme compression fiber.

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by

$$a_{\max} = \gamma k_u d \quad \text{where,} \qquad \text{(AS 8.1.3(b))}$$

$$\alpha_2 = 1.0 - 0.003 f'_c \ \text{where,} \ 0.67 \le \alpha_2 \le 0.85 \qquad \text{(AS 8.1.3(1))}$$

$$\gamma = 1.05 - 0.007 f'_c \ \text{where,} \ 0.67 \le \gamma \le 0.85 \qquad \text{(AS 8.1.3(2))}$$

$k_u = 0.36$ (AS 8.1.5)

The design procedure used by SAFE for both rectangular and flanged sections (L- and T-beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed ($A_{sc}f_{sy} > 0.15N^*$) (AS 10.7.1a); hence, all beams are designed for major direction flexure, shear, and torsion only.

11.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M^*(i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 11-1), where,

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_2 f'_c \varphi b}}$$ (AS 8.1.3)

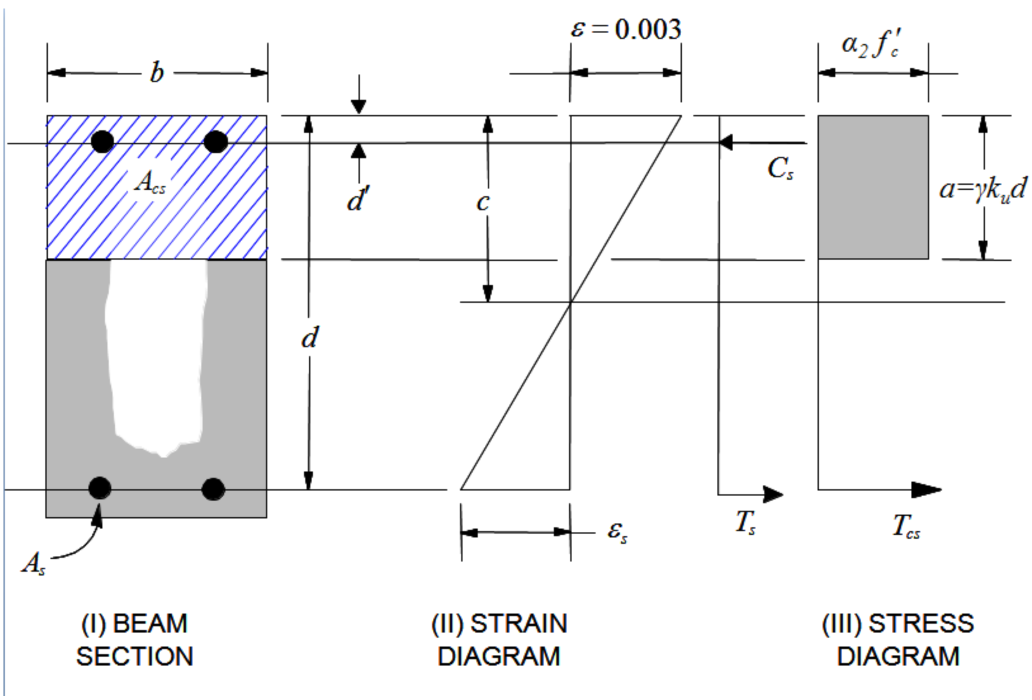


Figure 11-1 Rectangular Beam Design

where, the value of ϕ is taken as that for a tension controlled section ($k_u \leq 0.36$), which by default is 0.80 (AS 2.2.2) in the preceding and following equations. The selection of Reinforcement Class can be made using the Design Preferences.

- If $a \leq a_{\max}$, the area of tension reinforcement is then given by:

$$A_{st} = \frac{M^*}{\varphi f_{sy}\left(d - \frac{a}{2}\right)}$$

This reinforcement is to be placed at the bottom if M^* is positive, or at the top if M^* is negative.

- If $a > a_{\max}$, i.e., $k_u > 0.36$, compression reinforcement is required (AS 8.1.5) and is calculated as follows:

The compressive force developed in the concrete alone is given by:

$$C = \alpha_2 f'_c b a_{\max} \qquad \text{(AS 8.1.3)}$$

and the moment resisted by concrete compression and tension reinforcement is:

$$M_{uc} = C\left(d - \frac{a_{\max}}{2}\right)\phi$$

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M^* - M_{uc}$$

The required compression reinforcement is given by:

$$A_{sc} = \frac{M_{us}}{\left(f'_s - \alpha_2 f'_c\right)\left(d - d'\right)\phi}, \text{ where}$$

$$f'_s = 0.003 E_s \left[\frac{c - d'}{c}\right] \leq f_{sy} \qquad \text{(AS 8.1.2.1, 3.2.2)}$$

The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_{uc}}{f_{sy}\left[d - \frac{a_{\max}}{2}\right]\phi}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{us}}{f_{sy}\left(d - d'\right)\phi}$$

Therefore, the total tension reinforcement is $A_{st} = A_{s1} + A_{s2}$, and the total compression reinforcement is A_{sc}. A_{st} is to be placed at the bottom and A_{sc} is to be placed at the top if M^* is positive, and vice versa if M^* is negative.

11.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 11-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 11-1 is assumed on the compression side (AS 8.1.5).

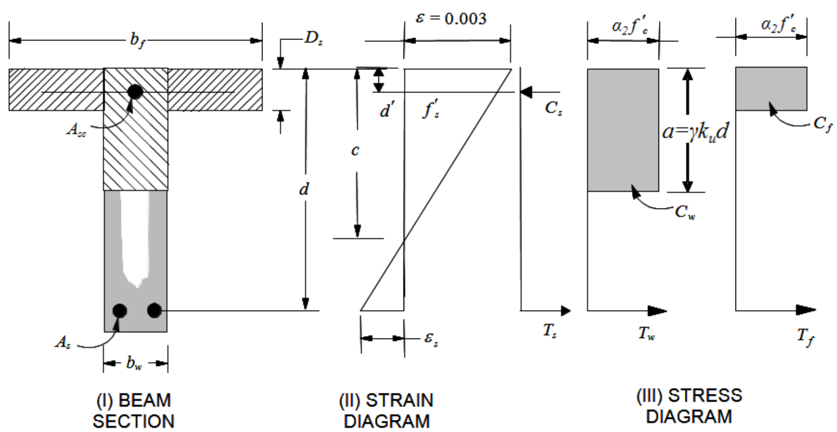


Figure 11-2 T-Beam Design

11.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M^* (i.e., designing top reinforcement), the calculation of the reinforcement is exactly the same as above, i.e., no flanged beam data is used.

11.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M^* > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{\alpha_2 f'_c \varphi b_f}}$$

where, the value of ϕ is taken as that for $k_u \leq 0.36$, which is 0.80 by default (AS 2.2.2) in the preceding and the following equations.

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \gamma k_u d \text{ where, } k_u = 0.36 \qquad \text{(AS 8.1.5)}$$

- If $a \leq D_s$, the subsequent calculations for A_{st} are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{\max}$.

- If $a > D_s$, the calculation for A_{st} has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 11-2. C_f is given by:

$$C_f = \alpha_2 f'_c \left(b_{ef} - b_w\right) \times \min\left(D_s, a_{\max}\right) \qquad \text{(AS 8.1.3(b))}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_{sy}}$ and the portion of M^* that is resisted by the flange is given by:

$$M_{uf} = \varphi C_f \left(d - \frac{\min\left(D_s, a_{\max}\right)}{2}\right)$$

Therefore, the balance of the moment M^* to be carried by the web is:

$$M_{uw} = M^* - M_{uf}$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{\alpha_2 f'_c \phi\, b_w}}$$

- If $a_1 \le a_{max}$, the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi\, f_{sy}\left(d - \dfrac{a_1}{2}\right)}, \text{ and}$$

$$A_{st} = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_{max}$, compression reinforcement is required and is calculated as follows:

The compression force in the web concrete alone is given by:

$$C_w = \alpha_2 f'_c\, b_w a_{max} \qquad \text{(AS 8.1.3)}$$

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{uc} = C_w\left(d - \frac{a_{max}}{2}\right)\phi$$

and the moment resisted by compression and tension reinforcement is:

$$M_{us} = M_{uw} - M_{uc}$$

Therefore, the compression reinforcement is computed as:

$$A_{sc} = \frac{M_{us}}{\left(f'_s - \alpha_2 f'_c\right)\left(d - d'\right)\phi}, \text{ where}$$

$$f'_s = 0.003E_s\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_{sy} \qquad \text{(AS 8.1.2.1, 3.2.2)}$$

The tension reinforcement for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{uc}}{f_{sy}\left[d - \frac{a_{\max}}{2}\right]\phi}$$

and the tension reinforcement for balancing the compression reinforcement is:

$$A_{s3} = \frac{M_{us}}{f_{sy}\left(d - d'\right)\phi}$$

The total tensile reinforcement is $A_{st} = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A_{sc}. A_{st} is to be placed at the bottom and A_{sc} is to be placed at the top.

11.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following limit (AS 8.1.6.1):

$$A_{st.\min} = \alpha_b\left(\frac{D}{d}\right)^2\frac{f'_{ct,f}}{f_{sy}}bd, \text{ where} \qquad \text{(AS 8.1.6.1(2))}$$

$$\alpha_b = 20, \quad \text{for Rectangular Section} \qquad \text{(AS8.1.6.1(2))}$$

for L- and T-Sections with the web in tension:

$$\alpha_b = 0.20 + \left(\frac{b_f}{b_w} - 1\right)\left(0.4\frac{D_s}{D} - 0.18\right) \ge 0.20\left(\frac{b_f}{b_w}\right)^{1/4}, \qquad \text{(AS8.1.6.1(2))}$$

for L- and T-Sections with the flange in tension:

$$\alpha_b = 0.20 + \left(\frac{b_f}{b_w} - 1\right)\left(0.25\frac{D_s}{D} - 0.08\right) \geq 0.20\left(\frac{b_f}{b_w}\right)^{2/3}, \qquad \text{(AS8.1.6.1(2))}$$

$$f'_{ct,f} = 0.6\sqrt{f'_c} \qquad \text{(AS 3.1.1.3(b))}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_{st} \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A_{sc} \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

11.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V^*.
- Determine the shear force, V_{uc}, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

11.5.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

11.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_{uc}, is calculated as:

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_w d_o f'_{cv} \left[\frac{A_{st}}{b_w d_o} \right]^{1/3} \quad \text{(AS 8.2.7.1)}$$

where,

$$f'_{cv} = \left(f'_c \right)^{1/3} \leq 4\text{MPa} \quad \text{(AS 8.2.7.1)}$$

$$\beta_1 = 1.1\left(1.6 - \frac{d_o}{1000}\right) \geq 1.1 \quad \text{(AS 8.2.7.1)}$$

β_2 = 1, or (AS 8.2.7.1)

$= 1 - \left(\frac{N^*}{3.5A_g}\right) \geq 0$ for members subject to significant axial tension, or

$= 1 + \left(\frac{N^*}{14A_g}\right)$ for members subject to significant axial compression.

β_3 = 1

11.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to:

$$V_{u.\min} = V_{uc} + 0.6 b_v d_o \quad \text{(AS 8.2.9)}$$

$$V_{u.\max} = 0.2 f'_c b d_o \quad \text{(AS 8.2.6)}$$

Given V^*, V_{uc}, and $V_{u.\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 by default (AS 2.2.2).

- If $V^* \leq \phi V_{uc}/2$,

$\frac{A_{sv}}{s} = 0$, if $D \leq 750$ mm; otherwise $A_{sv.\min}$ shall be provided. (AS 8.2.5).

- If $\left(\phi V_{uc}/2\right) < V^* \leq \phi V_{u.\min}$,

  $\frac{A_{sv}}{s} = 0$, if $D < b_w/2$ or 250 mm, whichever is greater (AS 8.2.5(c)(i)); otherwise $A_{sv.\min}$ shall be provided.

- If $\phi V_{u.\min} < V^* \leq \phi V_{u.\max}$,

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{\phi f_{sy.f}\, d_o \cot \theta_v}, \qquad \text{(AS 8.2.10)}$$

  and greater than $A_{sv.\min}$, defined as:

$$\frac{A_{sv.\min}}{s} = \left(0.35 \frac{b_w}{f_{sy.f}}\right) \qquad \text{(AS 8.2.8)}$$

  θ_v = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $V^*=\phi V_{u.\min}$ to 45 degrees when $V^*=\phi V_{u,\max}$.

- If $V^* > \phi V_{\max}$, a failure condition is declared. (AS 8.2.6)

- If V^* exceeds its maximum permitted value $\phi V_{\max}$, the concrete section size should be increased (AS 8.2.6).

Note that if torsion design is considered and torsion reinforcement is required, the calculated shear reinforcement is ignored. Closed stirrups are designed for combined shear and torsion according to AS 8.3.4(b).

The maximum of all of the calculated A_{sv}/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup require-

ments to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

11.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T^*.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

11.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T^* is permitted to be reduced in accordance with the code (AS 8.3.2). However, the program does not automatically redistribute the internal forces and reduce T^*. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

11.5.3.2 Determine Special Section Properties

For torsion design, special section properties such as A_t, J_t, and u_t are calculated. These properties are described in the following (AS 8.3).

A_t = Area of a polygon with vertices at the center of longitudinal bars at the corners of the cross-section

u_t = Perimeter of the polygon defined by A_t

J_t = Torsional modulus

In calculating the section properties involving reinforcement, such as A_{sw}/s and A_l, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm-diameter stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{uc} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_t = (b-2c)(h-2c), \quad \text{(AS 8.3.5)}$$

$$u_t = 2(b-2c)+2(h-2c), \quad \text{(AS 8.3.6)}$$

$$J_t = 0.33x^2y \quad \text{(AS 8.3.3)}$$

where, the section dimensions b, h and, c are shown in Figure 11-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_t = (b_w-2c)(h-2c), \quad \text{(AS 8.3.5)}$$

$$u_t = 2(h-2_c)+2(b_w-2_c), \quad \text{(AS 8.3.6)}$$

$$J_t = 0.33\Sigma x^2y \quad \text{(AS 8.3.3)}$$

where the section dimensions b_w, h, and c for a flanged beam are shown in Figure 11-3. The values x and y refer to the smaller and larger dimensions of a component rectangle, respectively.

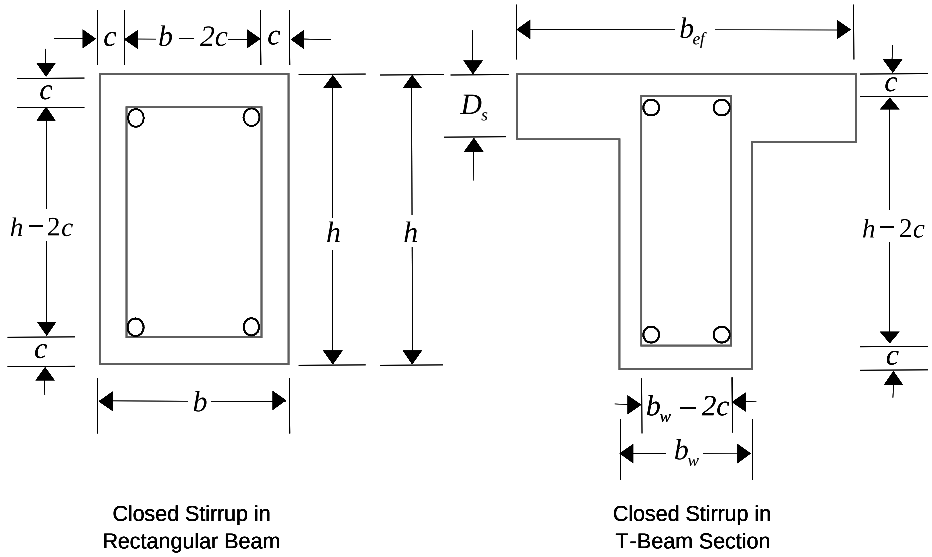


Figure 11-3 Closed stirrup and section dimensions for torsion design

11.5.3.3 Determine Torsion Reinforcement

The torsional strength of the section without torsion reinforcement, T_{uc}, is calculated as:

$$T_{uc} = 0.3 J_t \sqrt{f'_c} \qquad \text{(AS 8.3.5)}$$

where J_t is the torsion modulus of the concrete cross-section as described in detail in the previous section.

Torsion reinforcement also can be ignored if any one of the following is satisfied:

$$T^* \leq 0.25\phi T_{uc} \qquad \text{(AS 8.3.4(a)(i))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 0.5 \qquad \text{(AS 8.3.4(a)(ii))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1 \text{ and } D \leq \max(250\text{mm}, b/2) \qquad \text{(AS 8.3.4(a)(iii))}$$

If the factored torsion T^* alone or in combination with V^* does not satisfy any of the three conditions in the preceding description, torsion reinforcement is needed. It is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (AS 8.3).

- If $T^* > T_{cr}$, the required closed stirrup area per unit spacing, A_{sw}/s, is calculated as:

$$\frac{A_{sw}}{s} = \frac{T^* \tan\theta_t}{\phi 2 f_{sy.f} A_t} \qquad \text{(AS 8.3.5(b))}$$

where, the minimum value of A_{sw}/s is taken as follows:

$$\frac{A_{sw.\min}}{s} = \frac{0.35 b_w}{f_{sy.f}} \qquad \text{(AS 8.2.8)}$$

The value θ_t is the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $T^* = \phi T_{uc}$ to 45 degrees when $T^* = \phi T_{u,\max}$.

The following equation shall also be satisfied for combined shear and torsion by adding additional shear stirrups.

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} \le 1.0 \qquad \text{(AS 8.3.4(b))}$$

where,

$$T_{us} = f_{sy.f}\left(\frac{A_{sw}}{s}\right) 2 A_t \cot\theta_t \qquad \text{(AS 8.3.5(b))}$$

$$V_{us} = \left(A_{sv} f_{sy.f} d_o / s\right)\cot\theta_v \qquad \text{(AS 8.2.10(a))}$$

The required longitudinal rebar area is calculated as:

$$A_l = \frac{0.5 f_{sy.f}\left(\frac{A_{sw}}{s}\right) u_t \cot^2\theta_t}{f_{sy}} \qquad \text{(AS 8.3.6(a))}$$

An upper limit of the combination of V^* and T^* that can be carried by the section is also checked using the equation:

$$\frac{T^*}{\phi T_{u.\max}} + \frac{V^*}{\phi V_{u.\max}} \leq 1.0 \qquad \text{(AS 8.3.3)}$$

where,

$$V_{u.\max} = 0.2 f'_c\, b_w d_o \qquad \text{(AS 8.2.6)}$$

$$T_{u.\max} = 0.2 f'_c\, J_t \qquad \text{(AS 8.3.3)}$$

For rectangular sections, b_w is replaced with b. If the combination of V^* and T^* exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T^* > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups satisfy the following limit.

$$\left(\frac{A_{sv}}{s} + 2\frac{A_{sw}}{s}\right) \geq \frac{0.35b}{f_{sy.f}} \qquad \text{(AS 8.3.7, 8.2.8)}$$

If this equation is not satisfied with the originally calculated A_{sv}/s and A_{sw}/s, A_{sv}/s is increased to satisfy this condition. In that case, A_{sv}/s does not need to satisfy AS Section 8.2.8 independently.

The maximum of all the calculated A_l and A_{sw}/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

11.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (AS 3600-2001) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

11.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments and axial forces for each slab strip.

- Design flexural reinforcement for the strip.

These two steps, which are described in the following subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

11.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

11.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

11.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (AS 9.1.1):

$$A_s = 0.24\left(\frac{h}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy,f}} bh \text{ for flat slabs} \qquad \text{(AS 9.1.1(a))}$$

$$A_s = 0.19\left(\frac{h}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy,f}} bh$$

for slabs supported by beams/walls and slab footings. (AS 9.1.1(b))

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

11.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

11.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d_{om}/2$ from the face of the support (AS 9.2.1.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (AS 9.2.1.3). Figure 11-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

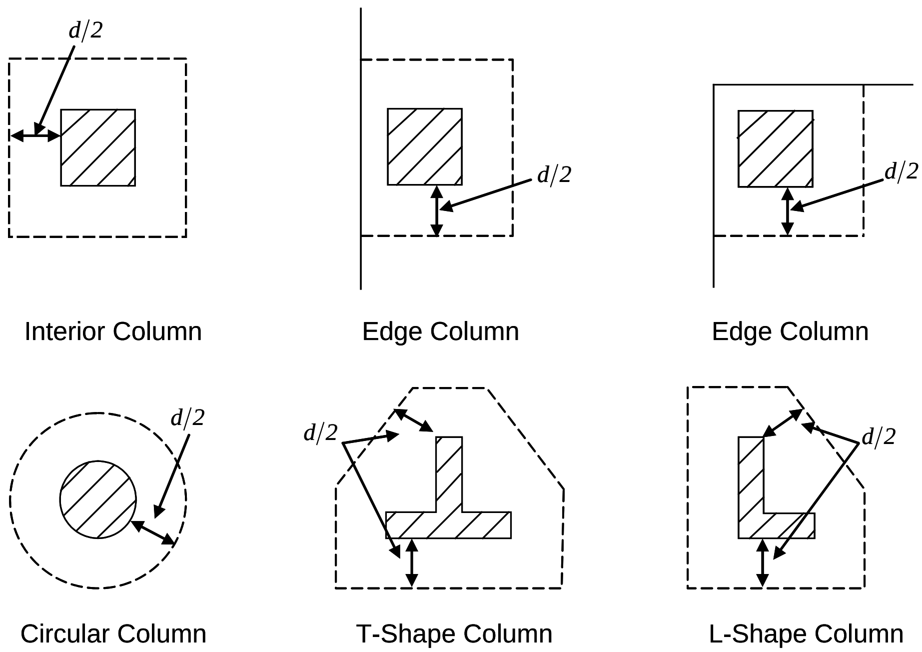


Figure 11-4 Punching Shear Perimeters

11.6.2.2 Determine Concrete Capacity

The shear capacity, f_{cv}, is calculated based on the minimum of the two expressions from AS 9.2.3, as shown, with the d_{om} and u terms removed to convert force to stress.

$$f_{cv} = \min \begin{cases} 0.17\left(1+\dfrac{2}{\beta_h}\right)\sqrt{f'_c} \\ 0.34\sqrt{f'_c} \end{cases} \qquad \text{(AS 9.2.3(a))}$$

where, β_h is the ratio of the longest dimension to the shortest dimension of the critical section.

11.6.2.3 Determine Maximum Shear Stress

The maximum design shear stress is computed along the major and minor axis of column separately using the following equation:

$$v_{\max} = \frac{V^*}{ud_{om}}\left[1.0 + \frac{uM_v}{8V^* ad_{om}}\right] \qquad \text{(AS 9.2.4(a))}$$

11.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

11.6.3 Design Punching Shear Reinforcement

The design guidelines for shear links or shear studs are not available in AS 3600-2009. SAFE uses the NZS 3101-06 guidelines to design shear studs or shear links.

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

11.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

11.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{max} = 3\ V_{u,min} = 3^{*}V_u \qquad \text{(AS 9.2.4(a), (d))}$$

where V_u is computed from AS 9.2.3 or 9.2.4. Given V^*, V_u, and $V_{u,max}$, the required shear reinforcement is calculated as follows, where, ϕ is the strength reduction factor.

$$\frac{A_{sv}}{s} = \frac{\left(V^{*} - \phi V_u\right)}{f_{sy} d_{om}}, \qquad \text{(AS 8.2.10)}$$

Minimum punching shear reinforcement should be provided such that:

$$V_s \geq \frac{1}{16}\sqrt{f'_c}\, u d_{om} \qquad \text{(NZS 12.7.4.3)}$$

- If $V^* > \phi V_{max}$, a failure condition is declared. (NZS 12.7.3.4)
- If V^* exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size.

11.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 11-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (NZS 12.7.4.4).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

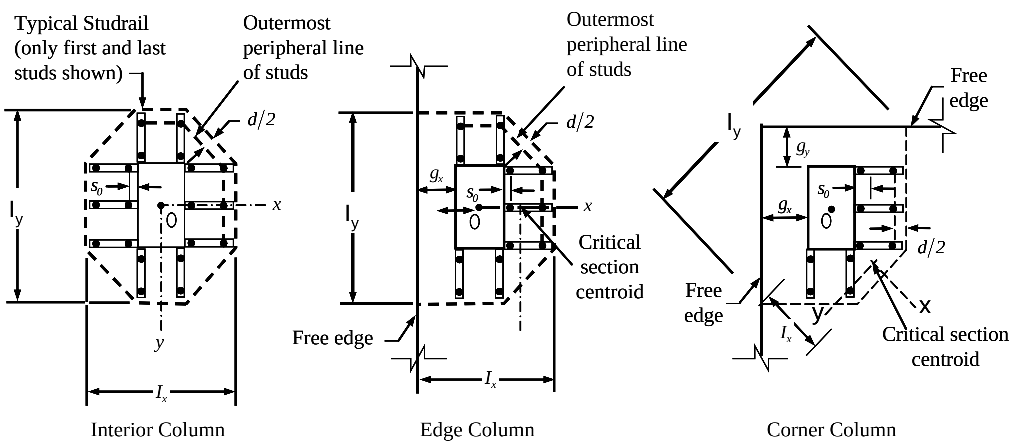


Figure 11-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

11.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus one-half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed $2d$ and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \quad \text{(NZS 12.7.4.4)}$$

$$s \leq 0.5d \quad \text{(NZS 12.7.4.4)}$$

$$g \leq 2d \quad \text{(NZS 12.7.4.4)}$$

Chapter 12
Design for ACI 318-11

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code ACI 318-11 [ACI 2011] is selected. Various notations used in this chapter are listed in Table 12-1. For referencing to the pertinent sections or equations of the ACI code in this chapter, a prefix “ACI” followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

12.1 Notations

Table 12-1 List of Symbols Used in the ACI 318-11 Code

A_{cp}	Area enclosed by the outside perimeter of the section, sq-in
A_g	Gross area of concrete, sq-in

Table 12-1 List of Symbols Used in the ACI 318-11 Code

A_l	Area of longitudinal reinforcement for torsion, sq-in
A_o	Area enclosed by the shear flow path, sq-in
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in
A_s	Area of tension reinforcement, sq-in
A'_s	Area of compression reinforcement, sq-in
A_t /s	Area of closed shear reinforcement per unit length of member for torsion, sq-in/in
A_v	Area of shear reinforcement, sq-in
A_v /s	Area of shear reinforcement per unit length, sq-in/in
a	Depth of compression block, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of section, in
b_f	Effective width of flange (flanged section), in
b_o	Perimeter of the punching shear critical section, in
b_w	Width of web (flanged section), in
b_1	Width of the punching shear critical section in the direction of bending, in
b_2	Width of the punching shear critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in
d	Distance from compression face to tension reinforcement, in
d'	Distance from compression face to compression reinforcement, in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, psi
f'_c	Specified compressive strength of concrete, psi

Table 12-1 List of Symbols Used in the ACI 318-11 Code

f'_s	Stress in the compression reinforcement, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{yt}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
h_f	Height of the flange, in
M_u	Factored moment at a section, lb-in
N_u	Factored axial load at a section occurring simultaneously with V_u or T_u, lb
P_u	Factored axial load at a section, lb
p_{cp}	Outside perimeter of concrete cross-section, in
p_h	Perimeter of centerline of outermost closed transverse torsional reinforcement, in
s	Spacing of shear reinforcement along the beam, in
T_{cr}	Critical torsion capacity, lb-in
T_u	Factored torsional moment at a section, lb-in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_s	Shear force resisted by transverse reinforcement, lb
V_u	Factored shear force at a section, lb
α_s	Punching shear scale factor based on column location
β_c	Ratio of the maximum to the minimum dimensions of the punching shear critical section
β_1	Factor for obtaining depth of the concrete compression block
ε_c	Strain in the concrete
$\varepsilon_{c\,\max}$	Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)

Table 12-1 List of Symbols Used in the ACI 318-11 Code

ε_s	Strain in the reinforcement
$\varepsilon_{s,min}$	Minimum tensile strain allowed in the reinforcement at nominal strength for tension controlled behavior (0.005 in/in)
ϕ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
λ	Shear strength reduction factor for light-weight concrete
θ	Angle of compression diagonals, degrees

12.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-11, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (ACI 9.2.1):

1.4D	(ACI 9-1)
1.2D + 1.6L	(ACI 9-2)
1.2D + 1.6 (0.75 PL)	(ACI 13.7.6.3, 9-2)
0.9D ± 1.0W	(ACI 9-6)
1.2D + 1.0L ± 1.0W	(ACI 9-4)
0.9D ± 1.0E	(ACI 9-7)
1.2D + 1.0L ± 1.0E	(ACI 9-5)
1.2D + 1.6L + 0.5S	(ACI 9-2)
1.2D + 1.0L + 1.6S	(ACI 9-3)
1.2D + 1.6S ± 0.5W	(ACI 9-3)
1.2D + 1.0L + 0.5S ± 1.0W	(ACI 9-4)
1.2D + 1.0L + 0.2S ± 1.0E	(ACI 9-5)

These are the default design load combinations in SAFE whenever the ACI 318-11 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

12.3 Limits on Material Strength

The concrete compressive strength, f'_c, should not be less than 2,500 psi (ACI 5.1.1). The upper limit of the reinforcement yield strength, f_y, is taken as 80 ksi (ACI 9.4) and the upper limit of the reinforcement shear strength, f_{yt}, is taken as 60 ksi (ACI 11.4.2).

If the input f'_c is less than 2,500 psi, SAFE continues to design the members based on the input f'_c and does not warn the user about the violation of the code. The user is responsible for ensuring that the minimum strength is satisfied.

12.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied to the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

ϕ = 0.90 for flexure (tension controlled) (ACI 9.3.2.1)

ϕ = 0.75 for shear and torsion (ACI 9.3.2.3)

These values can be overwritten; however, caution is advised.

12.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

12.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

12.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

12.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, f_y, is limited to 80 ksi (ACI 9.4), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 12-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 10.3.4) when the concrete in compression reaches its assumed strain limit of 0.003. When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed $\left(0.1 f'_c A_g\right)$ (ACI 10.3.5), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $\left(0.1 f'_c A_g\right)$ and axial tensions are always included in flexural and shear design.

12.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M_u (i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 12-1), where,

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f'_c\, \phi b}} \qquad \text{(ACI 10.2)}$$

and the value of ϕ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} d \qquad \text{(ACI 10.2.2)}$$

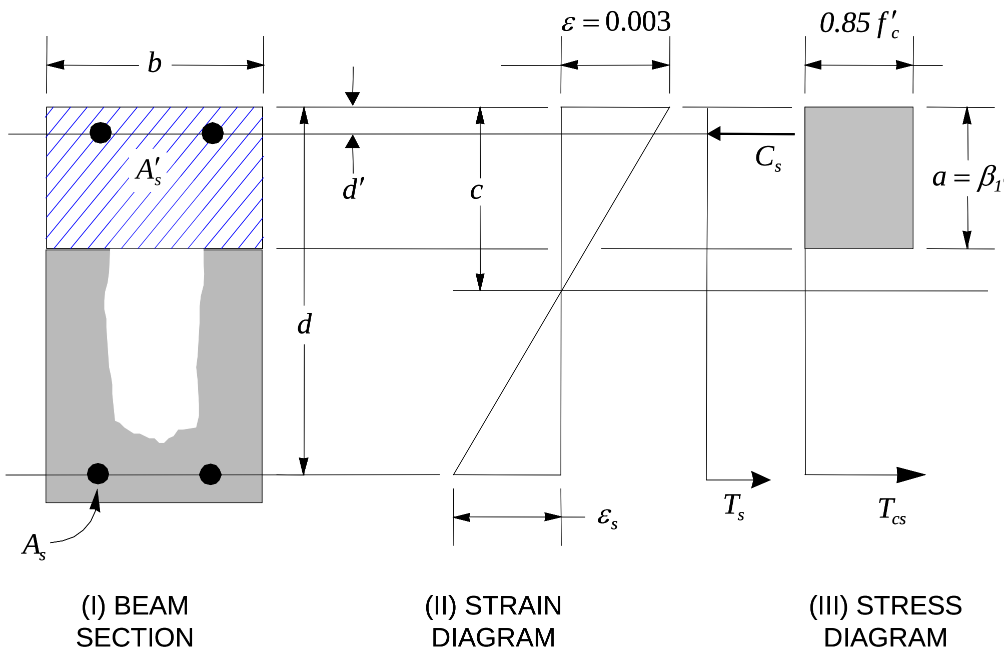


Figure 12-1 Rectangular Beam Design

where,

$\varepsilon_{c\max} = 0.003$ (ACI 10.2.3)

$\varepsilon_{s\min} = 0.005$ (ACI 10.3.4)

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$a_{\max} = \beta_1 c_{\max}$ (ACI 10.2.7.1)

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le a_{max}$ (ACI 10.3.4), the area of tension reinforcement is then given by:

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2} \right)}$$

This reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

- If $a > a_{max}$, compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

− The compressive force developed in the concrete alone is given by:

$$C = 0.85 f'_c \, b a_{max} \qquad \text{(ACI 10.2.7.1)}$$

and the moment resisted by concrete compression and tension reinforcement is:

$$M_{uc} = \phi C \left(d - \frac{a_{max}}{2} \right) \qquad \text{(ACI 9.3.2.1)}$$

− Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - M_{uc}$$

− The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\phi \left(f'_s - 0.85 f'_c \right) \left(d - d' \right)}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\,max} \left[\frac{c_{max} - d'}{c_{max}} \right] \le f_y \qquad \text{(ACI 10.2.2, 10.2.3, 10.2.4)}$$

− The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_{uc}}{\phi f_y \left[d - \frac{a_{\max}}{2} \right]}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{us}}{\phi f_y \left(d - d' \right)}$$

Therefore, the total tension reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

12.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 12-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 12-1 is assumed on the compression side.

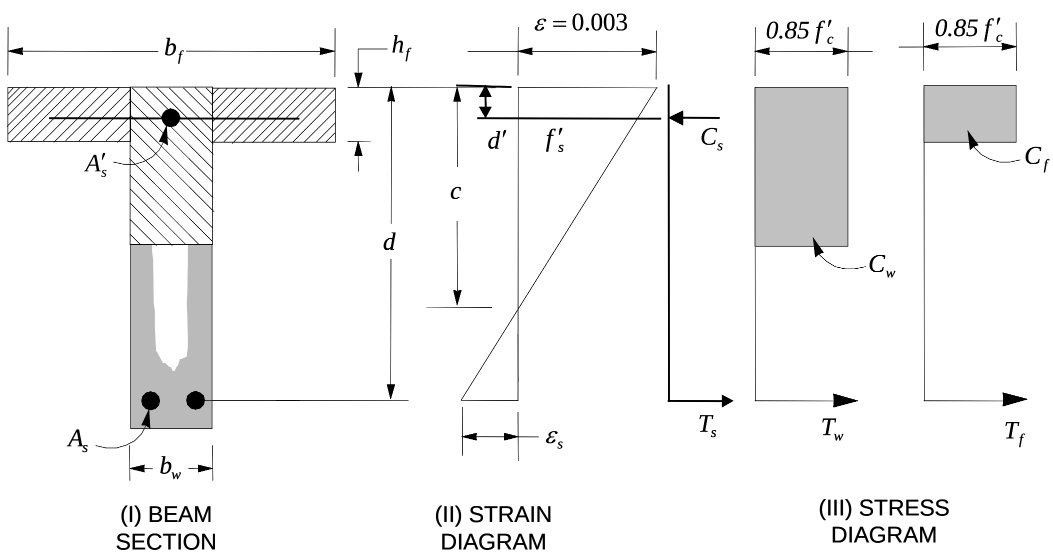


Figure 12-2 T-Beam Design

12.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

12.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2 M_u}{0.85 f'_c \phi b_f}} \qquad \text{(ACI 10.2)}$$

where, the value of ϕ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{max} = \frac{\varepsilon_{c\,max}}{\varepsilon_{c\,max} + \varepsilon_{s\,min}} d \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_{max}$.

- If $a > h_f$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 12-2. C_f is given by:

$$C_f = 0.85 f'_c \left(b_f - b_w\right) \min\left(h_f, a_{\max}\right) \qquad \text{(ACI 10.2.7.1)}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of M_u that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min\left(h_f, a_{\max}\right)}{2}\right) \qquad \text{(ACI 9.3.2.1)}$$

Again, the value for ϕ is 0.90 by default. Therefore, the balance of the moment, M_u , to be carried by the web is:

$$M_{uw} = M_u - M_{uf}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi b_w}} \qquad \text{(ACI 10.2)}$$

- If $a_1 \le a_{\max}$ (ACI 10.3.4), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi f_y \left(d - \frac{a_1}{2}\right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_{\max}$, compression reinforcement is required (ACI 10.3.5.1) and is calculated as follows:

  − The compressive force in the web concrete alone is given by:

  $$C_w = 0.85 f'_c b_w a_{\max} \qquad \text{(ACI 10.2.7.1)}$$

  Therefore the moment resisted by the concrete web and tension reinforcement is:

  $$M_{uc} = C_w \left(d - \frac{a_{\max}}{2} \right) \phi$$

  and the moment resisted by compression and tension reinforcement is:

  $$M_{us} = M_{uw} - M_{uc}$$

  Therefore, the compression reinforcement is computed as:

  $$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c \right)\left(d - d' \right) \phi}, \text{ where}$$

  $$f'_s = E_s \varepsilon_{c\max} \left[\frac{c_{\max} - d'}{c_{\max}} \right] \le f_y \qquad \text{(ACI 10.2.2, 10.2.3, 10.2.4)}$$

  The tension reinforcement for balancing compression in the web concrete is:

  $$A_{s2} = \frac{M_{uc}}{f_y \left[d - \frac{a_{\max}}{2} \right] \phi}$$

  and the tension reinforcement for balancing the compression reinforcement is:

  $$A_{s3} = \frac{M_{us}}{f_y \left(d - d' \right) \phi}$$

The total tension reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

12.5.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the two following limits:

$$A_{s,\min} = \max\left(\frac{3\sqrt{f'_c}}{f_y} b_w d, \frac{200}{f_y} b_w d\right) \qquad \text{(ACI 10.5.1)}$$

$$A_s \geq \frac{4}{3} A_{s(\text{required})} \qquad \text{(ACI 10.5.3)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

12.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

12.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

12.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = 2\lambda\sqrt{f'_c}\,b_w d \qquad \text{(ACI 11.2.1.2)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as $f'_c \le 100$ (ACI 11.1.2)

The value of λ should be specified in the material property definition.

12.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = V_c + \left(8\sqrt{f'_c}\right)b_w d \qquad \text{(ACI 11.4.7.9)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3). The flexural reinforcement strength, f_{yt}, is limited to 60 ksi (ACI 11.5.2) even if the material property is defined with a higher value.

- If $V_u \le 0.5\phi V_c$,

$$\frac{A_v}{s} = 0 \qquad \text{(ACI 11.4.6.1)}$$

- If $0.5\phi V_c < V_u \le \phi V_{\max}$,

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi\, f_{yt} d} \qquad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared. (ACI 11.4.7.9)

If V_u exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size (ACI 11.4.7.9).

Note that if torsion design is considered and torsion reinforcement is required, the equation given in ACI 11.4.6.3 does not need to be satisfied independently. See the subsequent section *Design of Beam Torsion Reinforcement* for details.

If the beam depth *h* is

$h \leq 10"$ for rectangular,

$h \leq \min\left\{24", \max\left(2.5h_f, \frac{b}{2}\right)\right.$ for T-beam,

the minimum shear reinforcement given by ACI 11.4.6.3 is not enforced (ACI 11.4.6.1).

$$\frac{A_v}{s} \geq \max\left(\frac{0.75\sqrt{f'_c}}{f_{yt}}b_w, \quad \frac{50b_w}{f_{yt}}\right) \qquad \text{(ACI 11.4.6.3)}$$

The maximum of all of the calculated A_v/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

12.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_u.
- Determine special section properties.

- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

12.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 11.6.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with the code (ACI 11.5.2.2). However, the program does not automatically redistribute the internal forces and reduce T_u. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

12.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{cp}, A_{oh}, A_o, p_{cp}, and p_h, are calculated. These properties are described in the following (ACI 2.1).

A_{cp} = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_{cp} = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to a 1.5 inch clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{cp} = bh \qquad \text{(ACI 11.5.1, 2.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1, R11.5.3.6(b))}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(ACI 11.5.3.6, 2.1)}$$

$$p_{cp} = 2b + 2h \qquad \text{(ACI 11.5.1, 2.1)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1)}$$

where, the section dimensions b, h, and c are shown in Figure 12-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_{cp} = b_w h + (b_f - b_w)h_f \qquad \text{(ACI 11.5.1, 2.1)}$$

$$A_{oh} = (b_w - 2c)(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1, R11.5.3.6(b))}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(ACI 11.5.3.6, 2.1)}$$

$$p_{cp} = 2b_f + 2h \qquad \text{(ACI11.5.1, 2.1)}$$

$$p_h = 2(h - 2c) + 2(b_w - 2c) \qquad \text{(ACI 11.5.3.1, 2.1)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 12-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h - h_f)$ (ACI 13.2.4).

12.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \phi 4\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{P_u}{4A_{cp}\lambda\sqrt{f'_c}}} \qquad \text{(ACI 11.5.2.2(c))}$$

where A_{cp} and p_{cp} are the area and perimeter of the concrete cross-section as described in the previous section, P_u is the factored axial force (compression positive), ϕ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 9.3.2.3), and f'_c is the specified concrete compressive strength.

12.5.3.4 Determine Torsion Reinforcement

If the factored torsion T_u is less than the threshold limit, T_{cr}, torsion can be safely ignored (ACI 11.5.1). In that case, the program reports that no torsion reinforcement is required. However, if T_u exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI R11.5.3.6). Note that the longitudinal reinforcement strength, f_y, is limited to 80 ksi (ACI 9.4) and the transverse reinforcement strength, f_{yt}, is limited to 60 ksi, even if the material property is defined with a higher value.

If $T_u > T_{cr}$, the required closed stirrup area per unit spacing, A_t /s, is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan\theta}{\phi 2 A_o f_{yt}} \qquad \text{(ACI 11.5.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{T_u p_h}{\phi 2 A_o f_y \tan\theta} \qquad \text{(ACI 11.5.3.7, 11.5.3.6)}$$

where, the minimum value of A_t /s is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{yt}} b_w \qquad \text{(ACI 11.5.5.3)}$$

and the minimum value of A_l is taken as:

$$A_l \geq \frac{5\sqrt{f'_c}\,A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{ys}}{f_y}\right) \qquad \text{(ACI 11.5.5.3)}$$

In the preceding expressions, θ is taken as 45 degrees. The code allows any value between 30 and 60 degrees (ACI 11.5.3.6).

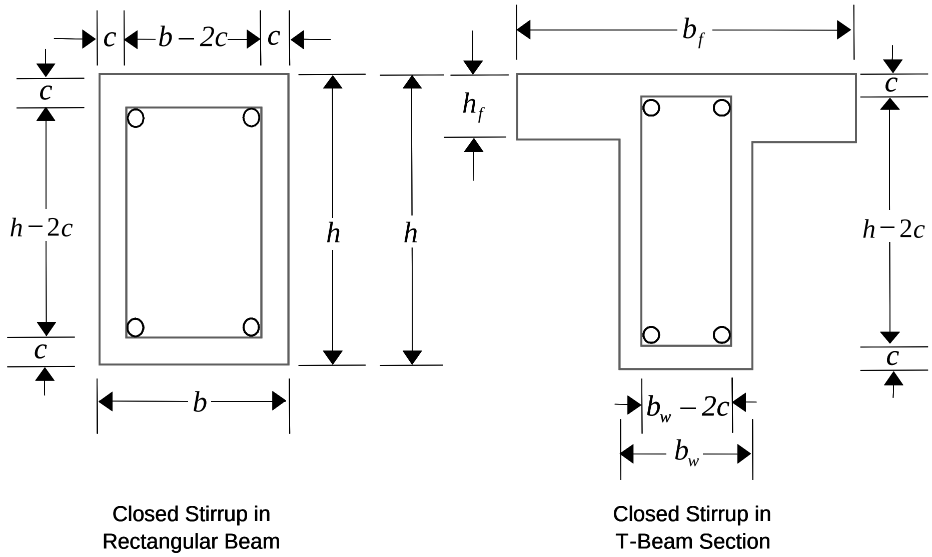


Figure 12-3 Closed stirrup and section dimensions for torsion design

An upper limit of the combination of V_u and T_u that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad \text{(ACI 11.5.3.1, 11.4.7.9)}$$

For rectangular sections, b_w is replaced with b. If the combination of V_u and T_u exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s} + 2\frac{A_t}{s}\right) \ge \max\left\{0.75\frac{\sqrt{f_c'}}{f_{ys}}b_w, \frac{50}{f_y}b_w\right\} \qquad \text{(ACI 11.5.5.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition. In that case, A_v/s does not need to satisfy the ACI Section 11.4.6.3 independently.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

12.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (ACI 318-11) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

12.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.

- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

12.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

12.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

12.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2, 13.3.1):

$$A_{s,\min} = 0.0020\, bh \text{ for } f_y = 40 \text{ ksi or } 50 \text{ ksi} \qquad \text{(ACI 7.12.2.1(a))}$$

$$A_{s,\min} = 0.0018\, bh \text{ for } f_y = 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(b))}$$

$$A_{s,\min} = \frac{0.0018 \times 60000}{f_y} bh \text{ for } f_y > 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(c))}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

12.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following sections.

12.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (ACI 11.11.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.11.1.3). Figure 12-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

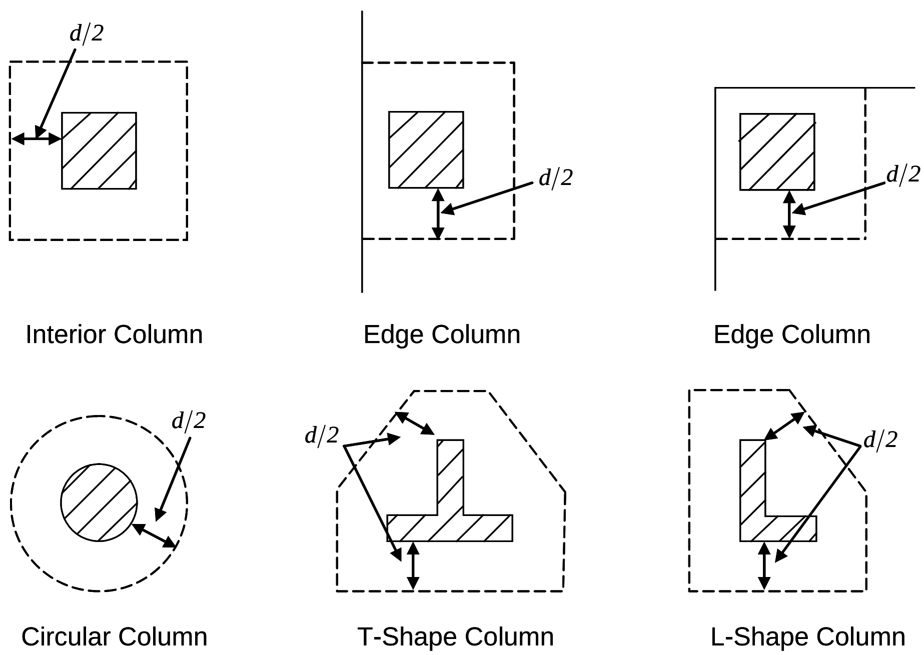


Figure 12-4 Punching Shear Perimeters

12.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$.

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(ACI 13.5.3.2)}$$

$$\gamma_v = 1 - \gamma_f \qquad \text{(ACI 13.5.3.1)}$$

For flat plates, γ_v is determined from the following equations taken from ACI 421.2R-07 *Seismic Design of Punching Shear Reinforcement in Flat Plates* [ACI 2007].

For interior columns,

$$\gamma_{vx} = 1 - \frac{1}{1+(2/3)\sqrt{l_y/l_x}} \qquad \text{(ACI 421.2 C-11)}$$

$$\gamma_{vy} = 1 - \frac{1}{1+(2/3)\sqrt{l_x/l_y}} \qquad \text{(ACI 421.2 C-12)}$$

For edge columns,

γ_{vx} = same as for interior columns (ACI 421.2 C-13)

$$\gamma_{vy} = 1 - \frac{1}{1+(2/3)\sqrt{l_x/l_y - 0.2}} \qquad \text{(ACI 421.2 C-14)}$$

γ_{vy} = 0 when $l_x/l_y \le 0.2$

For corner columns,

γ_{vx} = 0.4 (ACI 421.2 C-15)

γ_{vy} = same as for edge columns (ACI 421.2 C-16)

NOTE: Program uses ACI 421.2-12 and ACI 421.2-15 equations in lieu of ACI 421.2 C-14 and ACI 421.2 C-16 which are currently NOT enforced.

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span. The values l_x and l_y are the projections of the shear-critical section onto its principal axes, x and y, respectively.

12.6.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \left(2 + \dfrac{4}{\beta_c}\right)\lambda\sqrt{f'_c} \\ \left(2 + \dfrac{\alpha_s d}{b_o}\right)\lambda\sqrt{f'_c} \\ 4\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

where, β_c is the ratio of the maximum to the minimum dimensions of the critical section, b_o is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as:

$$\sqrt{f'_c} \le 100 \qquad \text{(ACI 11.1.2)}$$

12.6.2.4 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

$$v_U = \frac{V_U}{b_0 d} + \frac{\gamma_{V2}[M_{U2} - V_U(y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^{\ 2}} - \frac{\gamma_{V3}[M_{U3} - V_U(x_3 - x_1)][I_{22}(x_4 - x_3) - I_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}^{\ 2}} \qquad \text{Eq. 1}$$

$I_{22} = \sum_{sides=1}^{n} \bar{I}_{22}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 2

$I_{33} = \sum_{sides=1}^{n} \bar{I}_{33}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 3

$I_{23} = \sum_{sides=1}^{n} \bar{I}_{23}$, where "*sides*" refers to the sides of the critical section for punching shear Eq. 4

The equations for $\bar{I}_{22}, \bar{I}_{33}$, and $\bar{I}_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figure 12-5.

$\bar{I}_{22} = Ld(y_2 - y_3)^2$, for the side of the critical section parallel to the 2-axis Eq. 5a

$\bar{I}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2$, for the side of the critical section parallel to the 3-axis Eq. 5b

$\bar{I}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2$, for the side of the critical section parallel to the 2-axis Eq. 6a

$\bar{I}_{33} = Ld(x_2 - x_3)^2$, for the side of the critical section parallel to the 3-axis Eq. 6b

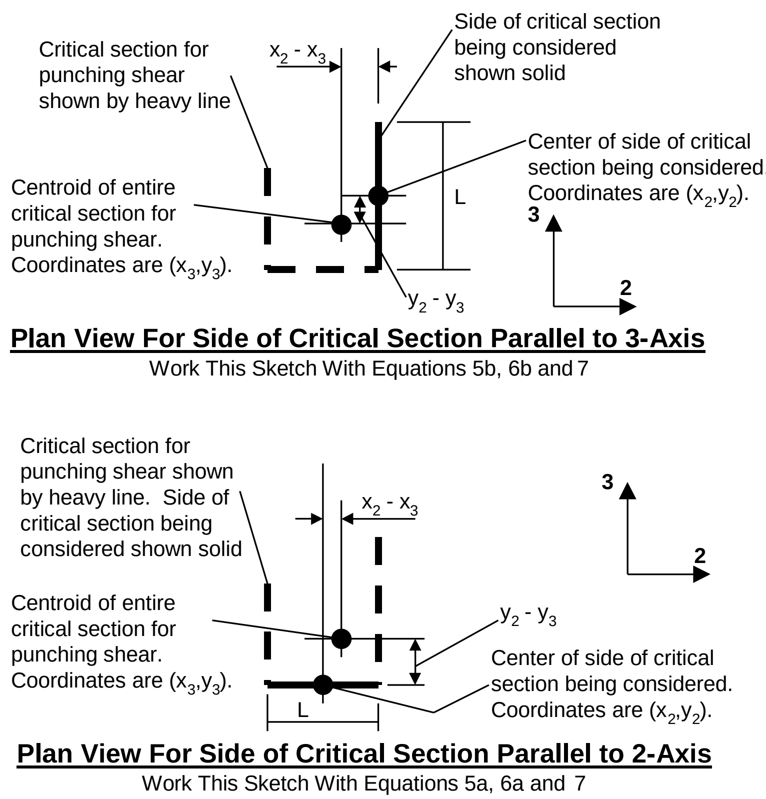


Figure 12-5 Shear Stress Calculations at Critical Sections

$$\bar{I}_{23} = Ld(x_2 - x_3)(y_2 - y_3),$$ for side of critical section parallel to 2-axis or 3-axis Eq. 7

NOTE: $\bar{I}_{23}$ is explicitly set to zero for corner condition.

where,

b_0 = Perimeter of the critical section for punching shear

d = Effective depth at the critical section for punching shear based on the average of d for 2 direction and d for 3 direction

I_{22} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis

I_{33} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis

I_{23} = Product of the inertia of the critical section for punching shear with respect to the 2 and 3 planes

L = Length of the side of the critical section for punching shear currently being considered

M_{U2} = Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)

M_{U3} = Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)

v_U = Punching shear stress

V_U = Shear at the center of the column (positive upward)

x_1, y_1= Coordinates of the column centroid

x_2, y_2= Coordinates of the center of one side of the critical section for punching shear

x_3, y_3= Coordinates of the centroid of the critical section for punching shear

x_4, y_4= Coordinates of the location where stress is being calculated

γ_{V2} = Percent of M_{U2} resisted by shear

γ_{V3} = Percent of M_{U3} resisted by shear

12.6.2.5 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

12.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 11.11.3). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

12.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

$$v_c \leq 2\lambda\sqrt{f'_c} \text{ for shear links} \qquad \text{(ACI 11.11.3.1)}$$

$$v_c \leq 3\lambda\sqrt{f'_c} \text{ for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

12.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = 6\sqrt{f'_c}\, b_o d \text{ for shear links} \qquad \text{(ACI 11.11.3.2)}$$

$$V_{\max} = 8\sqrt{f'_c}\, b_o d \text{ for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi f_{ys} d} \qquad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

$$\frac{A_v}{s} \geq \frac{2\sqrt{f'_c}b_o}{f_y} \quad \text{for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared. (ACI 11.11.3.2)
- If V_u exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size.

12.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 12-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

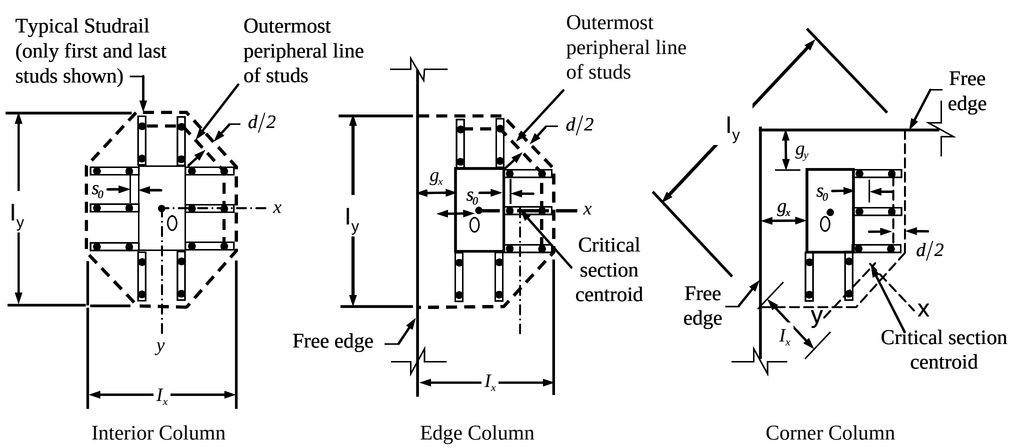


Figure 12-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$(ACI R11.11.3.3, 11.11.5.2. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 2d measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

12.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 2d, and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.5.2)}$$

$$s \leq \begin{cases} 0.75d & \text{for} \quad v_u \leq 6\phi\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.5.2)}$$

$$g \leq 2d \qquad \text{(ACI 11.11.5.3)}$$

The limits of s_o and the spacing, s, between for the links are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.3)}$$

$$s \leq 0.50d \qquad \text{(ACI 11.11.3)}$$

Chapter 13
Design for TS 500-2000

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code TS 500-2000 [TS 500] is selected. Various notations used in this chapter are listed in Table 13-1. For referencing to the pertinent sections or equations of the TS code in this chapter, a prefix "TS" followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

13.1 Notations

Table 13-1 List of Symbols Used in the TS 500-2000 Code

A_{cp}	Area enclosed by the outside perimeter of the section, mm^2
A_g	Gross area of concrete, mm^2

Table 13-1 List of Symbols Used in the TS 500-2000 Code

A_l	Area of longitudinal reinforcement for torsion, mm^2
A_o	Area enclosed by the shear flow path, mm^2
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, mm^2
A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{ot}/s	Area of transverse torsion reinforcement (closed stirrups) per unit length of the member, mm^2/mm
A_{ov}/s	Area of transverse shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
A_{sw}	Area of shear reinforcement, mm^2
A_{sw}/s	Area of shear reinforcement per unit length of the member, mm^2/mm
$a_{\max}$	Maximum allowed depth of compression block, mm
b	Width of section, mm
b_f	Effective width of flange (flanged section), mm
b_o	Perimeter of the punching shear critical section, mm
b_w	Width of web (flanged section), mm
b_1	Width of the punching shear critical section in the direction of bending, mm
b_2	Width of the punching shear critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
d	Distance from compression face to tension reinforcement, mm
d'	Distance from compression face to compression reinforcement, in
E_c	Modulus of elasticity of concrete, N/mm^2

Table 13-1 List of Symbols Used in the TS 500-2000 Code

E_s	Modulus of elasticity of reinforcement, N/mm^2
f_{cd}	Designed compressive strength of concrete, N/mm^2
f_{ck}	Characteristic compressive strength of concrete, N/mm^2
f_{ctk}	Characteristic tensile strength of concrete, N/mm^2
f_{yd}	Designed yield stress of flexural reinforcement, N/mm^2.
f_{yk}	Characteristic yield stress of flexural reinforcement, N/mm^2.
f_{ywd}	Designed yield stress of transverse reinforcement, N/mm^2.
h	Overall depth of a section, mm
h_f	Height of the flange, mm
M_d	Design moment at a section, N/mm
N_d	Design axial load at a section, N
p_{cp}	Outside perimeter of concrete cross-section, mm
p_h	Perimeter of centerline of outermost closed transverse torsional reinforcement, mm
s	Spacing of shear reinforcement along the beam, mm
T_{cr}	Critical torsion capacity, N/mm
T_d	Design torsional moment at a section, N/mm
V_c	Shear force resisted by concrete, N
$V_{\max}$	Maximum permitted total factored shear force at a section, N
V_s	Shear force resisted by transverse reinforcement, N
V_d	Design shear force at a section, N
α_s	Punching shear scale factor based on column location
β_c	Ratio of the maximum to the minimum dimensions of the punching shear critical section
k_1	Factor for obtaining depth of the concrete compression block

Table 13-1 List of Symbols Used in the TS 500-2000 Code

ε_c	Strain in the concrete
$\varepsilon_{c\ max}$	Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 mm / mm)
ε_s	Strain in the reinforcement
$\varepsilon_{cu,}$	Maximum usable compression strain allowed in extreme concrete fiber (0.003 mm/mm)
ε_s	Strain in reinforcing steel
γ_m	Material factor
γ_{mc}	Material factor for concrete
λ	Shear strength reduction factor for light-weight concrete

13.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For TS 500-2000, if a structure is subjected to dead (G), live (Q), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (TS 6.2.6):

$$1.4G + 1.6Q \qquad \text{(TS Eqn. 6.3)}$$

$$0.9G \pm 1.3W \qquad \text{(TS Eqn. 6.6)}$$
$$1.0G + 1.3Q \pm 1.3W \qquad \text{(TS Eqn. 6.5)}$$

$$0.9G \pm 1.0E \qquad \text{(TS Eqn. 6.8)}$$
$$1.0G + 1.0Q \pm 1.0E \qquad \text{(TS Eqn. 6.7)}$$

These are the default design load combinations in SAFE whenever the TS 500-2000 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

13.3 Limits on Material Strength

The characteristic compressive strength of concrete, f_{ck}, should not be less than 16 N/mm^2. The upper limit of the reinforcement yield stress, f_y, is taken as 420 N/mm^2 and the upper limit of the reinforcement shear strength, f_{yk} is taken as 420 N/mm^2.

The program enforces the upper material strength limits for flexure and shear design of beams and columns or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

13.4 Design Strength

The design strength for concrete and steel is obtained by dividing the characteristic strength of the material by a partial factor of safety, γ_{mc} and γ_{ms}. The values used in the program are as follows:

Partial safety factor for steel, γ_{ms} = 1.15, and (TS 6.2.5)

Partial safety factor for concrete, γ_{mc} = 1.5. (TS 6.2.5)

These factors are already incorporated in the design equations and tables in the code. Although not recommended, the program allows them to be overwritten. If they are overwritten, the program uses them consistently by modifying the code-mandated equations in every relevant place.

13.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

13.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

13.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

13.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 13-1 (TS 7.1). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by the program for both rectangular and flanged sections (T beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed $\left(0.1 f_{ck} A_g\right)$ (TS 7.3, Eqn. 7.2); hence, all of the beams are designed ignoring axial force.

13.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M_d (i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 13-1), where,

$$a = d - \sqrt{d^2 - \frac{2\left|M_d\right|}{0.85 f_{cd}\, b}}, \qquad \text{(TS 7.1)}$$

The maximum depth of the compression zone, c_b, is calculated based on the compressive strength of the concrete and the tensile steel tension using the following equation (TS 7.1):

$$c_b = \frac{\varepsilon_{cu} E_s}{\varepsilon_{cu} E_s + f_{yd}} d \qquad \text{(TS 7.1)}$$

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by

$$a_{\max} = 0.85 k_1 c_b \qquad \text{(TS 7.11, 7.3, Eqn. 7.4)}$$

where k_1 is calculated as follows:

$$k_1 = 0.85 - 0.006\left(f_{ck} - 25\right), \quad 0.70 \le k_1 \le 0.85. \qquad \text{(TS 7.1, Table 7.1)}$$

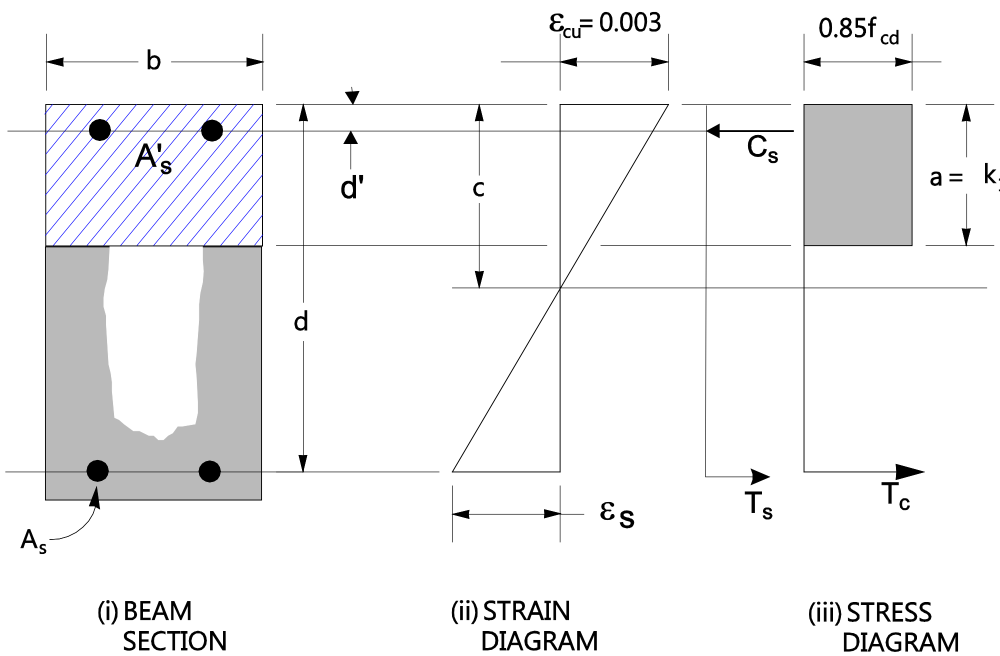


Figure 13-1 Rectangular Beam Design

- If $a \leq a_{max}$, the area of tension reinforcement is then given by:

$$A_s = \frac{M_d}{f_{yd}\left(d - \frac{a}{2}\right)}$$

This reinforcement is to be placed at the bottom if M_d is positive, or at the top if M_d is negative.

- If $a > a_{max}$, compression reinforcement is required (TS 7.1) and is calculated as follows:

− The compressive force developed in the concrete alone is given by:

$$C = 0.85 f_{cd} b a_{max}, \qquad \text{(TS 7.1)}$$

and the moment resisted by concrete compression and tension reinforcement is:

$$M_{dc} = C\left(d - \frac{a_{\max}}{2}\right).$$

− Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{ds} = M_d - M_{dc}.$$

− The required compression reinforcement is given by:

$$A_s' = \frac{M_{ds}}{\left(\sigma_s' - 0.85 f_{cd}\right)\left(d - d'\right)}, \text{ where}$$

$$\sigma_s' = E_s \varepsilon_{cu} \left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_{yd}. \qquad \text{(TS 7.1)}$$

− The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_{ds}}{f_{yd}\left[d - \frac{a_{\max}}{2}\right]},$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{ds}}{f_{yd}\left(d - d'\right)}$$

Therefore, the total tension reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_d is positive, and vice versa if M_d is negative.

13.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 13-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 13-1 is assumed on the compression side.

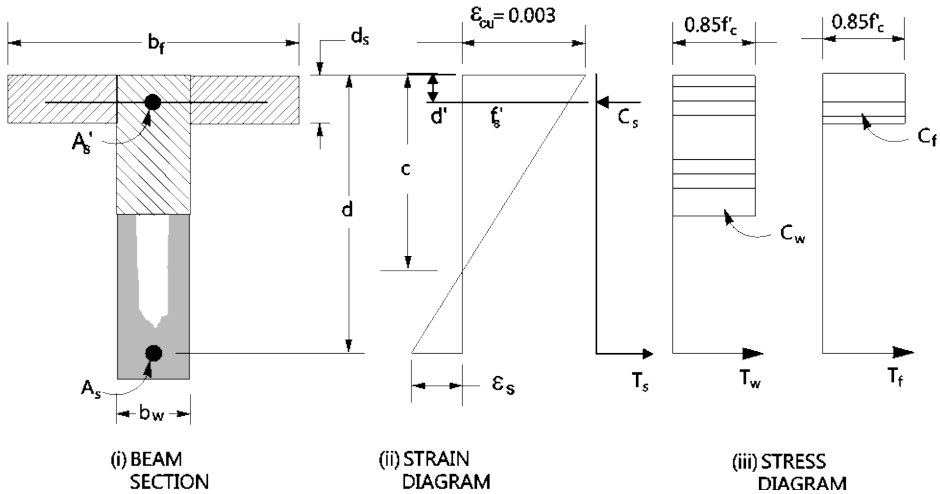


Figure 13-2 T-Beam Design

13.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_d (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

13.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M_d > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_d}{0.85 f_{cd} b_f}}$$

The maximum depth of the compression zone, c_b, is calculated based on the compressive strength of the concrete and the tensile steel tension using the following equation (TS 7.1):

$$c_b = \frac{\varepsilon_c E_s}{\varepsilon_{cu} E_s + f_{yd}} d \qquad \text{(TS 7.1)}$$

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{max} = 0.85 k_1 c_b \qquad \text{(TS 7.11, 7.3, Eqn. 7.4)}$$

where k_1 is calculated as follows:

$$k_1 = 0.85 - 0.006\left(f_{ck} - 25\right), \quad 0.70 \le k_1 \le 0.85. \qquad \text{(TS 7.1, Table 7.1)}$$

- If $a \le d_s$, the subsequent calculations for A_s are exactly the same as previously defined for the Rectangular section design. However, in that case, the width of the beam is taken as b_f, as shown in Figure 13-2. Compression reinforcement is required if $a > a_{max}$.

- If $a > d_s$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f , and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 13-2. C_f is given by:

$$C_f = 0.85 f_{cd}\left(b_f - b_w\right) \times \min\left(d_s, a_{max}\right) \qquad \text{(TS 7.1)}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_{yd}}$ and the portion of M_d that is resisted by the flange is given by:

$$M_{df} = C_f \left(d - \frac{\min\left(d_s, a_{max}\right)}{2} \right)$$

Therefore, the balance of the moment, M_d, to be carried by the web is given by:

$$M_{dw} = M_d - M_{df}$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{dw}}{0.85 f_{cd}\ b_w}} \qquad \text{(TS 7.1)}$$

- If $a_1 \le a_{max}$ (TS 7.1), the area of tensile steel reinforcement is then given by:

$$A_{s2} = \frac{M_{dw}}{f_{yd}\left(d - \frac{a_1}{2}\right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_{\max}$, compression reinforcement is required and is calculated as follows:

  The compression force in the web concrete alone is given by:

$$C = 0.85 f_{cd} b_w a_{\max} \qquad \text{(TS 7.1)}$$

Therefore the moment resisted by the concrete:

$$M_{dc} = C\left(d - \frac{a_{\max}}{2}\right),$$

The tensile steel for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{dc}}{f_{yd}\left[d - \frac{a_{\max}}{2}\right]},$$

The moment resisted by compression steel and tensile steel is:

$$M_{ds} = M_{dw} - M_{dc}$$

Therefore, the compression steel is computed as:

$$A'_s = \frac{M_{ds}}{\left(\sigma'_s - 0.85 f_{cd}\right)\left(d - d'\right)}, \text{ where}$$

$$\sigma'_s = E_s \varepsilon_{cu}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_{yd}, \text{ and} \qquad \text{(TS 7.1)}$$

the tensile steel for balancing the compression steel is:

$$A_{s3} = \frac{M_{ds}}{f_{yd}\left(d - d'\right)}$$

The total tensile reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

13.5.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required in a beam section is given by the following limit:

$$A_s \geq \frac{0.8 f_{ctd}}{f_{yd}} b_w d \qquad \text{(TS 7.3, Eqn. 7.3)}$$

The maximum flexural tensile steel required in a beam section is given by the following limit:

$$A_s - A_s^{'} \leq 0.85 \rho_b bd$$

An upper limit of 0.02 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed as follows:

$$A_s \leq \begin{cases} 0.02bd & \text{Rectangular Beam} \\ 0.02b_w d & \text{T-Beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.02bd & \text{Rectangular Beam} \\ 0.02b_w d & \text{T-Beam} \end{cases}$$

13.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_d.

- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

13.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

13.5.2.2 Determine Concrete Shear Capacity

Given the design force set N_d and V_d, the shear force carried by the concrete, V_c, is calculated as follows:

- If the beam is subjected to axial loading, N_d is positive in this equation regardless of whether it is a compressive or tensile force,

$$V_{cr} = 0.65 f_{ctd} b_w d \left(1 + \frac{\gamma N_d}{A_g}\right), \qquad \text{(TS 8.1.3, Eqn. 8.1)}$$

where,

$$\gamma = \begin{array}{l} 0.07 \text{ for axial compression} \\ \text{-}0.3 \text{ for axial tension} \\ 0 \text{ when tensile stress} < 0.5 \text{ MPa} \end{array}$$

$$V_c = 0.8 V_{cr}, \qquad \text{(TS 8.1.4, Eqn. 8.4)}$$

13.5.2.3 Determine Required Shear Reinforcement

Given V_d and V_c, the required shear reinforcement in the form of stirrups or ties within a spacing, s, is given for rectangular and circular columns by the following:

- The shear force is limited to a maximum of

$$V_{\max} = 0.22 f_{cd} A_w \qquad \text{(TS 8.1.5b)}$$

- The required shear reinforcement per unit spacing, A_v /s, is calculated as follows:

  If $V_d \le V_{cr}$,

  $$\frac{A_{sw}}{s} = 0.3\frac{f_{ctd}}{f_{ywd}} b_w, \qquad \text{(TS 8.1.5, Eqn. 8.6)}$$

  else if $V_{cr} < V_d \le V_{\max}$,

  $$\frac{A_{sw}}{s} = \frac{\left(V_d - V_c\right)}{f_{ywd}\, d}, \qquad \text{(TS 8.1.4, Eqn. 8.5)}$$

  $$\frac{A_{sw}}{s} \ge 0.3\frac{f_{ctd}}{f_{ywd}} b_w \qquad \text{(TS 8.1.5, Eqn. 8.6)}$$

  else if $V_d > V_{\max}$,

  a failure condition is declared. (TS 8.1.5b)

If V_d exceeds its maximum permitted value $V_{\max}$, the concrete section size should be increased (TS 8.1.5b).

Note that if torsion design is performed and torsion rebar is needed, the equation given in TS 8.1.5 does not need to be satisfied independently. See the next section *Design of Beam Torsion Reinforcement* for details.

The maximum of all of the calculated A_{sw}/s values, obtained from each design load combination, is reported along with the controlling shear force and associated design load combination name.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

13.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_d.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

13.5.3.1 Determine Factored Torsion

In the design of torsion reinforcement of any beam, the factored torsions for each design load combination at a particular design station are obtained by factoring the corresponding torsion for different analysis cases with the corresponding design load combination factors (TS 8.2).

In a statically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the design T_d is permitted to be reduced in accordance with code (TS 8.2.3). However, the program does not try to redistribute the internal forces and to reduce T_d. If redistribution is desired, the user should *release* the torsional DOF in the structural model.

13.5.3.2 Determine Special Section Properties

For torsion design, special section properties such as A_e, S and u_e are calculated. These properties are described as follows (TS 8.2.4).

A_e = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

S = Shape factor for torsion

u_e = Perimeter of area A_e

In calculating the section properties involving reinforcement, such as A_{ov}/s, A_{ot}/s, and u_e, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 30 mm. This is equivalent to 25-mm clear cover and a 10-mm-diameter stirrup placement. For torsion design of T beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a Rectangular beam section are given as follows:

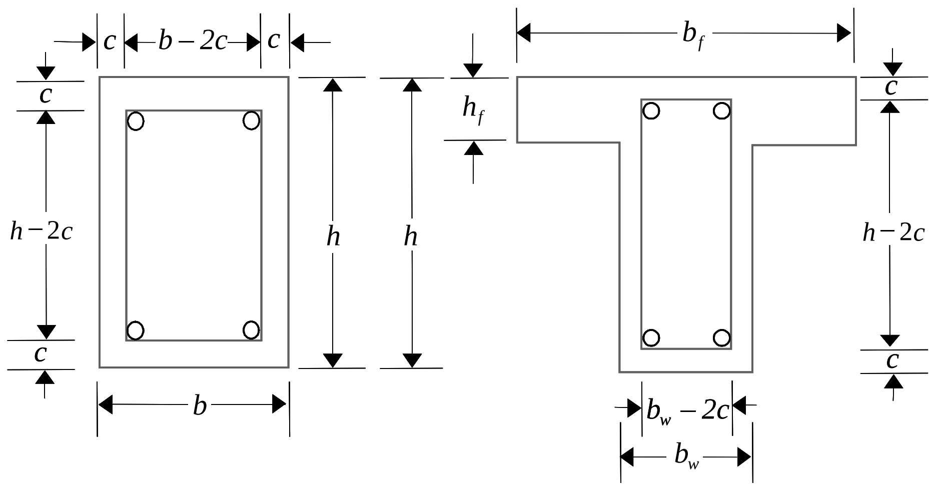


Figure 13-3 Closed stirrup and section dimensions for torsion design

$$A_e = (b-2c)(h-2c), \qquad \text{(TS 8.2.4)}$$

$$u_t = 2(b-2c)+2(h-2c), \qquad \text{(TS 8.2.4)}$$

$$S = x^2y/3 \qquad \text{(TS 8.2.4)}$$

where, the section dimensions b, h and c are shown in Figure 3-9. Similarly, the special section properties for a T beam section are given as follows:

$$A_e = (b_w - 2c)(h - 2c), \quad \text{(TS 8.2.4)}$$

$$u_t = 2(h - 2c) + 2(b_w - 2c), \quad \text{(TS 8.2.4)}$$

$$S = \Sigma x^2 y/3 \quad \text{(TS 8.2.4)}$$

where the section dimensions b_w, h and c for a T-beam are shown in Figure 13-3.

13.5.3.3 Determine Critical Torsion Capacity

Design for torsion may be ignored if either of the following is satisfied:

(i) The critical torsion limits, T_{cr}, for which the torsion in the section can be ignored, is calculated as follows:

$$T_d \leq T_{cr} = 0.65 f_{ctd} S \quad \text{(TS 8.2.3, Eqn 8.12)}$$

In that case, the program reports shear reinforcement based on TS 8.1.5, Eqn. 8.6. i.e.,

$$\frac{A_{sw}}{s} \geq 0.3 \frac{f_{ctd}}{f_{ywd}} b_w \quad \text{(TS 8.1.5, Eqn. 8.6)}$$

(ii) When design shear force and torsional moment satisfy the following equation, there is no need to compute torsional stirrups. However, the minimum stirrups and longitudinal reinforcement shown below must be provided:

$$\left(\frac{V_d}{V_{cr}}\right)^2 + \left(\frac{T_d}{T_{cr}}\right)^2 \leq 1 \quad \text{(TS 8.2.2, Eqn 8.10)}$$

where T_{cr} is computed as follows:

$$T_{cr} = 1.35 f_{ctd} S \quad \text{(TS 8.2.2, Eqn 8.11)}$$

The required minimum closed stirrup area per unit spacing, A_o/s, is calculated as:

$$\frac{A_o}{s} = 0.15 \frac{f_{ctd}}{f_{ywd}} \left(1 + \frac{1.3 T_d}{V_d b_w}\right) b_w \quad \text{(TS 8.2.4, Eqn. 8.17)}$$

In Eqn. 8.17, $\frac{T_d}{V_d b_w} \leq 1.0$ and for the case of statically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, minimum reinforcement will obtained by taking T_d equal to T_{cr}.

and the required minimum longitudinal rebar area, A_{sl}, is calculated as:

$$A_{sl} = \frac{T_d u_e}{2A_e f_{yd}} \qquad \text{(TS 8.2.5, Eqn. 8.18).}$$

13.5.3.4 Determine Torsion Reinforcement

If the factored torsion T_d is less than the threshold limit, T_{cr}, torsion can be safely ignored (TS 8.2.3), when the torsion is not required for equilibrium. In that case, the program reports that no torsion is required. However, if T_d exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (TS 8.2.4 and 8.2.5).

If $T_d > T_{cr}$, the required longitudinal rebar area, A_{sl}, is calculated as:

$$A_{sl} = \frac{T_d u_e}{2A_e f_{yd}} \qquad \text{(TS 8.2.4, Eqn. 8.16)}$$

and the required closed stirrup area per unit spacing, A_{ot}/s, is calculated as:

$$\frac{A_o}{s} = \frac{A_{ov}}{s} + \frac{A_{ot}}{s} \qquad \text{(TS 8.2.4, Eqn. 8.13)}$$

$$\frac{A_{ov}}{s} = \frac{\left(V_d - V_c\right)}{d f_{ywd}} \qquad \text{(TS 8.2.4, Eqn. 8.14)}$$

$$\frac{A_{ot}}{s} = \frac{T_d}{2A_e f_{ywd}} \qquad \text{(TS 8.2.4, Eqn. 8.15)}$$

where, the minimum value of A_o/s is taken as:

$$\frac{A_o}{s} = 0.15\frac{f_{ctd}}{f_{ywd}}\left(1 + \frac{1.3T_d}{V_d b_w}\right)b_w \qquad \text{(TS 8.2.4, Eqn. 8.17)}$$

where, $\frac{1.3T_d}{V_d b_w} \leq 1.0$

An upper limit of the combination of V_d and T_d that can be carried by the section also is checked using the following equation.

$$\frac{T_d}{S} + \frac{V_d}{b_w d} \leq 0.22\, f_{cd} \qquad \text{(TS 8.2.5b, Eqn. 8.19)}$$

The maximum of all the calculated A_{sl} and A_o/s values obtained from each design load combination is reported along with the controlling combination names.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

13.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (TS 500-2000) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

13.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

13.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

13.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

13.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (TS 11.4.5):

$A_{s,\min} = 0.0020\ bh$ for *steel grade S220* (TS 11.4.5)

$A_{s,\min} = 0.00175\ bh$ for *steel grade S420 and S500* (TS 11.4.5)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

13.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following sections.

13.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (TS 8.3.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (TS 8.3.1). Figure 13-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

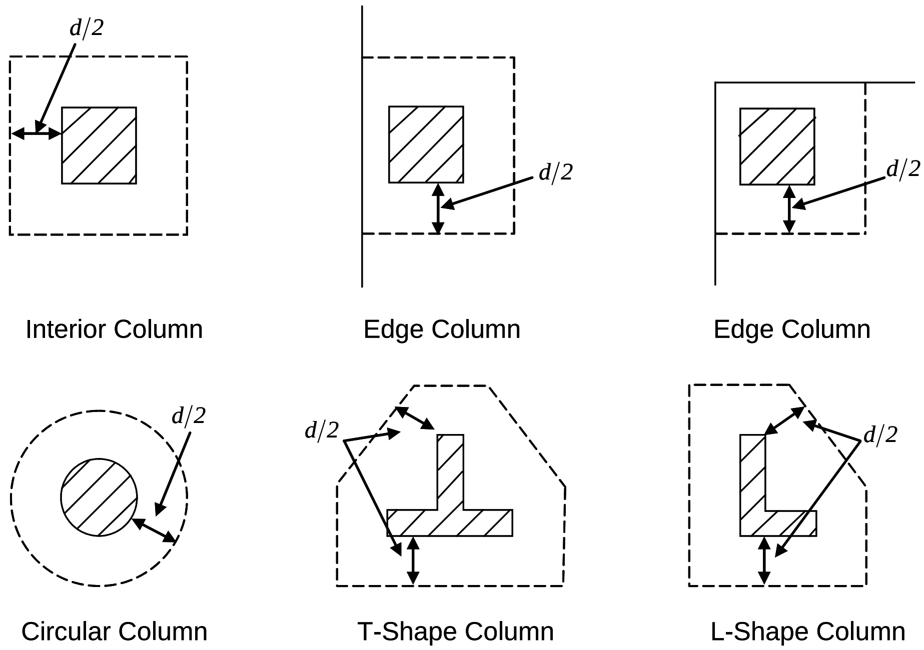


Figure 13-4 Punching Shear Perimeters

13.6.2.2 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the following limit:

$$v_{pr} = f_{ctd} = 0.35\sqrt{f_{ck}} / \gamma_c \quad \text{(TS 8.3.1)}$$

13.6.2.3 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, v_{pd}, is calculated as:

$$v_{pd} = \frac{V_{pd}}{u_p d}\left[1 + \eta \frac{0.4 M_{pd,2} u_p d}{V_{pd} W_{m,2}} + \eta \frac{0.4 M_{pd,3} u_p d}{V_{pd} W_{m,3}}\right], \text{ where} \quad \text{(TS 8.3.1)}$$

η factor to be used in punching shear check

$$\eta = \frac{1}{1+\sqrt{b_2 / b_1}} \text{ where } b_2 \geq 0.7b_1$$

When the aspect ratio of loaded area is greater than 3, the critical primeter is limited assuming $h = 3b$

u_p is the effective perimeter of the critical section

d is the mean effective depth of the slab

M_{pd} is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3

V_{pd} is the total punching shear force

W_m section modulus of area within critical punching perimeter (u_p) along bending axis 2 and 3.

13.6.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

13.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the slab thickness is greater than or equal to 250 mm, a (TS 8.3.2). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

13.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

$$v_{pr} = f_{ctd} = 0.35\sqrt{f_{ck}}/\gamma_c \qquad \text{(TS 8.3.1)}$$

13.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$v_{pr,\max} = 1.5\gamma\, f_{ctd} = 0.525\gamma\sqrt{f_{ck}}/\gamma_c \quad \text{for shear links/shear studs} \quad \text{(TS 8.3.1)}$$

Given V_{pd}, V_{pr}, and $V_{\text{pr,max}}$, the required shear reinforcement is calculated as follows,

$$\frac{A_v}{s} = \frac{\left(V_{pd} - V_{pr}\right)}{f_{yd}d} \qquad \text{(TS8.1.4)}$$

- If $V_{pd} > V_{\text{pr,max}}$, a failure condition is declared. (TS 8.3.1)
- If V_{pd} exceeds the maximum permitted value of $V_{\text{pr,max}}$, the concrete section should be increased in size.

13.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 13-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

NOTE: Shear Stud and shear links requirements are computed based on ACI 318-08 code as Turkish TS 500-2000 refers to special literature on this topic.

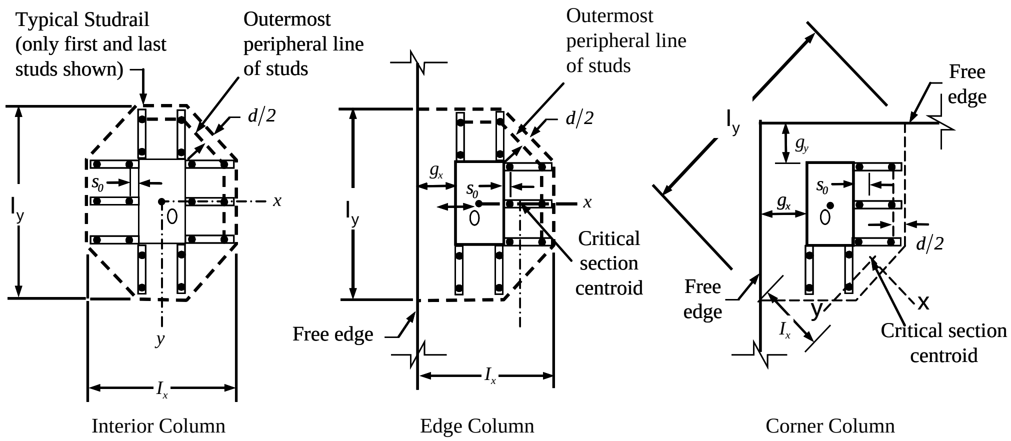


Figure 13-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$(ACI R11.3.3, 11.11.5.2. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

13.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 2*d*, and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.5.2)}$$

$$s \leq \begin{cases} 0.75d & \text{for} \quad v_u \leq 6\phi\lambda\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.5.2)}$$

$$g \leq 2d \qquad \text{(ACI 11.11.5.3)}$$

The limits of s_o and the spacing, s, between for the links are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.3)}$$

$$s \leq 0.50d \qquad \text{(ACI 11.11.3)}$$

Chapter 14
Design for Italian NTC 2008

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Italian code NTC2008 [D.M. 14/01/2008] is selected. For the load combinations, reference is also made to NTC2008. Various notations used in this chapter are listed in Table 14-1.

The design is based on user-specified loading combinations. However, the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

14.1 Notations

Table 14-1 List of Symbols Used in the NTC2008

A_c	Area of concrete section, mm^2
A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2

Table 14-1 List of Symbols Used in the NTC2008

A_{sl}	Area of longitudinal reinforcement for torsion, mm^2
A_{sw}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sw}/s_v	Area of shear reinforcement per unit length, mm^2/mm
A_t/s	Area of transverse reinforcement per unit length for torsion, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement
f_{cd}	Design concrete strength = $\alpha_{cc} f_{ck} / \gamma_c$, MPa
f_{ck}	Characteristic compressive concrete cylinder strength at 28 days, MPa
f_{ctm}	Mean value of concrete axial tensile strength, MPa
f_{cwd}	Design concrete compressive strength for shear design = $\alpha_{cc} \, {}^{f_{cwk}}\!/_{\gamma_c}$, MPa
f_{cwk}	Characteristic compressive cylinder strength for shear design, MPa
f'_s	Compressive stress in compression reinforcement, MPa
f_{yd}	Design yield strength of reinforcement = f_{yk}/γ_s, MPa
f_{yk}	Characteristic strength of shear reinforcement, MPa
f_{ywd}	Design strength of shear reinforcement = f_{ywk}/γ_s, MPa
f_{ywk}	Characteristic strength of shear reinforcement, MPa
h	Overall depth of section, mm

Table 14-1 List of Symbols Used in the NTC2008

h_f	Flange thickness, mm
M_{Ed}	Design moment at a section, N-mm
m	Normalized design moment, $M/bd^2\eta f_{cd}$
m_{lim}	Limiting normalized moment capacity as a singly reinforced beam
s_v	Spacing of the shear reinforcement, mm
T_{Ed}	Torsion at ultimate design load, N-mm
T_{Rdc}	Torsional cracking moment, N-mm
$T_{Rd,\max}$	Design torsional resistance moment, N-mm
u	Perimeter of the punch critical section, mm
V_{Rdc}	Design shear resistance from concrete alone, N
$V_{Rd,\max}$	Design limiting shear resistance of a cross-section, N
V_{Ed}	Shear force at ultimate design load, N
x	Depth of neutral axis, mm
x_{lim}	Limiting depth of neutral axis, mm
z	Lever arm, mm
α_{cc}	Coefficient accounting for long-term effects on the concrete compressive strength
α_{cw}	Coefficient accounting for the state of stress in the compression chord
δ	Redistribution factor
ε_c	Concrete strain
ε_s	Strain in tension reinforcement
ε'_s	Strain in compression steel
γ_c	Partial safety factor for concrete strength
γ_s	Partial safety factor for reinforcement strength
λ	Factor defining the effective depth of the compression zone
ν	Effectiveness factor for shear resistance without concrete crush-

Table 14-1 List of Symbols Used in the NTC2008

	ing
η	Concrete strength reduction factor for sustained loading and stress block
ρ_l	Tension reinforcement ratio
σ_{cp}	Axial stress in the concrete, MPa
θ	Angle of the concrete compression strut
ω	Normalized tension reinforcement ratio
ω'	Normalized compression reinforcement ratio
$\omega_{\lim}$	Normalized limiting tension reinforcement ratio

14.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be checked. NTC2008 allows load combinations to be defined based on NTC2008 Equation 2.5.1.

$$\sum_{j\geq 1}\gamma_{G1,j}G_{1k,j}+\sum_{l\geq 1}\gamma_{G2k,l}G_{2k,l}+P+\gamma_{Q,1}Q_{k,1}+\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(Eq. 2.5.1)}$$

Load combinations considering seismic loading are automatically generated based on NTC2008 Equation 2.5.5.

$$\sum_{j\geq 1}G_{1k,j}+\sum_{l\geq 1}G_{2k,l}+P+E+\sum_{i>1}\psi_{2,i}Q_{k,i} \qquad \text{(Eq. 2.5.5)}$$

For both sets of load combinations, the variable values are defined in the list that follows.

$\gamma_{G1,sup} = 1.30$ (NTC2008 Table 2.6.I)

$\gamma_{G1,inf} = 1.00$ (NTC2008 Table 2.6.I)

$\gamma_{G2,sup} = 1.50$ (NTC2008 Table 2.6.I)

$\gamma_{G2,inf} = 0.00$ (NTC2008 Table 2.6.I)

$\gamma_{Q,1,sup}$ = 1.5 (NTC2008 Table 2.6.I)

$\gamma_{Q,1,inf}$ = 0.0 (NTC2008 Table 2.6.I)

$\gamma_{Q,I,sup}$ = 1.5 (NTC2008 Table 2.6.I)

$\gamma_{Q,I,inf}$ = 0.0 (NTC2008 Table 2.6.I)

$\psi_{0,i}$ = 0.7 (live load, assumed not to be storage) (Table 2.5.I)

$\psi_{0,i}$ = 0.6 (wind load) (Table 2.5.I)

$\psi_{0,i}$ = 0.5 (snow load, assumed H ≤ 1,000 m) (Table 2.5.I)

$\psi_{2,i}$ = 0.3 (live, assumed office/residential space) (Table 2.5.I)

$\psi_{2,i}$ = 0 (snow, assumed H ≤ 1,000 m) (Table 2.5.I)

If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

14.3 Limits on Material Strength

The concrete compressive strength, f_{ck}, should not be greater than 90 MPa (NTC2008 Tab. 4.1.I). The reinforcement material should be B450C or B450A (NTC2008 §11.3.2).

NTC Table 11.3.Ia:

fy,nom	450 N/mm^2
ft,nom	540 N/mm^2

NTC Table 11.3.Ib: Material TYPE B450C Properties

Properties	**Prerequisite**	**Fracture %**
Characteristic yield stress, f_{yk}	$\geq f_{y,nom}$	5.0
Characteristic rupture stress, f_{tk}	$\geq f_{y,nom}$	5.0

$(f_{t/}\ f_y)_k$	≥ 1.15 < 1.35	10.0
Elongation at rupture $(f_{y/}\ f_{y,nom})_k$ $(A_{gt})_k$	 < 1.25 $\geq 7.5\ \%$	 10.0 10.0

NTC Table 11.3.Ic: Material TYPE B450A Properties

Properties	**Prerequisite**	**Fracture %**
Characteristic yield stress, f_{yk}	$\geq f_{y,nom}$	5.0
Characteristic rupture stress, f_{tk}	$\geq f_{y,nom}$	5.0
$(f_{t/}\ f_y)_k$	≥ 1.05 < 1.25	10.0
Elongation at rupture $(f_{y/}\ f_{y,nom})_k$ $(A_{gt})_k$	 < 1.25 $\geq 2.5\ \%$	 10.0 10.0

14.4 Partial Safety Factors

The design strengths for concrete and steel are obtained by dividing the characteristic strengths of the materials by the partial safety factors, γ_s and γ_c as shown here.

$$f_{cd} = \alpha_{cc}\, f_{ck} / \gamma_c \qquad \text{(NTC Eq. 4.1.4)}$$

$$f_{yd} = f_{yk} / \gamma_s \qquad \text{(NTC Eq. 4.1.6)}$$

$$f_{ywd} = f_{ywk} / \gamma_s \qquad \text{(NTC Eq. 4.1.6)}$$

α_{cc} is the coefficient taking account of long term effects on the compressive strength. α_{cc} is taken as 0.85 (NTC2008 4.1.2.1.1.1) by default and can be overwritten by the user.

The partial safety factors for the materials and the design strengths of concrete and reinforcement are given in the text that follows (NTC2008 4.1.2.1.1.1-3):

Partial safety factor for reinforcement, $\gamma_s = 1.15$

Partial safety factor for concrete, $\gamma_c = 1.5$

These values can be overwritten; however, caution is advised.

14.5 Beam Design

In the design of concrete beams, the program calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, torsions, design load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at a user-defined number of output stations along the beam span.

All beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

14.5.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at output stations along the beam span. The following steps are involved in designing the flexural reinforcement for the major moment for a particular beam, at a particular section:

Determine the maximum factored moments

- Determine the required reinforcing steel

14.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each design load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding design load combination factors.

The beam section is then designed for the factored moments obtained from each of the design load combinations. Positive moments produce bottom steel. In such cases, the beam may be designed as a rectangular or a T-beam section. Negative moments produce top steel. In such cases, the beam is always designed as a rectangular section.

14.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user can avoid the need for compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on a simplified rectangular stress block, as shown in Figure 14-7 (NTC Fig. 4.1.3). When the applied moment exceeds the moment capacity, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by the program for both rectangular and flanged sections (T-beams) is summarized in the following subsections. It is assumed that the design ultimate axial force is negligible, hence all beams are designed ignoring axial force.

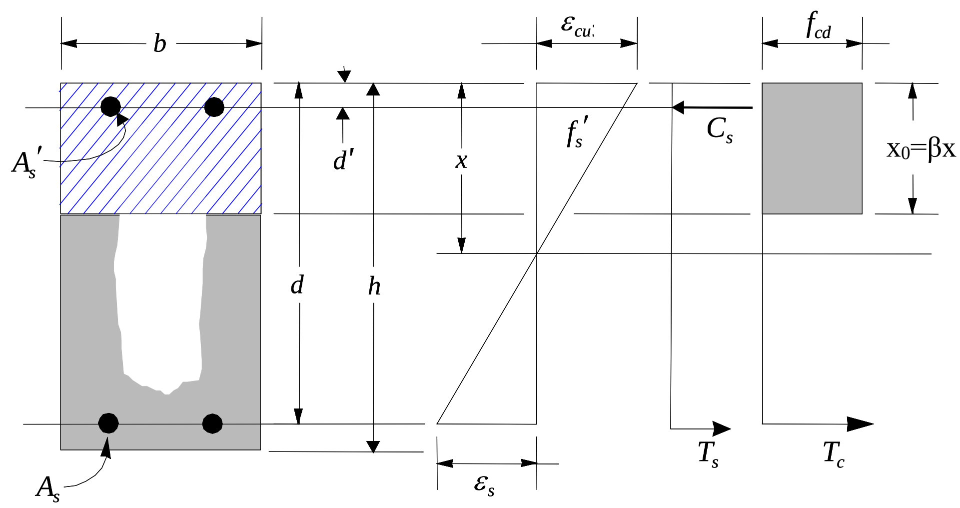


Figure 14-7 Rectangular beam design

In designing for a factored negative or positive moment, M_{Ed} (i.e., designing top or bottom steel), the effective strength and depth of the compression block are given by $\alpha_{cc} f_{cd}$ and βx (see Figure 14-7) respectively, where:

$\beta = 0.8$ (NTC § 4.1.2.1.2.2)

$\alpha_{cc} = 0.85$ (NTC § 4.1.2.1.2.2)

$$f_{cd} = \alpha_{cc} \frac{f_{ck}}{\gamma_c} \qquad \text{if} \quad f_{ck} \le 50 N/mm^2$$

$$\alpha_{cc} = 0.85$$

$$\gamma_c = 1.5$$

if $f_{ck} > 50 N/mm^2$ NTC 2008 refer to Eurocode 2:

$$f_{cd} = \alpha_{cc} \eta \frac{f_{ck}}{\gamma_c}$$

$\eta = 1.0 - (f_{ck} - 50)/200$ for $50 < f_{ck} \le 90$ MPa (EC2 Eq. 3.22)

For the design of the beams a ductility criterion, suggested in Eurocode 2 § 5.5, is followed.

The limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $(x/d)_{\text{lim}}$, is expressed as a function of the ratio of the redistributed moment to the moment before redistribution, δ, as follows:

$$\left(x/d\right)_{\text{lim}} = \left(\delta - k_1\right)/k_2 \text{ for } f_{ck} \leq 50 \text{ MPa} \quad \text{(EC2 Eq. 5.10a)}$$

$$\left(x/d\right)_{\text{lim}} = \left(\delta - k_3\right)/k_4 \text{ for } f_{ck} > 50 \text{ MPa} \quad \text{(EC2 Eq. 5.10b)}$$

No redistribution is assumed, such that δ is assumed to be 1. The four factors, k_1, k_2, k_3, and k_4 are defined as:

$$k_1 = 0.44 \quad \text{(EC2 5.5(4))}$$

$$k_2 = 1.25\left(0.6 + 0.0014/\varepsilon_{cu}\right) \quad \text{(EC2 5.5(4))}$$

$$k_3 = 0.54 \quad \text{(EC2 5.5(4))}$$

$$k_4 = 1.25\left(0.6 + 0.0014/\varepsilon_{cu}\right) \quad \text{(EC2 5.5(4))}$$

where the ultimate strain, ε_{cu2}, is determined from EC2 Table 3.1 as:

$$\varepsilon_{cu2} = 0.0035 \text{ for } f_{ck} < 50 \text{ MPa} \quad \text{(NTC § 4.1.2.1.2.2)}$$

$$\varepsilon_{cu2} = 2.6 + 35\left[\left(90 - f_{ck}\right)/100\right]^4 \text{ for } f_{ck} \geq 50 \text{ MPa} \quad \text{(NTC § 4.1.2.1.2.2)}$$

14.5.1.2.1 Rectangular Beam Flexural Reinforcement

For rectangular beams, the normalized moment, m, and the normalized section capacity as a singly reinforced beam, m_{lim}, are determined as:

$$m = \frac{M}{bd^2 f_{cd}}$$

$$m_{\text{lim}} = \beta\left(\frac{x}{d}\right)_{\text{lim}}\left[1 - \frac{\beta}{2}\left(\frac{x}{d}\right)_{\text{lim}}\right]$$

The reinforcing steel area is determined based on whether m is greater than, less than, or equal to m_{lim}.

- If $m \leq m_{\text{lim}}$, a singly reinforced beam will be adequate. Calculate the normalized steel ratio, ω, and the required area of tension reinforcement, A_s, as:

$$\omega = 1 - \sqrt{1-2m}$$

$$A_s = \omega \left[\frac{f_{cd} b d}{f_{yd}} \right]$$

This area of reinforcing steel is to be placed at the bottom if M_{Ed} is positive, or at the top if M_{Ed} is negative.

- If $m > m_{\text{lim}}$, compression reinforcement is required. Calculate the normalized steel ratios, ω', ω_{lim}, and ω, as:

$$\omega_{lim} = \beta \left(\frac{x}{d} \right)_{\text{lim}} = 1 - \sqrt{1-2m_{\text{lim}}}$$

$$\omega' = \frac{m - m_{\text{lim}}}{1 - d'/d}$$

$$\omega = \omega_{\text{lim}} + \omega'$$

where d' is the depth to the compression steel, measured from the concrete compression face.

Calculate the required area of compression and tension reinforcement, A_s' and A_s, as:

$$A_s' = \omega' \left[\frac{f_{cd} b d}{f_s'} \right]$$

$$A_s = \omega \left[\frac{f_{cd} b d}{f_{yd}} \right]$$

where f_s', the stress in the compression steel, is calculated as:

$$f_s' = E_s \varepsilon_c \left[1 - \frac{d'}{x_{lim}}\right] \le f_{yd}$$

A_s is to be placed at the bottom and A_s' is to be placed at the top if M_{Ed} is positive, and A_s' is to be placed at the bottom and A_s is to be placed at the top if M_{Ed} is negative.

14.5.1.2.2 T-Beam Flexural Reinforcement

In designing a T-beam, a simplified stress block, as shown in Figure 14-8, is assumed if the flange is in compression, i.e., if the moment is positive. If the moment is negative, the flange is in tension, and therefore ignored. In that case, a simplified stress block, similar to that shown in Figure 14-8, is assumed on the compression side.

14.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_{Ed} (i.e., designing top steel), the calculation of the reinforcing steel area is exactly the same as described for a rectangular beam, i.e., no specific T-beam data is used.

14.5.1.2.2.2 Flanged Beam Under Positive Moment

In designing for a factored positive moment, M_{Ed}, the program analyzes the section by considering the depth of the stress block. If the depth of the stress block is less than or equal to the flange thickness, the section is designed as a rectangular beam with a width b_f. If the stress block extends into the web, additional calculation is required.

For T-beams, the normalized moment, m, and the normalized section capacity as a singly reinforced beam, m_{lim}, are calculated as:

$$m = \frac{M}{b_f d^2 f_{cd}}$$

$$m_{\text{lim}} = \beta \left(\frac{x}{d}\right)_{\text{lim}} \left[1 - \frac{\beta}{2}\left(\frac{x}{d}\right)_{\text{lim}}\right]$$

Calculate the normalized steel ratios ω_{lim} and ω, as:

$$\omega_{\lim} = \beta\left(\frac{x}{d}\right)_{\lim}$$

$$\omega = 1 - \sqrt{1-2m}$$

Calculate the maximum depth of the concrete compression block, x_{max}, and the effective depth of the compression block, x, as:

$$x_{\max} = \omega_{\lim} d$$

$$x = \omega d$$

The reinforcing steel area is determined based on whether m is greater than, less than, or equal to m_{lim}.

- If $x \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for rectangular beam design. However, in this case, the width of the beam is taken as b_f, as shown in Figure 14-8. Compression reinforcement is required if $m > m_{lim}$.

- If $x > h_f$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, and the second part is for balancing the compressive force from the web, as shown in Figure 14-8.

- The required reinforcing steel area, A_{s2}, and corresponding resistive moment, M_2, for equilibrating compression in the flange outstands are calculated as:

$$A_{s2} = \frac{\left(b_f - b_w\right) h_f \, f_{cd}}{f_{yd}}$$

$$M_2 = A_{s2} f_{yd} \left(d - \frac{h_f}{2}\right)$$

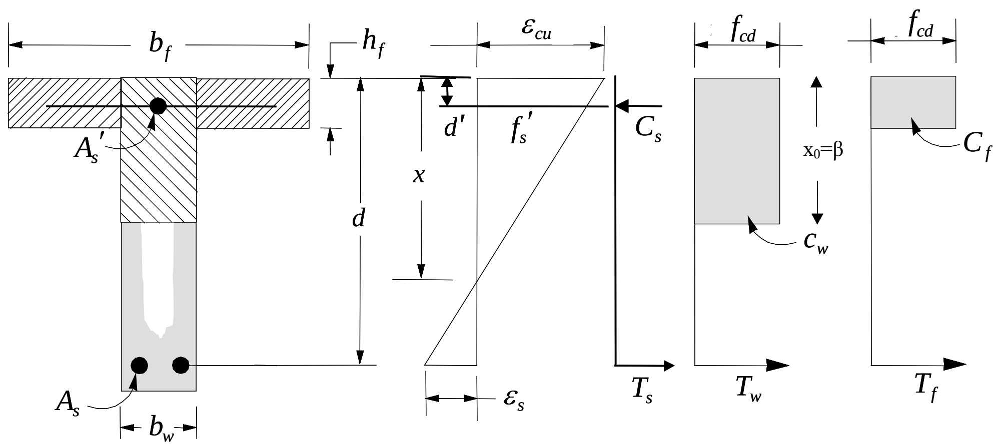


Figure 14-8 T-beam design

Now calculate the required reinforcing steel area A_{s1} for the rectangular section of width b_w to resist the remaining moment $M_1 = M_{Ed} - M_2$. The normalized moment, m_1 is calculated as:

$$m_1 = \frac{M_1}{b_w d^2 f_{cd}}$$

The reinforcing steel area is determined based on whether m_1 is greater than, less than, or equal to $m_{\lim}$.

- If $m_1 \leq m_{\lim}$, a singly reinforced beam will be adequate. Calculate the normalized steel ratio, ω_1, and the required area of tension reinforcement, A_{s1}, as:

  $$\omega_1 = 1 - \sqrt{1 - 2m}$$

  $$A_{s1} = \omega_1 \left[\frac{f_{cd} bd}{f_{yd}} \right]$$

- If $m_1 > m_{\lim}$, compression reinforcement is required. Calculate the normalized steel ratios, ω', $\omega_{\lim}$, and ω, as:

$$\omega_{\lim} = \beta \left(\frac{x}{d} \right)_{\lim}$$

$$\omega' = \frac{m - m_{\lim}}{1 - d'/d}$$

$$\omega_1 = \omega_{\lim} + \omega'$$

where d' is the depth to the compression steel, measured from the concrete compression face.

Calculate the required area of compression and tension reinforcement, A_s' and A_s, as:

$$A_s' = \omega' \left[\frac{f_{cd} bd}{f_s'} \right]$$

$$A_{s1} = \omega_1 \left[\frac{f_{cd} bd}{f_{yd}} \right]$$

where f_s', the stress in the compression steel, is calculated as:

$$f_s' = E_s \varepsilon_c \left[1 - \frac{d'}{x_{\lim}} \right] \le f_{yd}$$

The total tensile reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A_s'. A_s is to be placed at the bottom and A_s' is to be placed at the top of the section.

14.5.1.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel reinforcement, $A_{s,\min}$, required in a beam section is given as the maximum of the following two values:

$$A_{s,\min} = 0.26 \left(f_{ctm} / f_{yk} \right) b_t d \qquad \text{(NTC Eq. 4.1.43)}$$

$$A_{s,\min} = 0.0013 b_t d \qquad \text{(NTC Eq. 4.1.43)}$$

where b_t is the mean width of the tension zone, equal to the web width for T-beams, and f_{ctm} is the mean value of axial tensile strength of the concrete, calculated as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \quad \text{for } f_{ck} \le 50 \text{ MPa} \qquad \text{(NTC Eq. 11.2.3a)}$$

$$f_{ctm} = 2.12\, ln\left(1 + f_{cm}/10\right) \quad \text{for } f_{ck} > 50 \text{ MPa} \qquad \text{(NTC Eq. 11.2.3b)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(NTC Eq. 11.2.2)}$$

The maximum flexural steel reinforcement, $A_{s,\max}$, permitted as either tension or compression reinforcement is defined as:

$$A_{s,\max} = 0.04 A_c \qquad \text{(NTC § 4.1.6.1.1)}$$

where A_c is the gross cross-sectional area.

14.5.2 Design Beam Shear Reinforcement

The required beam shear reinforcement is calculated for each design load combination at each output station along the beam length. The following assumptions are made for the design of beam shear reinforcement:

- The beam section is assumed to be prismatic. The shear capacity is based on the beam width at the output station and therefore a variation in the width of the beam is neglected in the calculation of the concrete shear capacity at each particular output station.

- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement. Inclined shear steel is not handled.

The following steps are involved in designing the shear reinforcing for a particular beam, for a particular design load combination resulting from shear forces in a particular direction:

- Determine the design forces acting on the section, N_{Ed} and V_{Ed}. Note that N_{Ed} is needed for the calculation of V_{Rcd}.

- Determine the maximum design shear force that can be carried without crushing of the notional concrete compressive struts, V_{Rcd}.

- Determine the required shear reinforcement as area per unit length, A_{sw}/s.

The following three sections describe in detail the algorithms associated with this process.

14.5.2.1 Determine Design Shear Force

In the design of the beam shear reinforcement, the shear forces and moments for a particular design load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding design load combination factors.

14.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_{Rd,c}$, is calculated as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] b_w d \qquad \text{(EC2 6.2.2(1))}$$

with a minimum of:

$$V_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp} \right) b_w d \qquad \text{(EC2 6.2.2(1))}$$

where

f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.2.2(1))}$$

ρ_1 = tension reinforcement ratio = $\dfrac{A_{s1}}{b_w d} \leq 0.02$ (EC2 6.2.2(1))

A_{s1} = area of tension reinforcement (EC2 6.2.2(1))

$\sigma_{cp} = N_{Ed} / A_c < 0.2\ f_{cd}$ MPa (EC2 6.2.2(1))

The value of $C_{Rd,c}$, $v_{\min}$ and k_1 for use in a country may be found in its National Annex. The program default values for $C_{Rd,c}$, $v_{\min}$, and k_1 are given as follows (EC2 6.2.2(1)):

$C_{Rd,c}$= 0.18/γ_c (EC2 6.2.2(1))

v_{min} = 0.035 $k^{3/2}$ $f_{ck}^{1/2}$ (EC2 6.2.2(1))

k_1 = 0.15. (EC2 6.2.2(1))

For light-weight concrete:

$C_{Rd,c}$= 0.18/ γ_c (EC2 11.6.1(1))

v_{min} = 0.03 $k^{3/2}$ $f_{ck}^{1/2}$ (EC2 11.6.1(1))

k_1 = 0.15. (EC2 11.6.1(1))

14.5.2.3 Determine Maximum Design Shear Force

To prevent crushing of the concrete compression struts, the design shear force V_{Ed} is limited by the maximum sustainable design shear force, V_{Rcd}. If the design shear force exceeds this limit, a failure condition occurs. The maximum sustainable shear force is defined as:

$$V_{Rcd} = 0.9 \cdot d \cdot b_w \cdot \alpha_c f_{cd}^{'} \cdot \frac{\cot\alpha + \cot\vartheta}{1 + \cot^2\vartheta} \quad \text{(NTC Eq. 4.1.19)}$$

α_c = 1 for members not subjected to axial compression

$$= 1 + \frac{\sigma_{cp}}{f_{cd}} \text{ for } 0 \leq \sigma_{cp} \leq 0.25 f_{cd}$$

$$= 1.25 \text{ for } 0.25 f_{cd} \leq \sigma_{cp} \leq 0.5 f_{cd}$$

$$= 2.5\left(1 + \frac{\sigma_{cp}}{f_{cd}}\right) \text{ for } 0.5 f_{cd} \leq \sigma_{cp} \leq f_{cd}$$

$$f_{cd}^{'} = 0.5 f_{cd}$$

α angle between the shear reinforcement and the column axis. In the case of vertical stirrups $\alpha = 90$ degrees

θ angle between the concrete compression struts and the column axis. NTC 2008 allows θ to be taken between 21.8 and 45 degrees.

If torsion is significant i.e., $T_{Ed} > T_{cr}$ where T_{cr} is defined as:

$$T_{cr} = T_{Rd,c}\left(1 - \frac{\left|V_{Ed}\right|}{V_{Rd,c}}\right) \qquad \text{(EC2 Eq. 6.31)}$$

and if the load combination include seismic, the value of θ is taken as 45°. However, for other cases θ is optimized using the following relationship:

$$\left(\cot\theta + \tan\theta\right) = 0.9\alpha_{cw}v_1 f_{cd} / v_{Ed}$$

where

$$21.8° \le \theta \le 45°$$

14.5.2.4 Determine Required Shear Reinforcement

If V_{Ed} is less than V_{Rcd}, the required shear reinforcement in the form of stirrups or ties per unit spacing, A_{sw}/s, is calculated as:

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.9d \cdot f_{ywd}} \cdot \frac{1}{\left(\cot\alpha + \cot\theta\right)\sin\alpha} \qquad \text{(NTC Eq. 4.1.18)}$$

with $\alpha = 90$ degrees and ϑ given in the previous section.

The maximum of all of the calculated A_{sw}/s values, obtained from each design load combination, is reported for the major and minor directions of the beam, along with the controlling combination name.

The calculated shear reinforcement must be greater than the minimum reinforcement:

$$A_{sw,\min} = 1.5 \cdot b$$

with b in millimeters and $A_{sw,\,min}$ in mm^2/mm.

The beam shear reinforcement requirements reported by the program are based purely on shear strength consideration. Any minimum stirrup requirements to satisfy spacing considerations or transverse reinforcement volumetric considerations must be investigated independently by the user.

14.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at a user-defined number of output stations along the beam span. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to beam torsion:

- Determine the factored torsion, T_{Ed}.
- Determine torsion section properties.
- Determine the reinforcement steel required.

14.5.3.1 Determine Factored Torsion

In the design of torsion reinforcement of any beam, the factored torsions for each design load combination at a particular design station are obtained by factoring the corresponding torsion for different load cases with the corresponding design load combination factors.

14.5.3.2 Determine Torsion Section Properties

For torsion design, special torsion section properties, including A, t, u, and u_m are calculated. These properties are described as follows (NTC § 4.1.2.1.4).

A = area enclosed by centerlines of the connecting walls, where the centerline is located a distance of $t/2$ from the outer surface

$t = A_c / u$ = effective wall thickness

A_c = area of the section

u = outer perimeter of the cross-section

u_m = perimeter of the area A

For torsion design of T-beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during calculation of the torsion section properties.

With this assumption, the special properties for a Rectangular beam section are given as follows:

$A_c = bh$

$A = (b - t)(h - t)$

$u = 2b + 2h$

$u_m = 2(b - t) + 2(h - t)$

where, the section dimensions *b*, *h*, and *t* are shown in Figure 14-9. Similarly, the special section properties for a T-beam section are given as follows:

$A_c = b_w h + \left(b_f - b_w\right) d_s$

$A = (b_f - t)(h - t)$

$u = 2b_f + 2h$

$u_m = 2(b_f - t) + 2(h - t)$

where the section dimensions b_f, b_w, *h*, and d_s for a T-beam are shown in Figure 14-9.

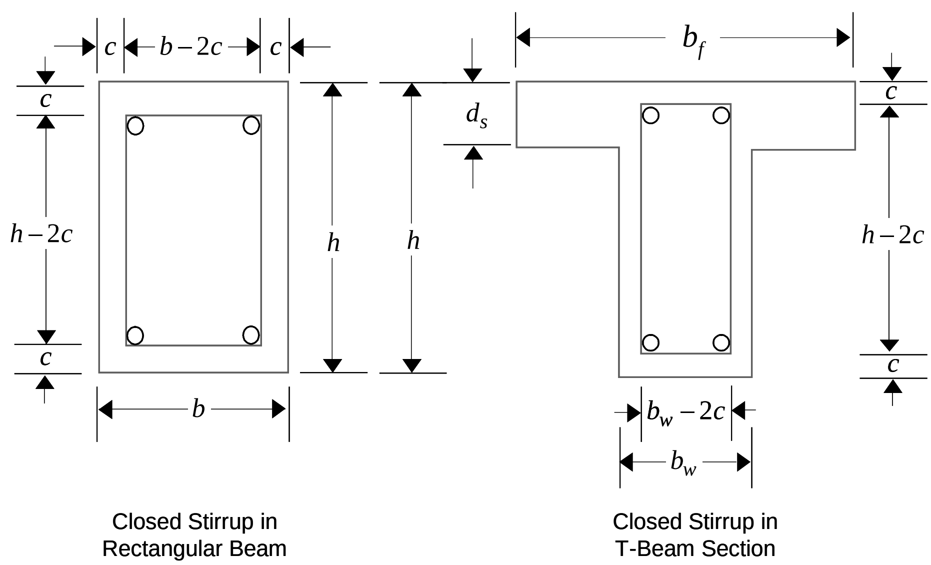


Figure 14-9 Closed stirrup and section dimensions for torsion design

14.5.3.3 Determine Torsion Reinforcement

It is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals.

The ultimate resistance of compression diagonals is:

$$T_{Rcd} = 2 \cdot A \cdot t \cdot f'_{cd} \cdot \frac{\cot\theta}{1+\cot^2\theta} \qquad \text{(NTC Eq. 4.1.27)}$$

$$f'_{cd} = 0.5 f_{cd}$$

θ is the angle of the compression diagonals, as previously defined for beam shear. The code allows any value between 21.8° and 68.2° (NTC Eq. 4.1.30), while the program assumes the conservative value of 45°.

An upper limit of the combination of V_{Ed} and T_{Ed} that can be carried by the section without exceeding the capacity of the concrete struts is checked using the following equation.

$$\frac{T_{Ed}}{T_{Rcd}} + \frac{V_{Ed}}{V_{Rcd}} \le 1.0 \qquad \text{(NTC Eq. 4.1.32)}$$

If the combination of V_{Ed} and T_{Ed} exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The ultimate resistance of the closed stirrups is:

$$T_{Rsd} = 2 \cdot A \cdot \frac{A_s}{s} \cdot f_{yd} \cdot \cot\theta \qquad \text{(NTC Eq. 4.1.28)}$$

A_s = stirrups' area

By reversing the equation 4.1.28 and imposing $T_{Ed} = T_{Rsd}$ it is possible to calculate the necessary steel area.

Finally the resistance of longitudinal bars is:

$$T_{Rld} = 2 \cdot A \cdot \frac{\sum A_1}{u_m} \cdot \frac{f_{yd}}{\cot \vartheta} \qquad \text{(NTC Eq. 4.1.28)}$$

$\sum A_1$ area of the total longitudinal bars

By reversing the equation 4.1.28 and imposing $T_{Ed} = T_{Rsd}$ it is possible to calculate the necessary steel area.

The maximum of all the calculated A_l and A_s / s values obtained from each design load combination is reported along with the controlling combination names.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

14.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (NTC2008) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

14.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, described in the subsections that follow, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

14.6.1.1 Determine Factored Moments for Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

14.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

14.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits:

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd \qquad \text{(NTC Eq. 4.1.43)}$$

$$A_{s,\min} = 0.0013bd \qquad \text{(NTC Eq. 4.1.43)}$$

where f_{ctm} is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \text{ for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(NTC Eq. 11.2.3a)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm}/10\right) \text{ for } f_{ck} > 50 \text{ MPa} \qquad \text{(NTC Eq. 11.2.3b)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(NTC Eq. 11.2.2)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (NTC § 4.1.6.1.1).

14.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

NTC2008 for the punching shear check refers to Eurocode2-2004.

14.6.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (EC2 6.4.1(4)) and at a critical section at a distance of 2.0d from the face of the support (EC2 6.4.2(1)). The perimeter of the critical section should be constructed such that its length is minimized. Figure 6-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

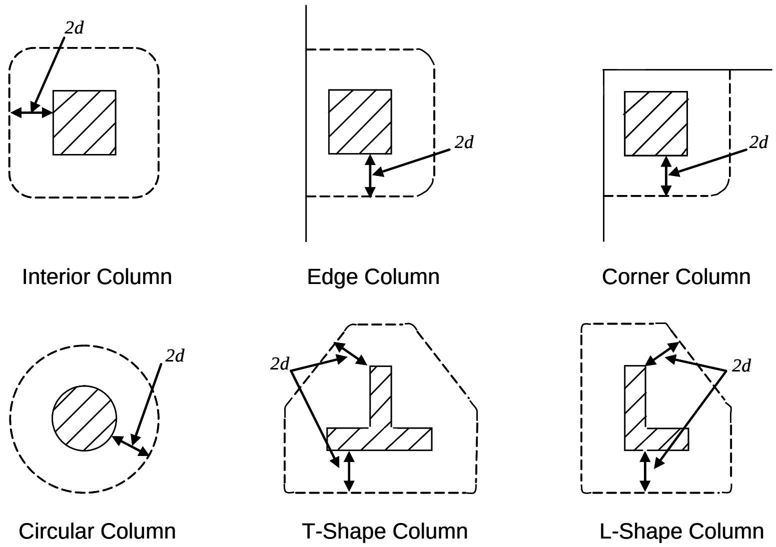


Figure 6-4 Punching Shear Perimeters

14.6.2.2 Determination of Concrete Capacity

The concrete punching shear stress capacity is taken as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] \qquad \text{(EC2 6.4.4(1))}$$

with a minimum of:

$$V_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp} \right) \qquad \text{(EC2 6.4.4(1))}$$

where f_{ck} is in MPa and

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.4.4(1))}$$

$$\rho_1 = \sqrt{\rho_{1x} \rho_{1y}} \leq 0.02 \qquad \text{(EC2 6.4.4(1))}$$

where ρ_{1x} and ρ_{1y} are the reinforcement ratios in the x and y directions respectively, conservatively taken as zeros, and

$$\sigma_{cp} = (\sigma_{cx} + \sigma_{cy})/2 \qquad \text{(EC2 6.4.4(1))}$$

where σ_{cx} and σ_{cy} are the normal concrete stresses in the critical section in the x and y directions respectively, conservatively taken as zeros.

$$C_{Rd,c} = 0.18/\gamma_c \qquad \text{(EC2 6.4.4(1))}$$

$$v_{\min} = 0.035k^{3/2} f_{ck}^{\ 1/2} \qquad \text{(EC2 6.4.4(1))}$$

$$k_1 = 0.15 \qquad \text{(EC2 6.4.4(1))}$$

14.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, v_{Ed}, is calculated as:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + k\frac{M_{Ed,2}u_1}{V_{Ed}W_{1,2}} + k\frac{M_{Ed,3}u_1}{V_{Ed}W_{1,3}}\right], \text{ where} \qquad \text{(EC2 6.4.4(2))}$$

k is the function of the aspect ratio of the loaded area in Table 14.1 of EN 1992-1-1

u_1 is the effective perimeter of the critical section

d is the mean effective depth of the slab

M_{Ed} is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3

V_{Ed} is the total punching shear force

W_1 accounts for the distribution of shear based on the control perimeter along bending axis 2 and 3.

14.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

14.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as described in the subsections that follow.

14.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

14.6.3.2 Determine Required Shear Reinforcement

The shear is limited to a maximum of V_{Rcd} calculated in the same manner as explained previously for beams.

Given v_{Ed}, $v_{Rd,c}$, and v_{Rcd}, the required shear reinforcement is calculated as follows (EC2 6.4.5).

- If $v_{R,dc} < v_{Ed} \leq v_{Rcd}$

$$A_{sw} = \frac{\left(v_{Ed} - 0.75 v_{Rd,c}\right)}{1.5 f_{ywd,ef}} \left(u_1 d\right) s_r \qquad \text{(EC2 6.4.5)}$$

- If $v_{Ed} > v_{Rcd,}$ a failure condition is declared. (EC2 6.2.3(3))
- If v_{Ed} exceeds the maximum permitted value of V_{Rcd} the concrete section should be increased in size.

14.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 6-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

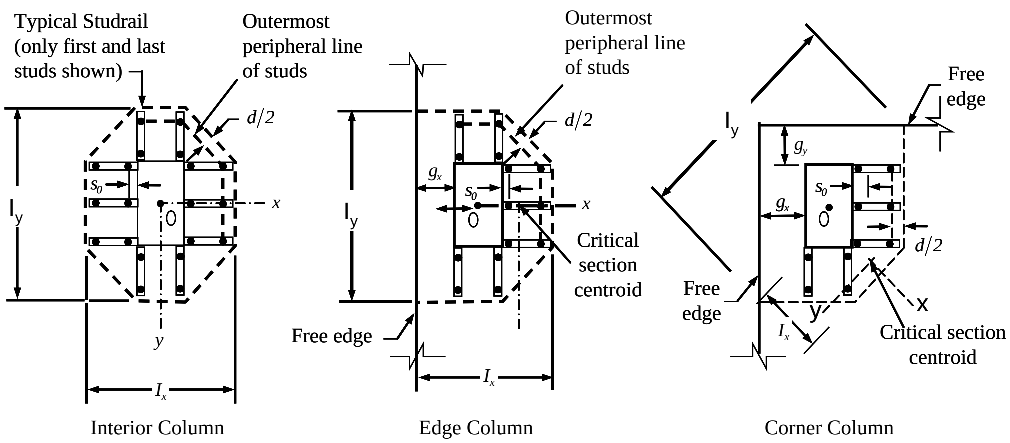


Figure 6-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed 2*d*. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5*d* measured in a direction parallel to the column face (EC2 9.4.3(1)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

14.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in EC2 4.4.1 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.3*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 1.5*d* and should not exceed 2*d* at additional perimeters. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$0.3d \leq s_o \leq 2d \quad \text{(EC2 9.4.3(1))}$$

$$s \leq 0.75d \quad \text{(EC2 9.4.3(1))}$$

$$g \leq 1.5d \text{ (first perimeter)} \quad \text{(EC2 9.4.3(1))}$$

$$g \leq 2d \text{ (additional perimeters)} \quad \text{(EC2 9.4.3(1))}$$

Chapter 15
Design for Hong Kong CP-2013

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Hong Kong limit state code CP-2013 [CP 2013] is selected. The various notations used in this chapter are listed in Table 15-1. For referencing to the pertinent sections of the Hong Kong code in this chapter, a prefix "CP" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

15.1 Notations

Table 15-1 List of Symbols Used in the CP-2013 Code

A_g	Gross area of cross-section, mm^2
A_l	Area of longitudinal reinforcement for torsion, mm^2
A_s	Area of tension reinforcement, mm^2

Table 15-1 List of Symbols Used in the CP-2013 Code

A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
$A_{sv,t}$	Total cross-sectional area of closed links for torsion, mm^2
A_{sv}/s_v	Area of shear reinforcement per unit length, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
C	Torsional constant, mm^4
d	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, N/mm^2
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 N/mm^2
f	Punching shear factor considering column location
f_{cu}	Characteristic cube strength, N/mm^2
f'_s	Stress in the compression reinforcement, N/mm^2
f_y	Characteristic strength of reinforcement, N/mm^2
f_{yv}	Characteristic strength of shear reinforcement, N/mm^2
h	Overall depth of a section in the plane of bending, mm
h_f	Flange thickness, mm
$h_{\min}$	Smaller dimension of a rectangular section, mm
$h_{\max}$	Larger dimension of a rectangular section, mm
K	Normalized design moment, $\frac{M_u}{bd^2 f_{cu}}$
K'	Maximum $\frac{M_u}{bd^2 f_{cu}}$ for a singly reinforced concrete section
k_1	Shear strength enhancement factor for support compression

Table 15-1 List of Symbols Used in the CP-2013 Code

k_2	Concrete shear strength factor, $[f_{cu}/25]^{1/3}$
M	Design moment at a section, N-mm
M_{single}	Limiting moment capacity as singly reinforced beam, N-mm
s_v	Spacing of the links along the length of the beam, mm
T	Design torsion at ultimate design load, N-mm
u	Perimeter of the punch critical section, mm
V	Design shear force at ultimate design load, N
v	Design shear stress at a beam cross-section or at a punching critical section, N/mm^2
v_c	Design concrete shear stress capacity, N/mm^2
$v_{\max}$	Maximum permitted design factored shear stress, N/mm^2
v_t	Torsional shear stress, N/mm^2
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm
β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain
ε_s	Strain in tension reinforcement
ε'_s	Strain in compression reinforcement

15.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (CP 2.3.1.3). For CP-2013, if a structure is subjected to dead (G), live (Q), pattern live (PQ), and wind (W) loads, and considering that wind forces are reversible, the following load combinations may need to be considered. (CP 2.3.2.1, Table 2.1).

1.4G
1.4G + 1.6Q (CP 2.3.2)

1.4G + 1.6(0.75PQ) (CP 2.3.2)

1.0G ± 1.4W
1.4G ± 1.4W (CP 2.3.2)
1.2G + 1.2Q ± 1.2W

These are also the default design load combinations in SAFE whenever the CP-2013 code is used. If roof live load is separately treated or other types of loads are present, other appropriate load combinations should be used. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

15.3 Limits on Material Strength

The concrete compressive strength, f_{cu}, should not be less than 20 N/mm^2 (CP 3.1.3). The program does not enforce this limit for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are used for design even if they are outside of the limits. It is the user's responsible to use the proper strength values while defining the materials.

15.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the following table, as taken from CP Table 2.2 (CP 2.4.3.2):

Values of γ_m for the Ultimate Limit State	
Reinforcement	1.15
Concrete in flexure and axial load	1.50
Concrete shear strength without shear reinforcement	1.25

These factors are incorporated into the design equations and tables in the code, but can be overwritten.

15.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the sections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

15.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the length of the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

15.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam is always designed as a rectangular or inverted flanged beam.

15.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 15-1 (CP 6.1.2.4(a)), where $\varepsilon_{c,\max}$ is defined as:

$$\varepsilon_{c,\max} = \begin{cases} 0.0035 & \text{if} \quad f_{cu} \le 60\text{N/mm}^2 \\ 0.0035 - 0.00006\left(f_{cu} - 60\right)^{1/2} & \text{if} \quad f_{cu} > 60\text{N/mm}^2 \end{cases}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \ge 0.9$; CP 6.1.2.4(b)). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \le \begin{cases} 0.5 & \text{for} & f_{cu} \le 45\text{ N/mm}^2 \\ 0.4 & \text{for} & 45 < f_{cu} \le 70\text{ N/mm}^2 \\ 0.33 & \text{for} & 70 < f_{cu} \le 100\text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(b))}$$

to safeguard against non-ductile failures (CP 6.1.2.4(b)). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a), Fig 6.1)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed ($0.1f_{cu}A_g$) (CP 6.1.2.4(a)), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $0.1f_{cu}A_g$ and axial tensions are always included in flexural and shear design.

15.5.1.2.1 Design of Rectangular Beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam, M_{single}, is obtained first for a section. The reinforcing is determined based on whether M is greater than, less than, or equal to M_{single}. See Figure 15-1

Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

$$M_{\text{single}} = K'f_{cu}\, bd^2, \text{ where} \qquad \text{(CP 6.1.2.4(c), Eqn. 6.8)}$$

$$K' = \begin{cases} 0.156 \text{ for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.120 \text{ for } 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.094 \text{ for } 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases}$$

- If $M \le M_{\text{single}}$, the area of tension reinforcement, A_s, is obtained from:

$$A_s = \frac{M}{0.87 f_y z}, \text{ where} \qquad \text{(CP 6.1.2.4(c), Eqn. 6.12)}$$

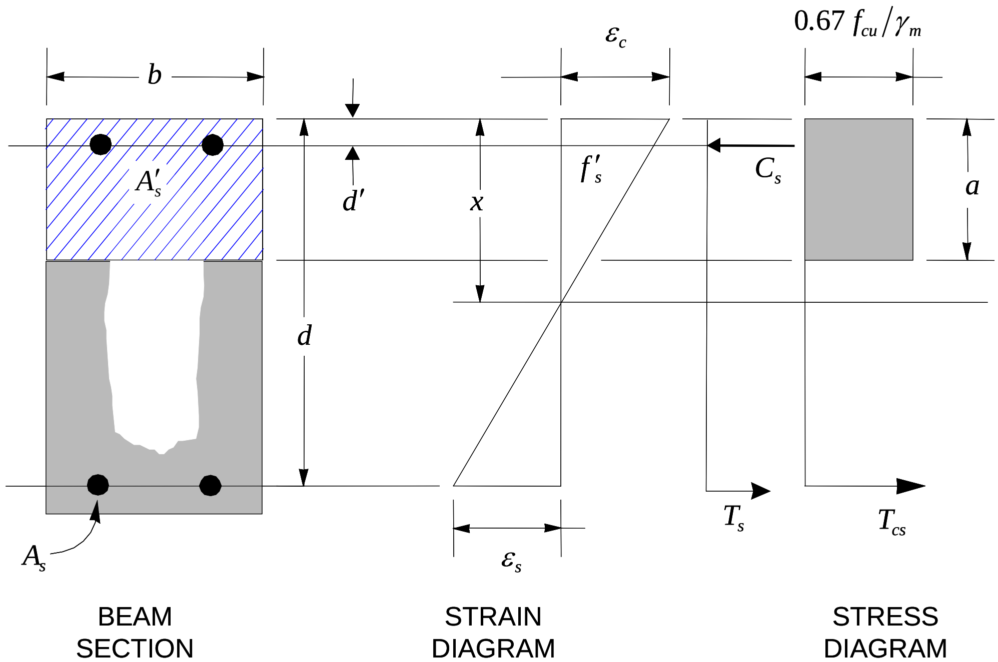


Figure 15-1 Rectangular Beam Design

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d \qquad \text{(CP 6.1.2.4(c), Eqn. 6.10)}$$

$$K = \frac{M}{f_{cu} b d^2} \qquad \text{(CP 6.1.2.4(c) , Eqn. 6.7)}$$

This reinforcement is to be placed at the bottom if *M* is positive, or at the top if *M* is negative.

- If $M > M_{\text{single}}$, compression reinforcement is required and calculated as follows:

$$A'_s = \frac{M - M_{\text{single}}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')} \qquad \text{(CP 6.1.2.4(c), , Eqn. 6.15)}$$

where d' is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = E_s \varepsilon_c \left(1 - \frac{d'}{x}\right) \le 0.87 f_y, \qquad \text{(CP 6.1.2.4(c), 3.2.6, Fig. 3.9)}$$

$$x = \begin{cases} \dfrac{d-z}{0.45}, & \text{for} \quad f_{cu} \le 45\,\text{N/mm}^2 \\ \dfrac{d-z}{0.40}, & \text{for} \quad 45 < f_{cu} \le 70\ \text{N/mm}^2 \\ \dfrac{d-z}{0.36}, & \text{for} \quad 70 < f_{cu} \le 100\ \text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a), Fig 6.1, Eqn. 6.11)}$$

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right\} \le 0.95d \qquad \text{(CP 6.1.2.4(c))}$$

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$A_s = \frac{M_{\text{single}}}{0.87 f_y z} + \frac{M - M_{\text{single}}}{0.87 f_y \left(d - d'\right)} \qquad \text{(CP 6.1.2.4(c))}$$

15.5.1.2.2 Design of Flanged Beams

15.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

15.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located in the flange. On the basis of this assumption, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange depth, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 15-2.

Assuming the neutral axis to lie in the flange, the normalized moment is given by:

$$K = \frac{M}{f_{cu}\, b_f\, d^2}. \qquad \text{(CP 6.1.2.4(c) , Eqn. 6.7)}$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \qquad \text{(CP 6.1.2.4(c), , Eqn. 6.10)}$$

the depth of the neutral axis is computed as:

$$x = \begin{cases} \dfrac{d-z}{0.45}, & \text{for} \quad f_{cu} \le 45\,\text{N/mm}^2 \\ \dfrac{d-z}{0.40}, & \text{for} \quad 45 < f_{cu} \le 70\,\text{N/mm}^2 \\ \dfrac{d-z}{0.36}, & \text{for} \quad 70 < f_{cu} \le 100\,\text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(c), Fig 6.1, , Eqn. 6.11)}$$

and the depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 45\ \text{N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \le 70\ \text{N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \le 100\ \text{N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a), Fig 6.1)}$$

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as b_f. Compression reinforcement is required when $K > K'$.

- If $a > h_f$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 15-2.

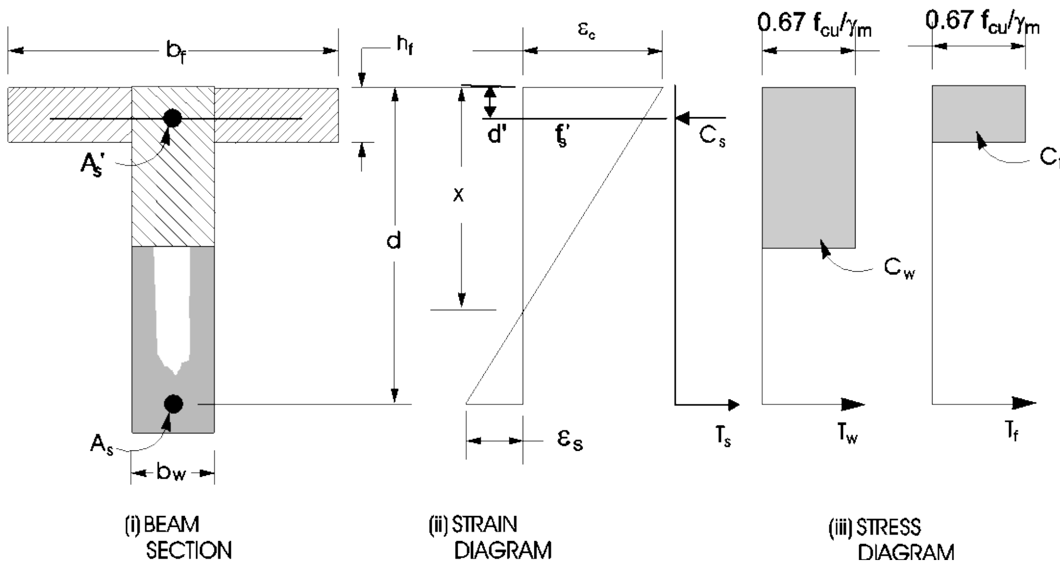


Figure 15-2 Design of a T-Beam Section

In that case, the ultimate resistance moment of the flange is given by:

$$M_f = \frac{0.67}{\gamma_c} f_{cu}\left(b_f - b_w\right)h_f\left(d - 0.5h_f\right)$$

The moment taken by the web is computed as:

$$M_w = M - M_f$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2}$$

- If $K_w \leq K'$ (CP 6.1.2.4(c)), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{0.87 f_y \left(d - 0.5 h_f\right)} + \frac{M_w}{0.87 f_y z}, \text{ where}$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \le 0.95d$$

- If $K_w > K'$, compression reinforcement is required and is calculated as follows:

  The ultimate moment of resistance of the web only is given by:

  $$M_{uw} = K f_{cu} b_w d^2$$

  The compression reinforcement is required to resist a moment of magnitude $M_w - M_{uw}$. The compression reinforcement is computed as:

  $$A'_s = \frac{M_w - M_{uw}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)\left(d - d'\right)}$$

  where, d' is the depth of the compression reinforcement from the concrete compression face, and

  $$f_s = E_s \varepsilon_c \left(1 - \frac{d}{x}\right) \le 0.87 f_y \qquad \text{(CP 6.1.2.4(c), 3.2.6, Fig 3.9)}$$

The area of tension reinforcement is obtained from equilibrium as:

$$A_s = \frac{1}{0.87 f_y}\left[\frac{M_f}{d - 0.5 h_f} + \frac{M_{uw}}{z} + \frac{M_w - M_{uw}}{d - d'}\right]$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right) \le 0.95d$$

15.5.1.2.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from CP Table 9.1(CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage	
			f_y = 250 MPa	f_y = 460 MPa
Rectangular	—	$100\ \dfrac{A_s}{bh}$	0.24	0.13
T or L-Beam with web in tension	$\dfrac{b_w}{b_f} < 0.4$	$100\ \dfrac{A_s}{b_w h}$	0.32	0.18
	$\dfrac{b_w}{b_f} \geq 0.4$	$100\ \dfrac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100\ \dfrac{A_s}{b_w h}$	0.48	0.26
L-Beam with web in compression	—	$100\ \dfrac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from CP Table 9.1 (CP 9.2.1.1).

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100\ \dfrac{A'_s}{bh}$	0.20
T or L-Beam	Web in tension	$100\ \dfrac{A'_s}{b_f h_f}$	0.40
	Web in compression	$100\ \dfrac{A'_s}{b_w h}$	0.20

An upper limit of 0.04 times the gross cross-sectional area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows (CP 9.2.1.3):

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$ (CP 9.2.1.3)

15.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (CP 6.1.2.5):

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

15.5.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors. The shear stress is then calculated as:

$$v = \frac{V}{bd}$$ (CP 6.1.2.5(a))

The maximum allowable shear stress, v_{max} is defined as:

$$v_{max} = \min(0.8\sqrt{f_{cu}}\,,\ 7\text{ MPa})$$ (CP 6.1.2.5(a))

15.5.2.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete, v_c, is calculated as:

$$v'_c = v_c + 0.6\frac{NVh}{A_c M} \le v_c\sqrt{1+\frac{N}{A_c v_c}}$$ (CP 6.1.2.5(k) , Eqn. 6.22)

$$v_c = \frac{0.79k_1k_2}{\gamma_m}\left(\frac{100A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}$$ (CP 6.1.2.5(c), Table 6.3)

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 (CP 6.1.2.5(g))

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3}, \qquad 1 \le k_2 \le \left(\frac{80}{25}\right)^{1/3}$$ (CP 6.1.2.5(c), Table 6.3)

$$\gamma_m = 1.25$$ (CP 6.1.2.5(c), Table 6.3)

However, the following limitations also apply:

$$0.15 \le \frac{100A_s}{bd} \le 3,$$ (CP 6.1.2.5(c), Table 6.3)

$$\left(\frac{400}{d}\right)^{1/4} \ge \begin{cases} 0.67, & \text{Members without shear reinforcement} \\ 1.00, & \text{Members with shear reinforcement} \end{cases}$$

(CP 6.1.2.5(c), Table 6.3)

$$\frac{Vh}{M} \le 1$$ (CP 6.1.2.5(k))

15.5.2.3 Determine Required Shear Reinforcement

Given v, v_c, and v_{max}, the required shear reinforcement is calculated as follows (CP Table 6.2, CP 6.1.2.5(b)):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, v_r, as:

$$v_r = \begin{cases} 0.4 & \text{if} \quad f_{cu} \le 40 \, {}^{\text{N}}\!/_{\text{mm}^2} \\ 0.4\left({}^{f_{cu}}\!/_{40} \right)^{2/3} & \text{if } 40 < f_{cu} \le 80 \, {}^{\text{N}}\!/_{\text{mm}^2} \\ 0.4\left({}^{80}\!/_{40} \right)^{2/3} & \text{if} \quad f_{cu} > 80 \, {}^{\text{N}}\!/_{\text{mm}^2} \end{cases} \qquad \text{(CP 6.1.2.5(b), Table 6.2)}$$

- If $v \le v'_c + v_r$, minimum reinforcement is required:

$$\frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}}, \qquad \text{(CP 6.1.2.5(b))}$$

- If $v > v'_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v'_c)b}{0.87 f_{yv}} \qquad \text{(CP 6.1.2.5(b))}$$

- If $v > v_{max}$, a failure condition is declared. (CP 6.1.2.5(b))

The maximum of all the calculated A_{sv}/s_v values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

15.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t.
- Determine special section properties.

- Determine critical torsion stress.
- Determine the torsion reinforcement required.

15.5.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given, as follows (CP 6.3.1).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2\left(h_{\max} - h_{\min}/3\right)} \qquad \text{(CP 6.3.3(a))}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{seg} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(CP 6.3.3(b))}$$

$h_{\max}$ = Larger dimension of a rectangular section

$h_{\min}$ = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated:

$$v_t \le \min(0.8\sqrt{f_{cu}}, 7\,\text{N/mm}^2) \times \frac{y_1}{550} \qquad \text{(CP 6.3.4, Table 6.17)}$$

Where y_1 is the larger center-to-center dimension of a rectangular link.

15.5.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.6 \text{ N/mm}^2\right) \qquad \text{(CP 6.3.4, Table 6.17)}$$

where f_{cu} is the specified concrete compressive strength.

15.5.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (CP 6.3.5). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP 6.3.5).

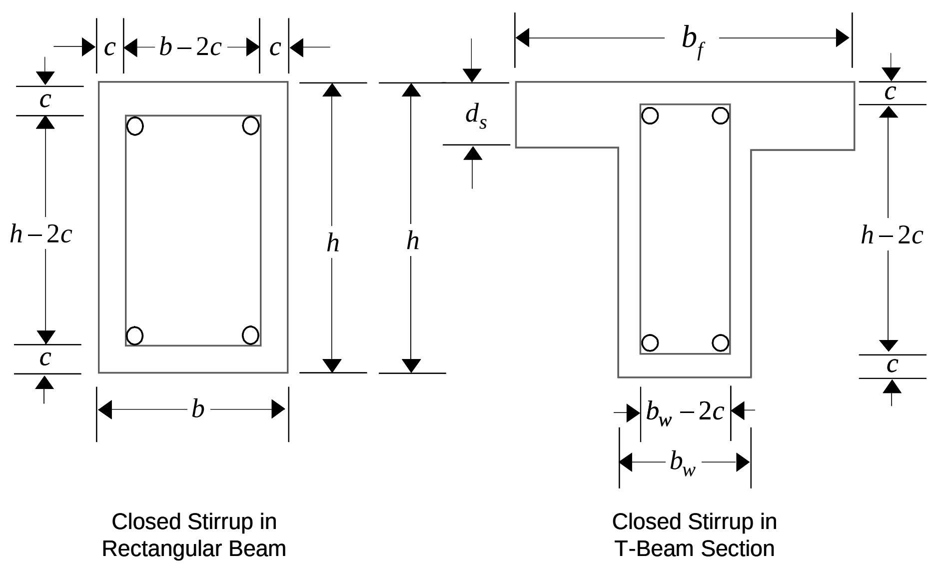


Figure 15-3 Closed stirrup and section dimensions for torsion design

- If $v_t > v_{t,\min}$, the required closed stirrup area per unit spacing, $A_{sv,t} / s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8 x_1 y_1 \left(0.87 f_{yv}\right)} \quad \text{(CP 6.3.6, Eqn. 6.67)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t} f_{yv} \left(x_1 + y_1\right)}{s_v f_y} \quad \text{(CP 6.3.6, Eqn. 6.68)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link, and y_1 is the larger center-to-center dimension of the closed link.

An upper limit of the combination of v and v_t that can be carried by the section also is checked using the equation:

$$v + v_t \leq \min\left(0.8\sqrt{f_{cu}}, 7 \text{ N/mm}^2\right) \quad \text{(CP 6.3.4)}$$

If the combination of v and v_t exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

15.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is performed based on the ultimate strength design method (CP-2013) for reinforced concrete, as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

15.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

15.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

15.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

15.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required in each direction of a slab is given by the following limits (CP 9.3.1.1), with interpolation for reinforcement of intermediate strength:

$$A_s \geq \begin{cases} 0.0024bh & \text{if} \quad f_y \leq 250 \text{ MPa} \\ 0.0013bh & \text{if} \quad f_y \geq 460 \text{ MPa} \end{cases} \qquad \text{(CP 9.3.1.1(a))}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 9.2.1.3).

15.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

15.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of 1.5d from the face of the support (CP 6.1.5.7(d)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CP 6.1.5.7). Figure 15-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

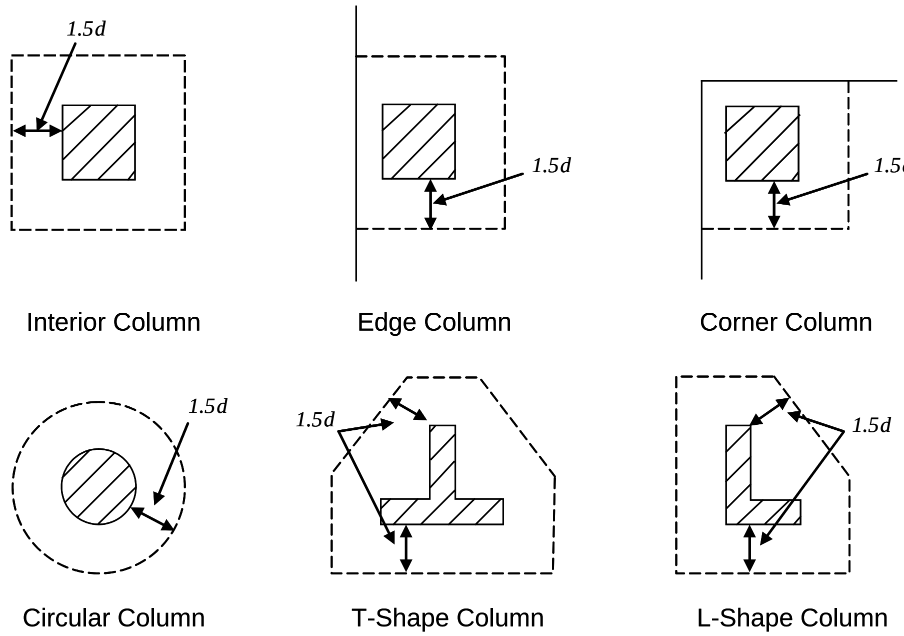


Figure 15-4 Punching Shear Perimeters

15.6.2.2 Determine Concrete Capacity

The concrete punching shear factored strength is taken as (CP 6.1.5.7(d), Table 6.3):

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} \qquad \text{(CP 6.1.2.5(d), Table 6.3)}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 (CP 6.1.2.5(g), 6.1.5.7(d))

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} \quad 1 \le k_2 \le \left(\frac{80}{25}\right)^{1/3} \qquad \text{(CP 6.1.2.5(c), Table 6.3)}$$

$$\gamma_m = 1.25 \qquad \text{(CP 2.4.3.2, Table 2.2)}$$

However, the following limitations also apply:

$$0.15 \le \frac{100A_s}{bd} \le 3, \qquad \text{(CP 6.1.2.5(c), Table 6.3)}$$

$$\left(\frac{400}{d}\right)^{1/4} \ge \begin{cases} 0.67, & \text{Members without shear reinforcement} \\ 1.00, & \text{Members with shear reinforcement} \end{cases}$$

(CP 6.1.2.5(c), Table 6.3)

A_s = area of tension reinforcement, which is taken as the average tension reinforcement of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement is taken as zero in the current implementation.

$$v \le \min(0.8\sqrt{f_{cu}}\text{, 7 MPa}) \qquad \text{(CP 6.1.5.7(b))}$$

$f_{cu} \le 80$ MPa (for calculation purpose only) (CP Table 6.3)

15.6.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, $v_{\max}$, is calculated as:

$$V_{eff,x} = V\left(f + \frac{1.5M_x}{Vy}\right) \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

$$V_{eff.y} = V\left(f + \frac{1.5M_y}{Vx}\right) \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

$$v_{\max} = \max\begin{cases} \dfrac{V_{eff,x}}{u\,d} \\ \dfrac{V_{eff,y}}{u\,d} \end{cases} \qquad \text{(CP 6.1.5.7)}$$

where,

u is the perimeter of the critical section,

x and *y* are the lengths of the sides of the critical section parallel to the axis of bending,

M_x and M_y are the design moments transmitted from the slab to the column at the connection,

V is the total punching shear force, and

f is a factor to consider the eccentricity of punching shear force and is taken as

$$f = \begin{cases} 1.00 & \text{for interior columns} \\ 1.25 & \text{for edge columns} \\ 1.25 & \text{for corner columns} \end{cases} \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

15.6.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

15.6.3 Design Punching Shear Reinforcement

The use of shear links as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 6.1.5.7(e)). The use of shear studs is not covered in Hong Kong CP 2013. However, program uses the identical clauses for shear studs when CP 20013 code is selected. If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

15.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

15.6.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum of:

$$v_{\max} = 2v_c \qquad \text{(CP 6.1.5.7(e))}$$

Given v, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows (CP 6.1.5.7(e)).

- If $v \leq 1.6v_c$,

$$\frac{A_v}{s} = \frac{(v - v_c)ud}{0.87\, f_{yv}} \geq \frac{v_r ud}{0.87\, f_{yv}}, \qquad \text{(CP 6.1.5.7(e))}$$

- If $1.6v_c \leq v < 2.0v_c$,

$$\frac{A_v}{s} = \frac{5(0.7v - v_c)ud}{0.87\, f_{yv}} \geq \frac{v_r ud}{0.87\, f_{yv}}, \qquad \text{(CP 6.1.5.7(e))}$$

$$v_r = \begin{cases} 0.4 \\ 0.4\left(\dfrac{f_{cu}}{40}\right)^{2/3} \end{cases} \text{N/mm}^2 \qquad \text{(CP 6.1.5.7, Table 6.2)}$$

- If $v > 2.0v_c$, a failure condition is declared. (CP 6.1.5.7(e))

If v exceeds the maximum permitted value of $v_{\max}$, the concrete section should be increased in size.

15.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 15-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 1.5*d* measured in a direction parallel to the column face (CP 6.1.5.7(f)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

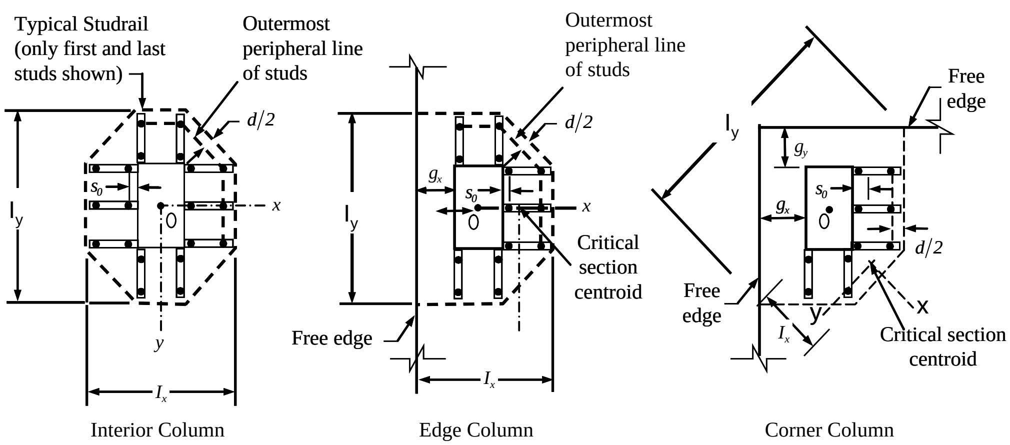


Figure 15-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

15.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 4.2.4 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 1.5d. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(CP 6.1.5.7(f))}$$

$$s \leq 0.75d \qquad \text{(CP 6.1.5.7(f))}$$

$$g \leq 1.5d \qquad \text{(CP 6.1.5.7(f))}$$

Stirrups are only permitted when slab thickness is greater than 200 mm (CP 6.1.5.7(e)).

Chapter 16
Design for ACI 318-14

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the American code ACI 318-14 [ACI 2014] is selected. Various notations used in this chapter are listed in Figure 16-1. For referencing to the pertinent sections or equations of the ACI code in this chapter, a prefix "ACI" followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

16.1 Notations

Table 16-1 List of Symbols Used in the ACI 318-14 Code

A_{cp}	Area enclosed by the outside perimeter of the section, sq-in
A_g	Gross area of concrete, sq-in

Table 16-1 List of Symbols Used in the ACI 318-14 Code

A_l	Area of longitudinal reinforcement for torsion, sq-in
A_o	Area enclosed by the shear flow path, sq-in
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in
A_s	Area of tension reinforcement, sq-in
A'_s	Area of compression reinforcement, sq-in
A_t /s	Area of closed shear reinforcement per unit length of member for torsion, sq-in/in
A_v	Area of shear reinforcement, sq-in
A_v /s	Area of shear reinforcement per unit length, sq-in/in
a	Depth of compression block, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of section, in
b_f	Effective width of flange (flanged section), in
b_o	Perimeter of the punching shear critical section, in
b_w	Width of web (flanged section), in
b_1	Width of the punching shear critical section in the direction of bending, in
b_2	Width of the punching shear critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in
d	Distance from compression face to tension reinforcement, in
d'	Distance from compression face to compression reinforcement, in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, psi
f'_c	Specified compressive strength of concrete, psi

Table 16-1 List of Symbols Used in the ACI 318-14 Code

f'_s	Stress in the compression reinforcement, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{yt}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
h_f	Height of the flange, in
M_u	Factored moment at a section, lb-in
N_u	Factored axial load at a section occurring simultaneously with V_u or T_u, lb
P_u	Factored axial load at a section, lb
p_{cp}	Outside perimeter of concrete cross-section, in
p_h	Perimeter of centerline of outermost closed transverse torsional reinforcement, in
s	Spacing of shear reinforcement along the beam, in
T_{cr}	Critical torsion capacity, lb-in
T_u	Factored torsional moment at a section, lb-in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_s	Shear force resisted by transverse reinforcement, lb
V_u	Factored shear force at a section, lb
α_s	Punching shear scale factor based on column location
β_c	Ratio of the maximum to the minimum dimensions of the punching shear critical section
β_1	Factor for obtaining depth of the concrete compression block
ε_c	Strain in the concrete
$\varepsilon_{c\,\max}$	Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)

Table 16-1 List of Symbols Used in the ACI 318-14 Code

ε_s	Strain in the reinforcement
$\varepsilon_{s,min}$	Minimum tensile strain allowed in the reinforcement at nominal strength for tension controlled behavior (0.005 in/in)
ϕ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
λ	Shear strength reduction factor for light-weight concrete
θ	Angle of compression diagonals, degrees

16.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-14, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (ACI 5.3.1):

$$1.4D \quad \text{(ACI Eqn. 5.3.1a)}$$

$$1.2D + 1.6L + 0.5L_r \quad \text{(ACI Eqn.5.3.1b)}$$
$$1.2D + 1.0L + 1.6L_r \quad \text{(ACI Eqn.5.3.1c)}$$

$$1.2D + 1.6(0.75\ PL) + 0.5L_r \quad \text{(ACI Eqn.5.3.1b, 6.4)}$$
$$1.2D + 1.6L + 0.5S \quad \text{(ACI Eqn.5.3.1b)}$$
$$1.2D + 1.0L + 1.6S \quad \text{(ACI Eqn.5.3.1c)}$$

$$0.9D \pm 1.0W \quad \text{(ACI Eqn.5.3.1f)}$$
$$1.2D + 1.0L + 0.5L_r \pm 1.0W \quad \text{(ACI Eqn.5.3.1d)}$$
$$1.2D + 1.6L_r \pm 0.5W \quad \text{(ACI Eqn.5.3.1c)}$$

$$1.2D + 1.6S \pm 0.5W \quad \text{(ACI Eqn.5.3.1c)}$$
$$1.2D + 1.0L + 0.5S \pm 1.0W \quad \text{(ACI Eqn.5.3.1d)}$$

0.9D ± 1.0E (ACI Eqn.5.3.1g)
1.2D + 1.0L + 0.2S ± 1.0E (ACI Eqn.5.3.1e)

These are the default design load combinations in SAFE whenever the ACI 318-14 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

16.3 Limits on Material Strength

The concrete compressive strength, f'_c, should not be less than 2,500 psi (ACI 19.2.1, Table 19.2.1.1). The upper limit of the reinforcement yield strength, f_y, is taken as 80 ksi (ACI 20.2.2.4a, Table 20.2.2.4a) and the upper limit of the reinforcement shear strength, f_{yt}, is taken as 60 ksi (ACI 21.2.2.4a, Table 21.2.2.4a).

If the input f'_c is less than 2,500 psi, SAFE continues to design the members based on the input f'_c and does not warn the user about the violation of the code. The user is responsible for ensuring that the minimum strength is satisfied.

16.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied to the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

ϕ = 0.90 for flexure (tension controlled) (ACI 21.2.1, Table 21.2.1)

ϕ = 0.75 for shear and torsion (ACI 21.2.1, Table 21.2.1)

These values can be overwritten; however, caution is advised.

16.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

16.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

16.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Calculation of bottom reinforcement is based on positive beam moments. In such cases, the beam may be designed as a rectangular or flanged beam. Calculation of top reinforcement is based on negative beam moments. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

16.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete. Note that the flexural reinforcement strength, f_y, is limited to 80 ksi (ACI 20.2.2.4a), even if the material property is defined using a higher value.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 16-1 (ACI 22.2.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 21.2.2, Table 21.2.2) when the concrete in compression reaches its assumed strain limit of 0.003. When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed $\left(0.1f'_cA_g\right)$ (ACI 9.5.2.1), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $\left(0.1f'_cA_g\right)$ and axial tensions are always included in flexural and shear design.

16.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M_u (i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 16-1), where,

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f'_c\, \phi b}} \qquad \text{(ACI 22.2.2)}$$

and the value of ϕ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 21.2.2, Table 21.2.2):

$$c_{max} = \frac{\varepsilon_{cmax}}{\varepsilon_{cmax} + \varepsilon_{smin}} d \qquad \text{(ACI 22.2.1.2)}$$

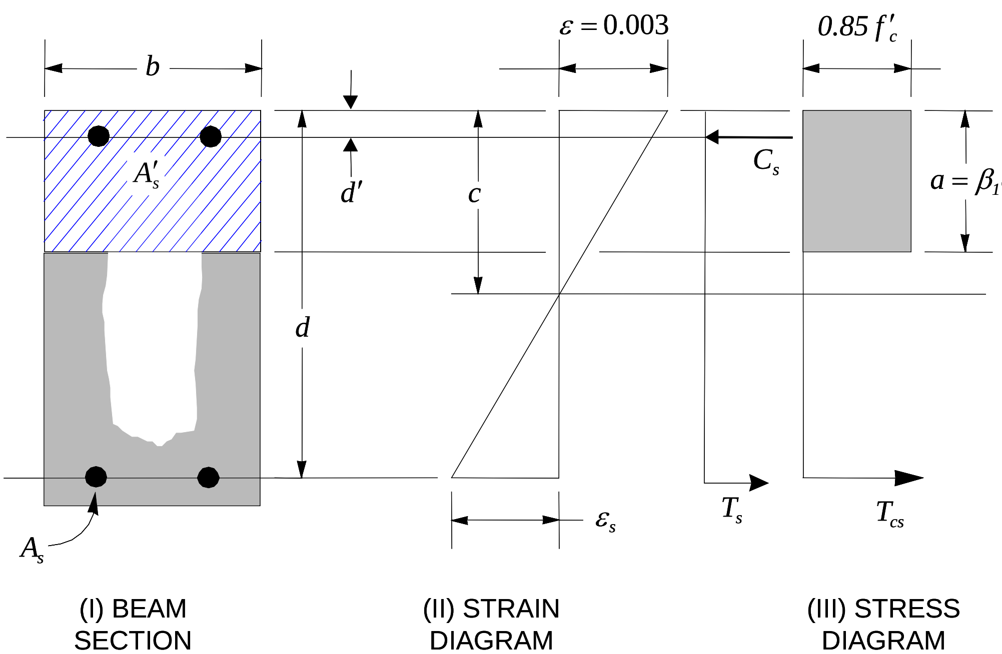


Figure 16-1 Rectangular Beam Design

where,

$\varepsilon_{cmax} = 0.003$ (ACI 21.2.2, Fig R21.2)

$\varepsilon_{smin} = 0.005$ (ACI 21.2.2, Fig R21.2.26)

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 22.2.2.4.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 22.2.2.4.3)}$$

- If $a \le a_{\max}$ (ACI 10.3.4), the area of tension reinforcement is then given by:

$$A_s = \frac{M_u}{\phi\, f_y\left(d - \frac{a}{2}\right)}$$

This reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

- If $a > a_{\max}$, compression reinforcement is required (ACI 9.3.3.1, 21.2.2, Fig 21.2.26, 22.2.2.4.1) and is calculated as follows:

− The compressive force developed in the concrete alone is given by:

$$C = 0.85 f'_c\, b a_{\max} \qquad \text{(ACI 22.2.2.4.1)}$$

and the moment resisted by concrete compression and tension reinforcement is:

$$M_{uc} = \phi C\left(d - \frac{a_{\max}}{2}\right)$$

− Therefore the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - M_{uc}$$

− The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\phi\left(f'_s - 0.85 f'_c\right)\left(d - d'\right)}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y \quad \text{(ACI 9.2.1.2, 9.5.2.1, 20.2.2, 22.2.1.2)}$$

− The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_{uc}}{\phi f_y \left[d - \frac{a_{\max}}{2} \right]}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{us}}{\phi f_y \left(d - d' \right)}$$

Therefore, the total tension reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

16.5.1.2.2 Design of Flanged Beams

In designing a flanged beam, a simplified stress block, as shown in Figure 16-2, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 16-1 is assumed on the compression side.

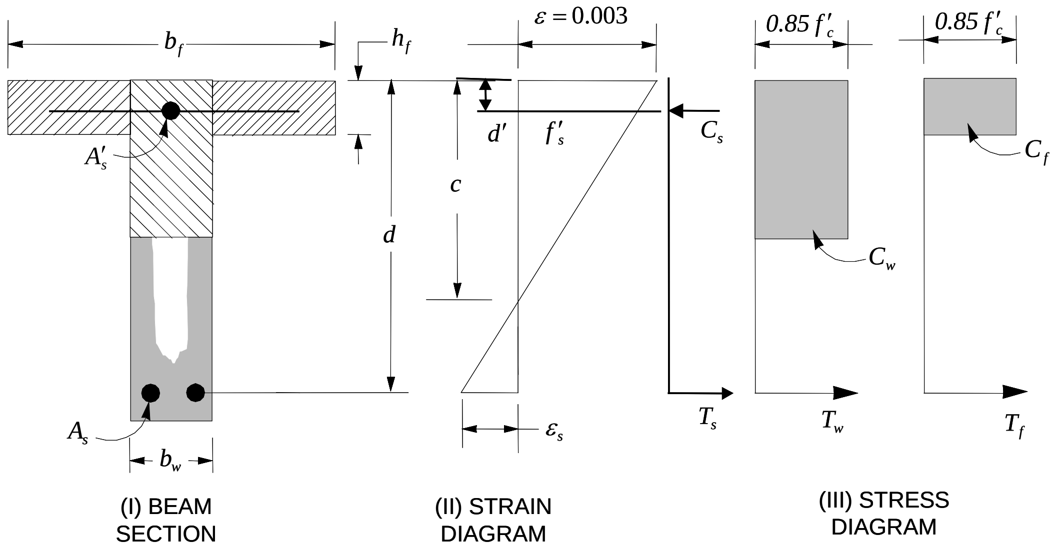


Figure 16-2 T-Beam Design

16.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

16.5.1.2.2.2 Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2 M_u}{0.85 f'_c \phi b_f}} \qquad \text{(ACI 22.2)}$$

where, the value of ϕ is taken as that for a tension-controlled section, which by default is 0.90 (ACI 21.2.1, 21.2.2, Table 21.2.1, Table 21.2.2) in the preceding and the following equations.

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension controlled behavior (ACI 9.3.3.1, 21.2.2, Fig 21.2.26):

$$c_{\max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} d \qquad \text{(ACI 22.2.1.2)}$$

where,

$\varepsilon_{c\max} = 0.003$ (ACI 21.2.2, Fig 21.2.26)

$\varepsilon_{s\min} = 0.005$ (ACI 21.2.2, Fig 21.2.26)

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \beta_1 c_{\max} \qquad \text{(ACI 22.2.2.4.11)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \quad \text{(ACI 22.2.2.4.3)}$$

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_{max}$.

- If $a > h_f$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 16-2. C_f is given by:

$$C_f = 0.85 f'_c \left(b_f - b_w\right) \min\left(h_f, a_{\max}\right) \qquad \text{(ACI 22.2.2.4.1)}$$

Therefore, $A_{s1} = \frac{C_f}{f_y}$ and the portion of M_u that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min\left(h_f, a_{\max}\right)}{2} \right)$$

Again, the value for ϕ is 0.90 by default. Therefore, the balance of the moment, M_u , to be carried by the web is:

$M_{uw} = M_u - M_{uf}$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi b_w}} \quad \text{(ACI 22.2)}$$

- If $a_1 \leq a_{\max}$ (ACI 9.3.3.1, 21.2.2), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$A_s = A_{s1} + A_{s2}$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_{\max}$, compression reinforcement is required (ACI 9.3.3.1, 21.2.2, Fig 21.2.2b, 22.2.2.4.1) and is calculated as follows:

− The compressive force in the web concrete alone is given by:

$$C_w = 0.85 f'_c b_w a_{\max} \quad \text{(ACI 22.2.2.4.1)}$$

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{uc} = C_w \left(d - \frac{a_{\max}}{2} \right) \phi$$

and the moment resisted by compression and tension reinforcement is:

$M_{us} = M_{uw} - M_{uc}$

Therefore, the compression reinforcement is computed as:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c \right)\left(d - d' \right) \phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max} \left[\frac{c_{\max} - d'}{c_{\max}} \right] \leq f_y \qquad \text{(ACI 9.2.1.2, 9.5.2.1, 20.2.2, 22.2.1.2)}$$

The tension reinforcement for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{uc}}{f_y \left[d - \frac{a_{\max}}{2} \right] \phi}$$

and the tension reinforcement for balancing the compression reinforcement is:

$$A_{s3} = \frac{M_{us}}{f_y (d - d') \phi}$$

The total tension reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

16.5.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the two following limits:

$$A_{s,\min} = \max \left(\frac{3\sqrt{f'_c}}{f_y} b_w d, \frac{200}{f_y} b_w d \right) \qquad \text{(ACI 9.6.1.2)}$$

$$A_s \geq \frac{4}{3} A_{s(\text{required})} \qquad \text{(ACI 9.6.1.3)}$$

For T-beam in negative moment b_w in the above expression is substituted by b_w', where:

$$b_w' = min\{b_f, 2b_w\} \qquad \text{(ACI 9.6.1.2)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \le \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \le \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

16.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

16.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

16.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = 2\lambda\sqrt{f'_c}\,b_w d \qquad \text{(ACI 22.5.5.1)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as $f'_c \le 100$ (ACI 22.5.3.1)

The value of λ should be specified in the material property definition.

16.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = V_c + \left(8\sqrt{f'_c}\right) b_w d \qquad \text{(ACI 22.5.1.2)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3). The flexural reinforcement strength, f_{yt}, is limited to 60 ksi (ACI 11.5.2) even if the material property is defined with a higher value.

- If $V_u \leq 0.5\phi V_c$,

$$\frac{A_v}{s} = 0 \qquad \text{(ACI 9.6.3.1)}$$

- If $0.5\phi V_c < V_u \leq \phi V_{\max}$,

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi\, f_{yt} d} \qquad \text{(ACI 22.5.1.1, 22.5.10.1, 20.5.10.5.3)}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared. (ACI 22.5.1.2)

If V_u exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size (ACI 22.5.1.2).

Note that if torsion design is considered and torsion reinforcement is required, the equation given in ACI 9.6.3.3 does not need to be satisfied independently. See the subsequent section *Design of Beam Torsion Reinforcement* for details.

If the beam depth h is

$h \leq 10"$ for rectangular,

$h \leq \min\left\{24", \max\left(2.5h_f, \frac{b}{2}\right)\right.$ for T-beam, the minimum shear reinforcement given by ACI 9.6.3.3 does not need to be enforced.

$$\frac{A_v}{s} \geq \max\left(\frac{0.75\sqrt{f'_c}}{f_{yt}}b_w, \quad \frac{50b_w}{f_{yt}}\right) \qquad \text{(ACI 9.6.3.3, Table 9.6.3.3)}$$

The maximum of all of the calculated A_v/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

16.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_u.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

16.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 9.4.4.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with the code (ACI 22.7.3.3). However, the program does not automatically redistribute the internal forces and reduce T_u. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

16.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{cp}, A_{oh}, A_o, p_{cp}, and p_h, are calculated. These properties are described in the following (ACI 2.2).

A_{cp} = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_{cp} = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to a 1.5 inch clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{cp} = bh \quad \text{(ACI 2.2, R22.7.5)}$$

$$A_{oh} = (b - 2c)(h - 2c) \quad \text{(ACI 2.2, R22.7, Fig R22.7.6.1.1)}$$

$$A_o = 0.85\, A_{oh} \quad \text{(ACI 22.7.6.1.1, Fig R22.7.6.1.1)}$$

$$p_{cp} = 2b + 2h \quad \text{(ACI 2.2, R22.7.5)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \quad \text{(ACI 22.7.6.1.1, Fig R22.7.6.1.1)}$$

where, the section dimensions b, h, and c are shown in Figure 16-3. Similarly, the special section properties for a flanged beam section are given as:

A_{cp} = $b_w h + (b_f - b_w)h_f$ (ACI 2.2, R22.7.5)

A_{oh} = $(b_w - 2c)(h - 2c)$ (ACI 2.2, R22.7, Fig R22.7.6.1.1)

A_o = $0.85\, A_{oh}$ (ACI 22.7.6.1.1, Fig R22.7.6.1.1)

p_{cp} = $2b_f + 2h$ (ACI 2.2, R22.7.5)

p_h = $2(h - 2c) + 2(b_w - 2c)$ (ACI 2.2, R22.7.5)

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 16-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h - h_f)$.

16.5.3.3 Determine Critical Torsion Capacity

The threshold torsion limit, T_{th}, and the cracking torsion limits, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{th} = \lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$ (ACI 22.7.4.1, Table 22.7.4.1a)

$$T_{th} = 4\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$ (ACI 22.7.5.1g, Table 22.7.5.2)

where A_{cp} and p_{cp} are the area and perimeter of the concrete cross-section as described in detail in the previous section; f_{pc} is the concrete compressive stress at the centroid of the section; ϕ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 21.2.1g, Table 21.2.1c); and f'_c is the specified concrete compressive strength.

16.5.3.4 Determine Torsion Reinforcement

If the factored torsion T_u is less than the threshold limit, T_{cr}, torsion can be safely ignored (ACI 22.7.1.1, 9.6.4.1). In that case, the program reports that no torsion reinforcement is required. However, if T_u exceeds the threshold limit,

T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI 22.7.1, 22.7.6.1). Note that the longitudinal reinforcement strength, f_y, and the transverse reinforcement strength, f_{yt}, are limited to 60 ksi (ACI 20.2.2.4a), even if the material property is defined with a higher value.

If $T_u > T_{cr}$, the required closed stirrup area per unit spacing, A_t/s, is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan\theta}{\phi 2 A_o f_{yt}} \qquad \text{(ACI 22.7.6.1)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{T_u p_h}{\phi 2 A_o f_y \tan\theta} \qquad \text{(ACI 22.7.6.1)}$$

where, the minimum value of A_t/s is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{yt}} b_w \qquad \text{(ACI 22.7.6.1)}$$

the minimum value of A_l is taken as the least of the following:

$$A_{l,\min} \geq \frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{ys}}{f_y}\right) \qquad \text{(ACI 9.6.4.3a)}$$

$$A_{l,\min} = \frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{25}{f_{ys}} b_w\right) p_h \left(\frac{f_{ys}}{f_y}\right) \qquad \text{(ACI 9.6.4.3b)}$$

In the preceding expressions, θ is taken as 45 degrees. The code allows any value between 30 and 60 degrees.

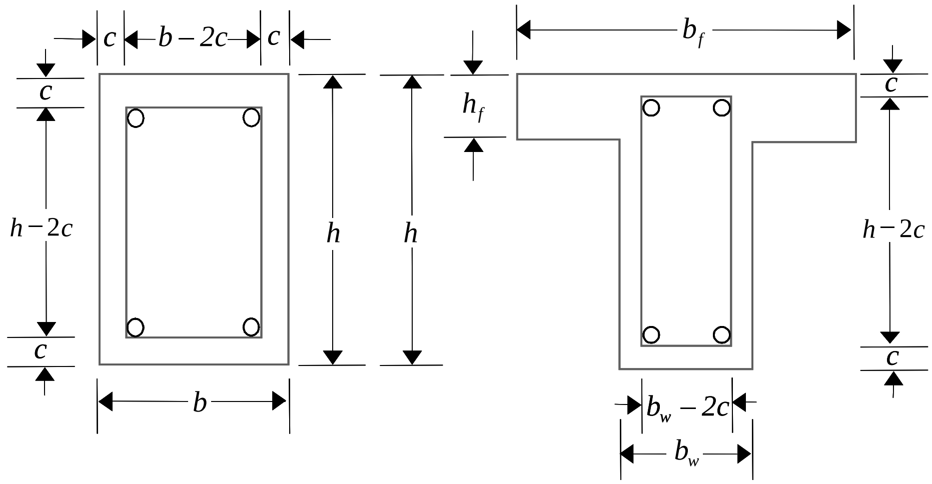


Figure 16-3 Closed stirrup and section dimensions for torsion design

An upper limit of the combination of V_u and T_u that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f'_c}\right) \qquad \text{(ACI 22.7.7.1a)}$$

For rectangular sections, b_w is replaced with b. If the combination of V_u and T_u exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s}+2\frac{A_t}{s}\right) \ge \max\left\{0.75\frac{\sqrt{f'_c}}{f_{ys}}b_w, \frac{50}{f_y}b_w\right\} \qquad \text{(ACI 9.6.4.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition. In that case, A_v/s does not need to satisfy the ACI Section 9.6.6.3 independently.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

16.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (ACI 318-14) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

16.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.

- Design flexural reinforcement for the strip.

These two steps, described in the text that follows, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

16.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

16.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This is the method used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

16.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.6.1.1, 8.6.1.1):

$$A_{s,\min} = 0.0020\, bh \text{ for } f_y < 60 \text{ ksi} \qquad \text{(ACI Table 7.6.1.1, Table 8.6.1.1)}$$

$$A_{s,\min} = 0.0018\, bh \text{ for } f_y = 60 \text{ ksi} \qquad \text{(ACI Table 7.6.1.1, Table 8.6.1.1)}$$

$$A_{s,\min} = \frac{0.0018 \times 60000}{f_y} bh \text{ for } f_y > 60 \text{ ksi} \qquad \text{(ACI Table 7.6.1.1, Table 8.6.1.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

16.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following sections.

16.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (ACI 22.6.4.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 22.6.4.3). Figure 16-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge or corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

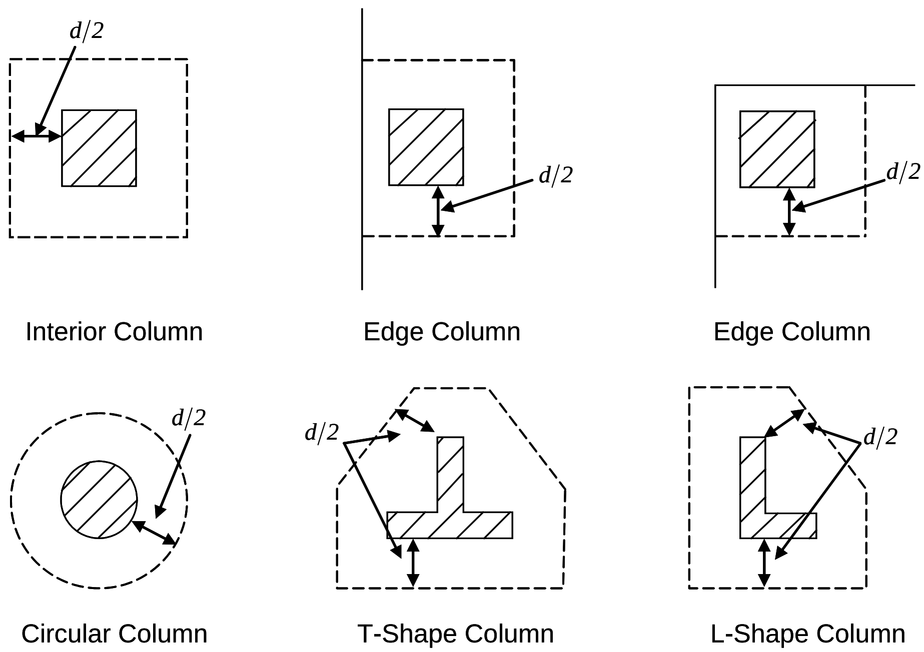


Figure 16-4 Punching Shear Perimeters

16.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_{sc}$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_{sc}$.

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(ACI 8.4.2.3)}$$

$$\gamma_v = 1 - \gamma_f \qquad \text{(ACI 8.4.4.2.2)}$$

For reinforced concrete slabs, γ_f is permitted to increase to the maximum modified values provided in ACI Table 8.4.2.3.4 provided that the limitations on v_{ug} and ε_t given in ACI Table 8.4.2.3.4 are satisfied .

Column Location	Span Direction	v_{ug}	ε_t	Maximum modified γ_f
Corner column	Either direction	$\leq 0.5\phi v_c$	≥ 0.004	1.0
Edge column	Perpendicular to the edge	$\leq 0.75\phi v_c$	≥ 0.004	1.0
	Parallel to the edge	$\leq 0.4\phi v_c$	≥ 0.010	$\gamma_f = \frac{1.25}{1+(2/3)\sqrt{b_1/b_2}} \leq 1.0$
Interior column	Either direction	$\leq 0.4\phi v_c$	≥ 0.010	$\gamma_f = \frac{1.25}{1+(2/3)\sqrt{b_1/b_2}} \leq 1.0$

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span.

16.6.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \left(2+\dfrac{4}{\beta_c}\right)\lambda\sqrt{f'_c} \\ \left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda\sqrt{f'_c} \\ 4\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 22.6.5.2)}$$

where, β_c is the ratio of the maximum to the minimum dimensions of the critical section, b_o is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 22.6.65.3)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as:

$$\sqrt{f'_c} \leq 100 \qquad \text{(ACI 22.5.3.1)}$$

16.6.2.4 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

$$v_U = \frac{V_U}{b_0 d} + \frac{\gamma_{V2}[M_{U2} - V_U(y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^{\,2}} - \frac{\gamma_{V3}[M_{U3} - V_U(x_3 - x_1)][I_{22}(x_4 - x_3) - I_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}^{\,2}} \qquad \text{Eq. 1}$$

$$I_{22} = \sum_{sides=1}^{n} \overline{I}_{22}, \text{ where "}sides\text{" refers to the sides of the critical section for punching shear} \qquad \text{Eq. 2}$$

$$I_{33} = \sum_{sides=1}^{n} \overline{I}_{33}, \text{ where "\textit{sides}" refers to the sides of the critical section for punching shear} \qquad \text{Eq. 3}$$

$$I_{23} = \sum_{sides=1}^{n} \overline{I}_{23}, \text{ where "\textit{sides}" refers to the sides of the critical section for punching shear} \qquad \text{Eq. 4}$$

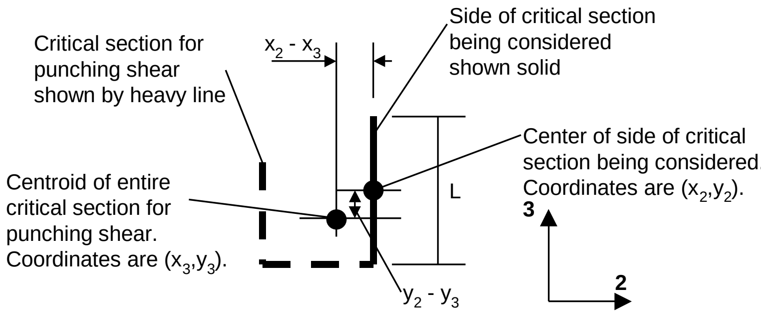
The equations for $\overline{I}_{22}, \overline{I}_{33}$, and $\overline{I}_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figure 16-5.

$$\overline{I}_{22} = Ld(y_2 - y_3)^2, \text{ for the side of the critical section parallel to the 2-axis} \qquad \text{Eq. 5a}$$

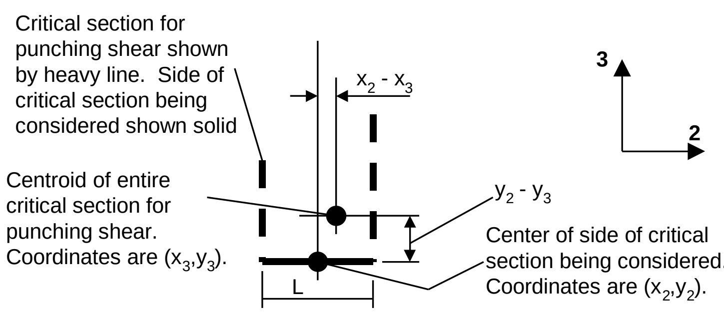
$$\overline{I}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2, \text{ for the side of the critical section parallel to the 3-axis} \qquad \text{Eq. 5b}$$

$$\overline{I}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2, \text{ for the side of the critical section parallel to the 2-axis} \qquad \text{Eq. 6a}$$

$$\overline{I}_{33} = Ld(x_2 - x_3)^2, \text{ for the side of the critical section parallel to the 3-axis} \qquad \text{Eq. 6b}$$



Plan View For Side of Critical Section Parallel to 3-Axis
Work This Sketch With Equations 5b, 6b and 7



Plan View For Side of Critical Section Parallel to 2-Axis
Work This Sketch With Equations 5a, 6a and 7

Figure 16-5 Shear Stress Calculations at Critical Sections

$$\bar{I}_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \qquad \text{Eq. 7}$$

NOTE: $\bar{I}_{23}$ is explicitly set to zero for corner condition.

where,

b_0 = Perimeter of the critical section for punching shear

d = Effective depth at the critical section for punching shear based on the average of d for 2 direction and d for 3 direction

I_{22} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis

I_{33} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis

I_{23} = Product of the inertia of the critical section for punching shear with respect to the 2 and 3 planes

L = Length of the side of the critical section for punching shear currently being considered

M_{U2} = Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)

M_{U3} = Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)

v_U = Punching shear stress

V_U = Shear at the center of the column (positive upward)

x_1, y_1= Coordinates of the column centroid

x_2, y_2= Coordinates of the center of one side of the critical section for punching shear

x_3, y_3= Coordinates of the centroid of the critical section for punching shear

x_4, y_4= Coordinates of the location where stress is being calculated

γ_{V2} = Percent of M_{U2} resisted by shear

γ_{V3} = Percent of M_{U3} resisted by shear

16.6.2.5 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

16.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 22.6.7.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

16.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

$$v_c \le 2\lambda\sqrt{f'_c} \text{ for shear links} \qquad \text{(ACI 22.6.6.1)}$$

$$v_c \le 3\lambda\sqrt{f'_c} \text{ for shear studs} \qquad \text{(ACI 22.6.6.1)}$$

16.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = 6\sqrt{f'_c}\, b_o d \text{ for shear links} \qquad \text{(ACI 22.6.6.2)}$$

$$V_{\max} = 8\sqrt{f'_c}\, b_o d \text{ for shear studs} \qquad \text{(ACI 22.6.6.2)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi f_{ys} d} \qquad \text{(ACI 22.5.1.1, 22.5.10.1, 20.5.10.5.3)}$$

$$\frac{A_v}{s} \geq \frac{2\sqrt{f'_c}b_o}{f_y} \quad \text{for shear studs}$$

- If $V_u > \phi V_{max}$, a failure condition is declared. (ACI 22.5.1.2)
- If V_u exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size.

16.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 16-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

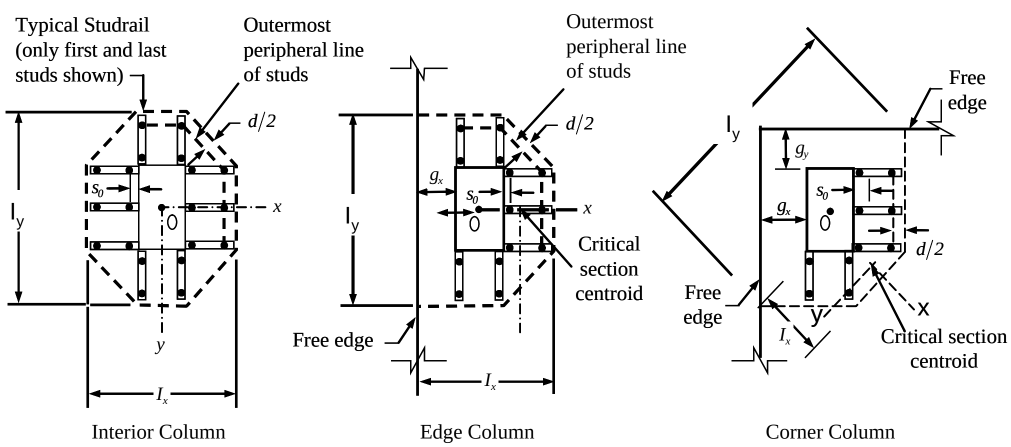


Figure 16-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$(ACI 8.7.6.3, Table 8.7.6.3) and the spacing between shear reinforcement shall not exceed $d/2$(ACI 8.7.6.3, Table 8.7.6.3). The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (ACI 8.7.6.3, Table 8.7.6.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

16.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in 20.6.1.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 2d, and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 8.7.7.1.2)}$$

$$s \leq \begin{cases} 0.75d & \text{for} \quad v_u \leq 6\phi\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\sqrt{f'_c} \end{cases} \qquad \text{(ACI 8.7.7.1.2)}$$

$$g \leq 2d \qquad \text{(ACI 8.7.7.1.2)}$$

The limits of s_o and the spacing, s, between for the links are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 8.7.6.3)}$$

$$s \leq 0.50d \qquad \text{(ACI 8.7.6.3)}$$

Chapter 17
Design for CSA A23.3-14

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the Canadian code CSA A23.3-14 [CSA 14] is selected. Various notations used in this chapter are listed in Table 5-1. For referencing to the pertinent sections of the Canadian code in this chapter, a prefix "CSA" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

17.1 Notations

Table 17-1 List of Symbols Used in the CSA A23.3-14 Code

A_c	Area enclosed by outside perimeter of concrete cross-section, sq-mm
A_{ct}	Area of concrete on flexural tension side, sq-mm

Table 17-1 List of Symbols Used in the CSA A23.3-14 Code

A_l	Area of longitudinal reinforcement for torsion, sq-mm
A_o	Gross area enclosed by shear flow path, sq-mm
A_{oh}	Area enclosed by centerline of outermost closed transverse torsional reinforcement, sq-mm
A_s	Area of tension reinforcement, sq-mm
A'_s	Area of compression reinforcement, sq-mm
$A_{s(\text{required})}$	Area of steel required for tension reinforcement, sq-mm
A_t / s	Area of closed shear reinforcement for torsion per unit length, sq-mm/mm
A_v	Area of shear reinforcement, sq-mm
A_v / s	Area of shear reinforcement per unit length, sq-mm/mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
b	Width of member, mm
b_f	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
c_b	Depth to neutral axis at balanced conditions, mm
d	Distance from compression face to tension reinforcement, mm
d_v	Effective shear depth, mm
d'	Distance from compression face to compression reinforcement, mm
h_s	Thickness of slab (flanged section), mm
E_c	Modulus of elasticity of concrete, MPa

Table 17-1 List of Symbols Used in the CSA A23.3-14 Code

E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f'_c	Specified compressive strength of concrete, MPa
f'_s	Stress in the compression reinforcement, psi
f_y	Specified yield strength of flexural reinforcement, MPa
f_{yt}	Specified yield strength of shear reinforcement, MPa
h	Overall depth of a section, mm
I_g	Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.
M_f	Factored moment at section, N-mm
N_f	Factored axial force at section, N
p_c	Outside perimeter of concrete cross-section, mm
p_h	Perimeter of area A_{oh}, mm
s	Spacing of the shear reinforcement along the length of the beam, mm
s_z	Crack spacing parameter
T_f	Factored torsion at section, N-mm
V_c	Shear resisted by concrete, N
$V_{r,\max}$	Maximum permitted total factored shear force at a section, N
V_f	Factored shear force at a section, N
V_s	Shear force at a section resisted by steel, N
α_1	Ratio of average stress in rectangular stress block to the specified concrete strength
β	Factor accounting for shear resistance of cracked concrete
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
ε_s	Strain in reinforcing steel

Table 17-1 List of Symbols Used in the CSA A23.3-14 Code

ε_x	Longitudinal strain at mid-depth of the section
ϕ_c	Strength reduction factor for concrete
ϕ_s	Strength reduction factor for steel
ϕ_m	Strength reduction factor for member
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
θ	Angle of diagonal compressive stresses, degrees
λ	Shear strength factor

17.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For CSA A23.3-14, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (CSA 8.3.2, Table C.1a)

1.4D (CSA 8.3.2, Table C.1a Case 1)

1.25D + 1.5L
1.25D + 1.5L + 1.0S
1.25D + 1.5L ± 0.4W
0.9D + 1.5L (CSA 8.3.2, Table C.1a Case 2)
0.9D + 1.5L + 1.0S
0.9D + 1.5L ± 0.4W

1.25D + 1.5(0.75 PL) (CSA 13.8.4.3)

1.25D + 1.5S
1.25D + 1.5S + 0.5L
1.25D + 1.5S ± 0.4W (CSA 8.3.2, Table C.1a Case 3)
0.9D + 1.5S
0.9D + 1.5S + 0.5L

$0.9D + 1.5S \pm 0.4W$

$1.25D \pm 1.4W$
$1.25D + 0.5L \pm 1.4W$
$1.25D + 0.5S \pm 1.4W$
$0.9D \pm 1.4W$ (CSA 8.3.2, Table C.1a Case 4)
$0.9D + 0.5L \pm 1.4W$
$0.9D + 0.5S \pm 1.4W$

$1.0D \pm 1.0E$
$1.0D + 0.5L \pm 1.0E$
$1.0D + 0.25S \pm 1.0E$ (CSA 8.3.2, Table C.1a Case 5)
$1.0D + 0.5L + 0.25S \pm 1.0E$

These are also the default design load combinations in SAFE whenever the CSA A23.3-14 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

17.3 Limits on Material Strength

The upper and lower limits of f'_c are 80 MPa and 20 MPa, respectively, for all framing types (CSA 8.6.1.1).

$$20 \text{ MPa} \leq f'_c \leq 80 \text{ MPa} \quad \text{(CSA 8.6.1.1)}$$

The upper limit of f_y is 500 MPa for all frames (CSA 8.5.1).

$$f_y \leq 500 \text{ MPa} \quad \text{(CSA 8.5.1)}$$

SAFE enforces the upper material strength limits for flexure and shear design of beams and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

17.4 Strength Reduction Factors

The strength reduction factors, ϕ, are material dependent and defined as:

ϕ_c = 0.65 for concrete (CSA 8.4.2)

ϕ_s = 0.85 for reinforcement (CSA 8.4.3a)

These values can be overwritten; however, caution is advised.

17.5 Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

17.5.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

17.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom reinforcement. In such cases the beam may be designed as a rectangular or flanged beam. Negative beam moments produce top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

17.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 17-1 (CSA 10.1.7). Furthermore, it is assumed that the compression carried by the concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the text that follows. For reinforced concrete design where design ultimate axial compression load does not exceed (0.1 $f'_c A_g$), axial force is ignored; hence, all beams are designed for major direction flexure, shear, and torsion only. Axial compression greater than $0.1f'_c A_g$ and axial tensions are always included in flexural and shear design.

17.5.1.2.1 Design of Rectangular Beams

In designing for a factored negative or positive moment, M_f (i.e., designing top or bottom reinforcement), the depth of the compression block is given by a (see Figure 17-1), where,

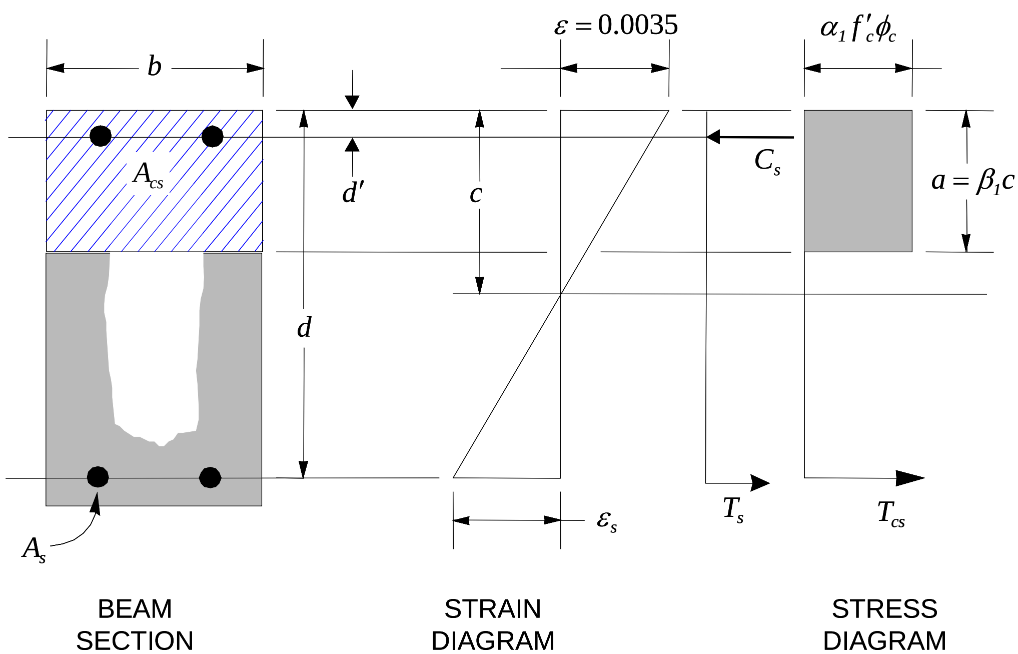


Figure 17-1 Rectangular Beam Design

$$a = d - \sqrt{d^2 - \frac{2\left|M_f\right|}{\alpha_1 f'_c \phi_c b}} \qquad \text{(CSA 10.1)}$$

where the value of ϕ_c is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters α_1, β_1, and c_b are calculated as:

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d \qquad \text{(CSA 10.5.2)}$$

The balanced depth of the compression block is given by:

$$a_b = \beta_1 c_b \qquad \text{(CSA 10.1.7)}$$

- If $a \le a_b$ (CSA 10.5.2), the area of tension reinforcement is given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2} \right)}$$

This reinforcement is to be placed at the bottom if M_f is positive, or at the top if M_f is negative.

- If $a > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The factored compressive force developed in the concrete alone is given by:

$$C = \phi_c \alpha_1 f'_c b a_b \qquad \text{(CSA 10.1.7)}$$

and the factored moment resisted by concrete compression and tension reinforcement is:

$$M_{fc} = \text{C}\left(d - \frac{a_b}{2} \right)$$

Therefore, the moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{fs} = M_f - M_{fc}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{fs}}{\left(\phi_s f'_s - \phi_c \alpha_1 f'_c \right)\left(d - d' \right)}, \text{ where}$$

$$f'_s = 0.0035\, E_s \left[\frac{c - d'}{c} \right] \le f_y \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The required tension reinforcement for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{2}\right)\phi_s}$$

and the tension reinforcement for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_{fs}}{f_y\left(d - d'\right)\phi_s}$$

Therefore, the total tension reinforcement, $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa if M_f is negative.

17.5.1.2.2 Design of Flanged Beams

17.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_f (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

17.5.1.2.2.2 Flanged Beam Under Positive Moment

- If $M_f > 0$, the depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_f}{\alpha_1 f'_c \phi_c b_f}} \qquad \text{(CSA 10.1)}$$

where, the value of ϕ_c is 0.65 (CSA 8.4.2) in the preceding and the following equations. The parameters α_1, β_1, and c_b are calculated as:

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d \qquad \text{(CSA 10.5.2)}$$

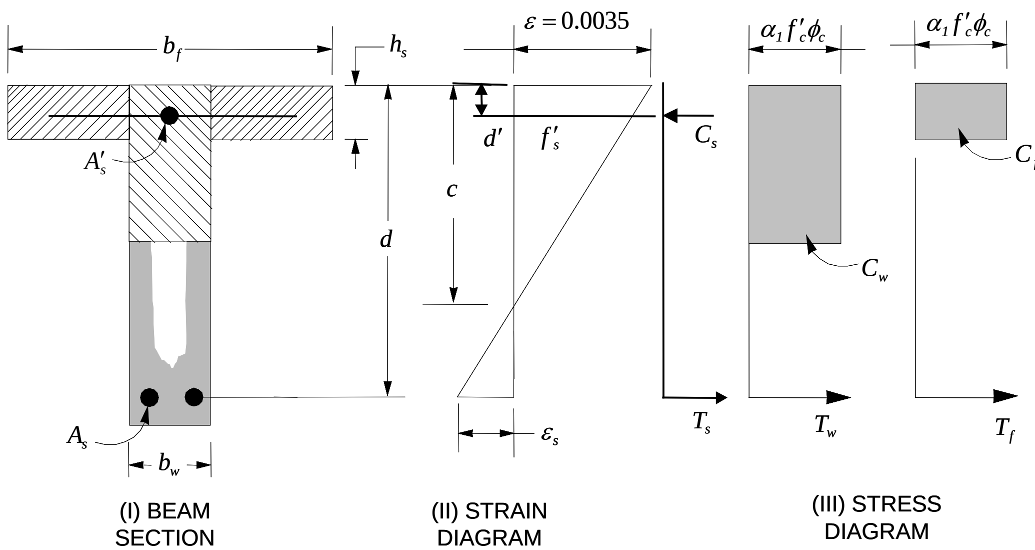


Figure 17-2 Design of a T-Beam Section

The balanced depth of the compression block is given by:

$$a_b = \beta_1 c_b \qquad \text{(CSA 10.1.4,10.1.7)}$$

- If $a \le h_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_b$.

- If $a > h_s$, calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w as shown in Figure 17-2. C_f is given by:

$$C_f = \alpha_1 f'_c \left(b_f - b_w\right) \min\left(h_s, a_b\right) \qquad \text{(CSA 10.1.7)}$$

Therefore, $A_{s1} = \dfrac{C_f \phi_c}{f_y \phi_s}$ and the portion of M_f that is resisted by the flange is given by:

$$M_{ff} = C_f \left(d - \frac{\min(h_s, a_b)}{2} \right) \phi_c$$

Therefore, the balance of the moment, M_f to be carried by the web is:

$$M_{fw} = M_f - M_{ff}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f'_c \phi_c b_w}} \qquad \text{(CSA 10.1)}$$

- If $a_1 \le a_b$ (CSA 10.5.2), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{fw}}{\phi_s f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}$$

This reinforcement is to be placed at the bottom of the flanged beam.

- If $a_1 > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

The compressive force in the web concrete alone is given by:

$$C = \phi_c \alpha_1 f'_c b_w a_b \qquad \text{(CSA 10.1.7)}$$

Therefore the moment resisted by the concrete web and tension reinforcement is:

$$M_{fc} = C \left(d - \frac{a_b}{2} \right)$$

and the moment resisted by compression and tension reinforcement is:

$$M_{fs} = M_{fw} - M_{fc}$$

Therefore, the compression reinforcement is computed as:

$$A'_s = \frac{M_{fs}}{\left(\phi_s f'_c - \phi_c \alpha_1 f'_c\right)\left(d - d'\right)}, \text{ where}$$

$$f'_s = \varepsilon_c E_s \left[\frac{c - d'}{c}\right] \le f_y \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The tension reinforcement for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{2}\right)\phi_s}$$

and the tension reinforcement for balancing the compression reinforcement is:

$$A_{s3} = \frac{M_{fs}}{f_y\left(d - d'\right)\phi_s}$$

The total tension reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top.

17.5.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the minimum of the following two limits:

$$A_s \ge \frac{0.2\sqrt{f'_c}}{f_y} b_w h \qquad \text{(CSA 10.5.1.2)}$$

$$A_s \ge \frac{4}{3} A_{s(\text{required})} \qquad \text{(CSA 10.5.1.3)}$$

In addition, the minimum flexural tension reinforcement provided in a flanged beam with the flange under tension in an ordinary moment resisting frame is given by the limit:

$$A_s \geq 0.004\,(b - b_w)\,h_s \qquad \text{(CSA 10.5.3.1)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

17.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_f.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Deter*mine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

17.5.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

17.5.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v \qquad \text{(CSA 11.3.4)}$$

$$\sqrt{f'_c} \le 8 \text{ MPa} \qquad \text{(CSA 11.3.4)}$$

ϕ_c is the resistance factor for concrete. By default it is taken as 0.65 (CSA 8.4.2).

λ is the strength reduction factor to account for low density concrete (CSA 2.2). For normal density concrete, its value is 1 (CSA 8.6.5), which is taken by the program as the default value. For concrete using lower density aggregate, the user can change the value of λ in the material property data. The recommended value for λ is as follows (CSA 8.6.5):

$$\lambda = \begin{cases} 1.00, & \text{for normal density concrete,} \\ 0.85, & \text{for semi-low-density concrete} \\ & \text{in which all of the fine aggregate is natural sand,} \\ 0.75, & \text{for semi-low-density concrete} \\ & \text{in which none of the fine aggregate is natural sand.} \end{cases} \qquad \text{(CSA 8.6.5)}$$

β is the factor for accounting for the shear resistance of cracked concrete (CSA 2.2) and should be greater or equal to 0.05. Its value is normally between 0.1 and 0.4. It is determined according to CSA 11.3.6, and described further in the following sections.

b_w is the effective web width. For rectangular beams, it is the width of the beam. For flanged beams, it is the width of the web of the beam.

d_v is the effective shear depth. It is taken as the greater of 0.9*d* or 0.72*h*(CSA 2.3), where *d* is the distance from the extreme compression fiber to the centroid of the tension reinforcement, and *h* is the overall depth of the cross-section in the direction of the shear force (CSA 2.3).

The value of β is preferably taken as the special value (CSA 11.3.6.2) or it is determined using the simplified method (CSA 11.3.6.3), if applicable. When the conditions of the special value or simplified method do not apply, the general method is used (CSA 11.3.6.4).

If the overall beam depth, *h*, is less than 250 mm or if the depth of a flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, β is taken as 0.21 (CSA 11.3.6.2).

When the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, the specified concrete strength f'_c does not exceed 60 MPa, and the tensile force is negligible, β is determined in accordance with the simplified method, as follows (CSA 11.3.6.3):

- When the section contains at least the minimum transverse reinforcement, β is taken as 0.18 (CSA 11.6.3.3a).

$$\beta = 0.18 \qquad \text{(CSA 11.3.6.3(a))}$$

- When the section contains no transverse reinforcement, β is determined based on the specified maximum nominal size of coarse aggregate, a_g.

For a maximum size of coarse aggregate not less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + d_v} \qquad \text{(CSA 11.3.6.3(b))}$$

where d_v is the effective shear depth expressed in millimeters.

For a maximum size of coarse aggregate less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + s_{ze}} \qquad \text{(CSA 11.3.6.3 c)}$$

$$\text{where, } s_{ze} = \frac{35 s_z}{15 + a_g} \geq 0.85 s_z \qquad \text{(CSA 11.3.6.3.c)}$$

In the preceding expression, the crack spacing parameter, s_{ze}, shall be taken as the minimum of d_v and the maximum distance between layers of distributed longitudinal reinforcement. However, s_{ze} is conservatively taken as equal to d_v.

In summary, for simplified cases, β can be expressed as follows:

$$\beta = \begin{cases} 0.18, & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{230}{1000 + d_v}, & \text{if no transverse reinforcement is provided, and } a_g \geq 20\text{mm,} \\ \dfrac{230}{1000 + S_{ze}}, & \text{if no transverse reinforcement is provided, and } a_g < 20\text{mm.} \end{cases}$$

- When the specified yield strength of the longitudinal reinforcing f_y is greater than 400 MPa, the specified concrete strength f'_c is greater than 60 MPa, or tension is not negligible, β is determined in accordance with the general method as follows (CSA 11.3.6.1, 11.3.6.4):

$$\beta = \frac{0.40}{(1+1500\varepsilon_x)} \bullet \frac{1300}{(1000+S_{ze})} \qquad \text{(CSA 11.3.6.4)}$$

In the preceding expression, the equivalent crack spacing parameter, s_{ze} is taken equal to 300 mm if minimum transverse reinforcement is provided (CSA 11.3.6.4). Otherwise it is determined as stated in the simplified method.

$$S_{ze} = \begin{cases} 300 & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{35}{15 + a_g} S_z \geq 0.85 S_z & \text{otherwise.} \end{cases}$$

(CSA 11.3.6.3, 11.3.6.4)

The value of a_g in the preceding equations is taken as the maximum aggregate size for f'_c of 60 MPa, is taken as zero for f'_c of 70 MPa, and linearly interpolated between these values (CSA 11.3.6.4).

The longitudinal strain, ε_x at mid-depth of the cross-section is computed from the following equation:

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5N_f}{2(E_s A_s)} \qquad \text{(CSA 11.3.6.4)}$$

In evaluating ε_x the following conditions apply:

- ε_x is positive for tensile action.

- V_f and M_f are taken as positive quantities. (CSA 11.3.6.4(a))
- M_f is taken as a minimum of $V_f\, d_v$. (CSA 11.3.6.4(a))
- N_f is taken as positive for tension. (CSA 2.3)

A_s is taken as the total area of longitudinal reinforcement in the beam. It is taken as the envelope of the reinforcement required for all design load combinations. The actual provided reinforcement might be slightly higher than this quantity. The reinforcement should be developed to achieve full strength (CSA 11.3.6.3(b)).

If the value of ε_x is negative, it is recalculated with the following equation, in which A_{ct} is the area of concrete in the flexural tensile side of the beam, taken as half of the total area.

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5N_f}{2\left(E_s A_s + E_c A_{ct}\right)} \qquad \text{(CSA 11.3.6.4(c))}$$

$$E_s = 200,000 \text{ MPa} \qquad \text{(CSA 8.5.4.1)}$$

$$E_c = 4500\sqrt{f_c'} \text{ MPa} \qquad \text{(CSA 8.6.2.3)}$$

If the axial tension is large enough to induce tensile stress in the section, the value of ε_x is doubled (CSA 11.3.6.4(e)).

For sections closer than d_v from the face of the support, ε_x is calculated based on M_f and V_f at a section at a distance d_v from the face of the support (CSA 11.3.6.4(d)). This condition currently is not checked by SAFE.

An upper limit on ε_x is imposed as:

$$\varepsilon_x \le 0.003 \qquad \text{(CSA 11.3.6.4(f))}$$

In both the simplified and general methods, the shear strength of the section due to concrete, v_c depends on whether the minimum transverse reinforcement is provided. To check this condition, the program performs the design in two passes. In the first pass, it assumes that no transverse shear reinforcement is needed. When the program determines that shear reinforcement is required, the program performs the second pass assuming that at least minimum shear reinforcement is provided.

17.5.2.3 Determine Required Shear Reinforcement

The shear force is limited to $V_{r,\max}$ where:

$$V_{r,\max} = 0.25\phi_c f'_c b_w d \qquad \text{(CSA 11.3.3)}$$

Given V_f, V_c, and $V_{r,\max}$, the required shear reinforcement is calculated as follows:

- If $V_f \le V_c$,

$$\frac{A_v}{s} = 0 \qquad \text{(CSA 11.3.5.1)}$$

- If $V_c < V_f \le V_{r,\max}$,

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)\tan\theta}{\phi_s f_{yt} d_v} \qquad \text{(CSA 11.3.3, 11.3.5.1)}$$

- If $V_f > V_{r,\max}$, a failure condition is declared. (CSA 11.3.3)

A minimum area of shear reinforcement is provided in the following regions (CSA 11.2.8.1):

(a) in regions of flexural members where the factored shear force V_f exceeds V_c

(b) in regions of beams with an overall depth greater than 750 mm

(c) in regions of beams where the factored torsion T_f exceeds $0.25T_{cr}$.

Where the minimum shear reinforcement is required by CSA 11.2.8.1, or by calculation, the minimum area of shear reinforcement per unit spacing is taken as:

$$\frac{A_v}{s} \geq 0.06 \frac{\sqrt{f'_c}}{f_{yt}} b_w \qquad \text{(CSA 11.2.8.2)}$$

In the preceding equations, the term θ is used, where θ is the angle of inclination of the diagonal compressive stresses with respect to the longitudinal axis of the member (CSA 2.3). The θ value is normally between 22 and 44 degrees. It is determined according to CSA 11.3.6.

Similar to the β factor, which was described previously, the value of θ is preferably taken as the special value (CSA 11.3.6.2), or it is determined using the simplified method (CSA 11.3.6.3), whenever applicable. The program uses the general method when conditions for the simplified method are not satisfied (CSA 11.3.6.4).

- If the overall beam depth, h, is less than 250 mm or if the depth of the flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, θ is taken as 42 degrees (CSA 11.3.6.2).

- If the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, or the specified concrete strength f'_c does not exceed 60 MPa, θ is taken to be 35 degree (CSA 11.3.6.3).

  $$\theta = 35^\circ \text{ for } P_f \leq 0 \text{ or } f_y \leq 400 \text{ MPa or } f'_c \leq 60 \text{ MPa} \qquad \text{(CSA11.3.6.3)}$$

- If the axial force is tensile, the specified yield strength of the longitudinal reinforcing f_y > 400 MPa, and the specified concrete strength f'_c > 60 MPa, θ is determined using the general method as follows (CSA 11.3.6.4),

  $$\theta = 29 + 7000\varepsilon_x \text{ for } P_f < 0, \quad f_y > 400 \text{ MPa}, f'_c \leq 60 \text{ MPa} \qquad \text{(CSA11.3.6.4)}$$

where ε_x is the longitudinal strain at the mid-depth of the cross-section for the factored load. The calculation procedure is described in preceding sections.

The maximum of all of the calculated A_v/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup require-

ments to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

17.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_f.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

17.5.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_f is permitted to be reduced in accordance with the code (CSA 11.2.9.2). However, the program does not automatically redistribute the internal forces and reduce T_f. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

17.5.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_c, A_{oh}, A_o, p_c, and p_h, are calculated. These properties are described in the following (CSA 2.3).

A_c = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_c = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 millimeters. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as follows:

$$A_c = bh \quad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \quad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\,A_{oh} \quad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b + 2h \quad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \quad \text{(CSA 11.3.10.4)}$$

where, the section dimensions b, h, and c are shown in Figure 17-3. Similarly, the special section properties for a flanged beam section are given as follows:

$$A_c = b_w h + (b_f - b_w)h_s \quad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b_w - 2c)(h - 2c) \quad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\,A_{oh} \quad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b_f + 2h \quad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(h - 2c) + 2(b_w - 2c) \quad \text{(CSA 11.3.10.4)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 17-3. Note that the flange width on either side of the beam web is limited to the smaller of $6h_s$ or 1/12 the span length (CSA 10.3.4).

17.5.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored, is calculated as:

$$T_{cr} = \frac{0.38\lambda\phi_c\sqrt{f'_c}\left(\dfrac{A_c^2}{p_c}\right)}{4} \qquad \text{(CSA 11.2.9.1)}$$

where A_{cp} and p_c are the area and perimeter of the concrete cross-section as described in the previous section, λ is a factor to account for low-density concrete, ϕ_c is the strength reduction factor for concrete, which is equal to 0.65, and f'_c is the specified concrete compressive strength.

17.5.3.4 Determine Torsion Reinforcement

If the factored torsion T_f is less than the threshold limit, T_{cr}, torsion can be safely ignored (CSA 11.2.9.1). In that case, the program reports that no torsion reinforcement is required. However, if T_f exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CSA 11.3).

- If $T_f > T_{cr}$, the required closed stirrup area per unit spacing, A_t /s, is calculated as:

$$\frac{A_t}{s} = \frac{T_f \tan\theta}{\phi_s 2A_o f_{yt}} \qquad \text{(CSA 11.3.10.3)}$$

  and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{\dfrac{M_f}{d_v} + 0.5N_f + \sqrt{\left(V_f - 0.5V_s\right)^2 + \left(\dfrac{0.45 p_h T_f}{2A_o}\right)^2}\cot\theta}{\phi_s f_y}$$

(CSA 11.3.10.6, 11.3.9)

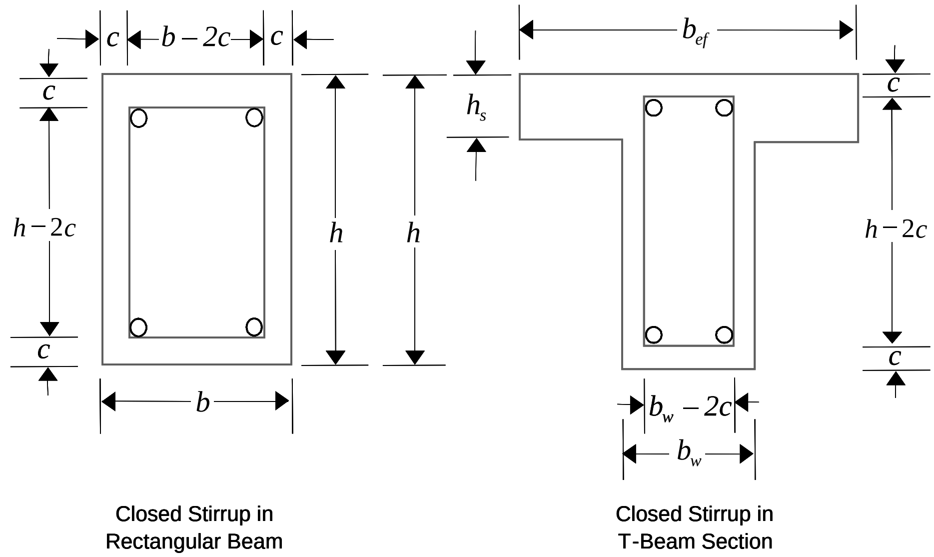


Figure 17-3 Closed stirrup and section dimensions for torsion design

In the preceding expressions, θ is computed as previously described for shear, except that if the general method is being used, the value ε_x, calculated as specified in CSA 11.3.6.4, is replaced by:

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{V_f^{\,2} + \left(\dfrac{0.9 p_h T_f}{2A_o}\right)^2} + 0.5N_f}{2\left(E_s A_s\right)} \qquad \text{(CSA 11.3.10.5)}$$

An upper limit of the combination of V_u and T_u that can be carried by the section also is checked using the equation:

$$\sqrt{\left(\frac{V_f}{b_w d_v}\right)^2 + \left(\frac{T_f p_h}{1.7A_{oh}^2}\right)^2} \le 0.25\phi_c f'_c \qquad \text{(CSA 11.3.10.4(b))}$$

For rectangular sections, b_w is replaced with b. If the combination of V_f and T_f exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_f > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s}+2\frac{A_t}{s}\right)\geq 0.06\sqrt{f'_c}\,\frac{b_w}{f_{yt}} \qquad \text{(CSA 11.2.8.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

17.6 Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is performed based on the ultimate strength design method (CSA A23.3-14) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "Design Strips" in the *Key Features and Terminology* manual.

17.6.1 Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

17.6.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

17.6.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. Where openings occur, the slab width is adjusted accordingly.

17.6.1.3 Minimum and Maximum Slab Reinforcement

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limit (CSA 13.10.1):

$A_s \geq 0.002\ bh$ (CSA 7.8.1)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

17.6.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code-specific items are described in the following subsections.

17.6.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (CSA 13.3.3.1 and CSA 13.3.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.3.3.3). Figure 17-4 shows the auto punching perimeters considered by SAFE for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

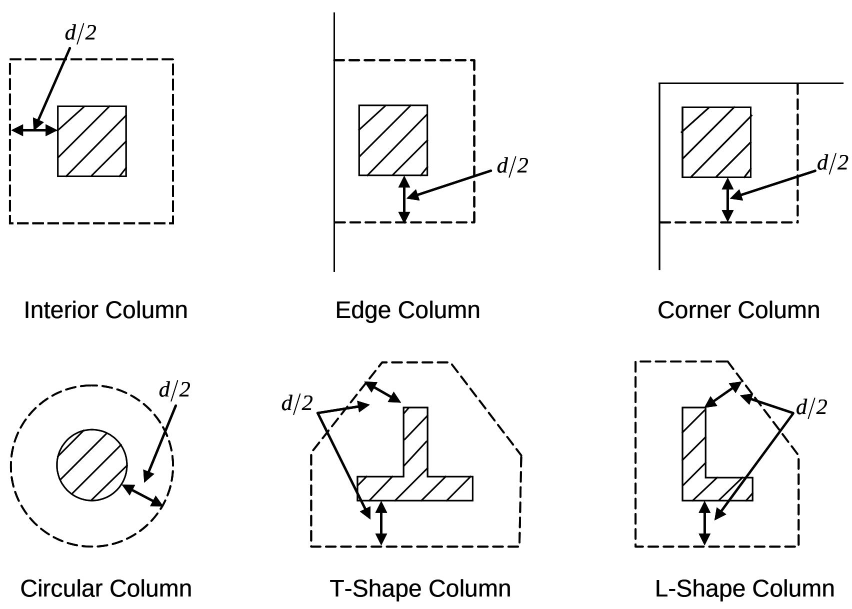


Figure 17-4 Punching Shear Perimeters

17.6.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(CSA 13.10.2)}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \qquad \text{(CSA 13.3.5.3)}$$

where b_1 is the width of the critical section measured in the direction of the span, and b_2 is the width of the critical section measured in the direction perpendicular to the span.

17.6.2.3 Determination of Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_v = \min \begin{cases} \phi_c \left(1+\dfrac{2}{\beta_c}\right) 0.19\lambda\sqrt{f'_c} \\ \phi_c \left(0.19+\dfrac{\alpha_s d}{b_0}\right)\lambda\sqrt{f'_c} \\ \phi_c 0.38\lambda\sqrt{f'_c} \end{cases} \qquad \text{(CSA 13.3.4.1)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4, \text{ for interior columns} \\ 3, \text{ for edge colums, and} \\ 2, \text{ for corner columns.} \end{cases} \qquad \text{(CSA 13.3.4.1(b))}$$

The value of $\sqrt{f'_c}$ is limited to 8 MPa for the calculation of the concrete shear capacity (CSA 13.3.4.2).

If the effective depth, *d*, exceeds 300 mm, the value of v_c is reduced by a factor equal to 1300/(1000 + *d*) (CSA 13.3.4.3).

17.6.2.4 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section.

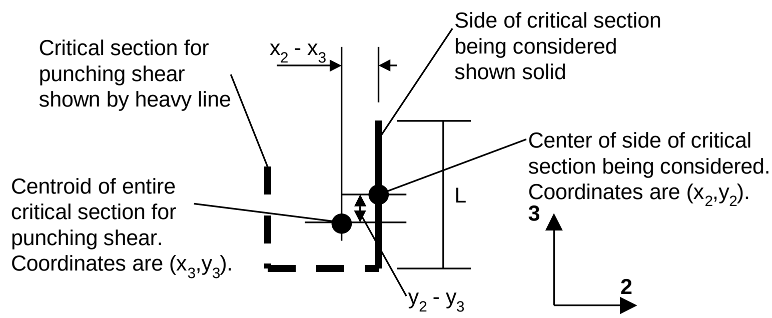
$$v_f = \frac{V_f}{b_0 d} + \frac{\gamma_{V2}[M_{f2} - V_f(y_3 - y_1)][I_{33}(y_4 - y_3) - I_{23}(x_4 - x_3)]}{I_{22}I_{33} - I_{23}^2} - \frac{\gamma_{V3}[M_{f3} - V_f(x_3 - x_1)][I_{22}(x_4 - x_3) - I_{23}(y_4 - y_3)]}{I_{22}I_{33} - I_{23}^2} \qquad \text{Eq. 1}$$

$$I_{22} = \sum_{sides=1}^{n} \overline{I}_{22}, \text{ where "}\textit{sides}\text{" refers to the sides of the critical section for punching shear} \qquad \text{Eq. 2}$$

$$I_{33} = \sum_{sides=1}^{n} \overline{I}_{33}, \text{ where "}\textit{sides}\text{" refers to the sides of the critical section for punching shear} \qquad \text{Eq. 3}$$

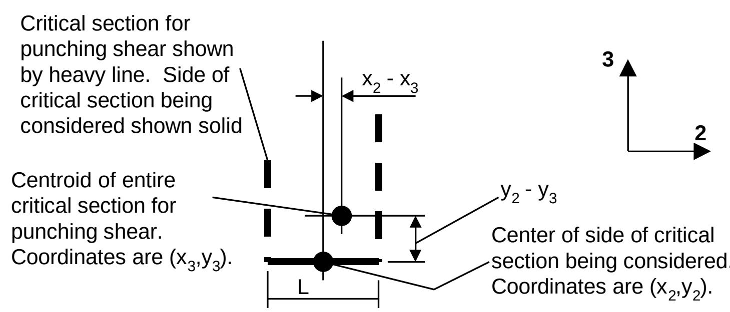
$$I_{23} = \sum_{sides=1}^{n} \overline{I}_{23}, \text{ where "}\textit{sides}\text{" refers to the sides of the critical section for punching shear} \qquad \text{Eq. 4}$$

The equations for $\overline{I}_{22}, \overline{I}_{33}$, and $\overline{I}_{23}$ are different depending on whether the side of the critical section for punching shear being considered is parallel to the 2-axis or parallel to the 3-axis. Refer to Figures 5-5.



Plan View For Side of Critical Section Parallel to 3-Axis

Work This Sketch With Equations 5b, 6b and 7



Plan View For Side of Critical Section Parallel to 2-Axis

Work This Sketch With Equations 5a, 6a and 7

Figure 17-5 Shear Stress Calculations at Critical Sections

$$\bar{I}_{22} = Ld(y_2 - y_3)^2, \text{ for side of critical section parallel to 2-axis} \qquad \text{Eq. 5a}$$

$$\bar{I}_{22} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(y_2 - y_3)^2, \text{ for side of critical section parallel to 3-axis} \qquad \text{Eq. 5b}$$

$$\bar{I}_{33} = \frac{Ld^3}{12} + \frac{dL^3}{12} + Ld(x_2 - x_3)^2, \text{ for side of critical section parallel to 2-axis} \qquad \text{Eq. 6a}$$

$$\bar{I}_{33} = Ld(x_2 - x_3)^2, \text{ for side of critical section parallel to 3-axis} \qquad \text{Eq. 6b}$$

$$\overline{I}_{23} = Ld(x_2 - x_3)(y_2 - y_3), \text{ for side of critical section parallel to 2-axis or 3-axis} \quad \text{Eq. 7}$$

NOTE: $\overline{I}_{23}$ is explicitly set to zero for corner condition.

where,

b_0 = Perimeter of the critical section for punching shear

d = Effective depth at the critical section for punching shear based on the average of d for 2 direction and d for 3 direction

I_{22} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 2-axis

I_{33} = Moment of inertia of the critical section for punching shear about an axis that is parallel to the local 3-axis

I_{23} = Product of inertia of the critical section for punching shear with respect to the 2 and 3 planes

L = Length of the side of the critical section for punching shear currently being considered

M_{f2} = Moment about the line parallel to the 2-axis at the center of the column (positive in accordance with the right-hand rule)

M_{f3} = Moment about the line parallel to the 3-axis at the center of the column (positive in accordance with the right-hand rule)

V_f = Punching shear stress

V_f = Shear at the center of the column (positive upward)

x_1, y_1 = Coordinates of the column centroid

x_2, y_2 = Coordinates of the center of one side of the critical section for punching shear

x_3, y_3 = Coordinates of the centroid of the critical section for punching shear

x_4, y_4 = Coordinates of the location where stress is being calculated

γ_{V2} = Percent of M_{f2} resisted by shear

γ_{V3} = Percent of M_{f3} resisted by shear

17.6.2.5 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by SAFE. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

17.6.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CSA 13.2.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed, and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is performed as explained in the subsections that follow.

17.6.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is taken as:

$$v_c = 0.28\lambda\phi_c\sqrt{f'_c} \quad \text{for shear studs} \qquad \text{(CSA 13.3.8.3)}$$

$$v_c = 0.19\lambda\phi_c\sqrt{f'_c} \quad \text{for shear stirrups} \qquad \text{(CSA 13.3.9.3)}$$

17.6.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of $v_{r,\max}$, where

$$v_{r,\max} = 0.75\lambda\phi_c\sqrt{f'_c} \text{ for shear studs} \qquad \text{(CSA 13.3.8.2)}$$

$$v_{r,\max} = 0.55\lambda\phi_c\sqrt{f'_c} \text{ for shear stirrups} \qquad \text{(CSA 13.3.9.2)}$$

Given v_f, v_c, and $v_{f,,\max}$, the required shear reinforcement is calculated as follows, where, ϕ_s, is the strength reduction factor.

- If $v_f > v_{r,\max}$,

$$\frac{A_v}{s} = \frac{(v_f - v_c)}{\phi_s f_{yv}} b_o \qquad \text{(CSA 13.3.8.5, 13.3.9.4)}$$

- If $v_f > v_{r,\max}$, a failure condition is declared. (CSA 13.3.8.2)
- If v_f exceeds the maximum permitted value of $v_{r\max}$, the concrete section should be increased in size.

17.6.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 17-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

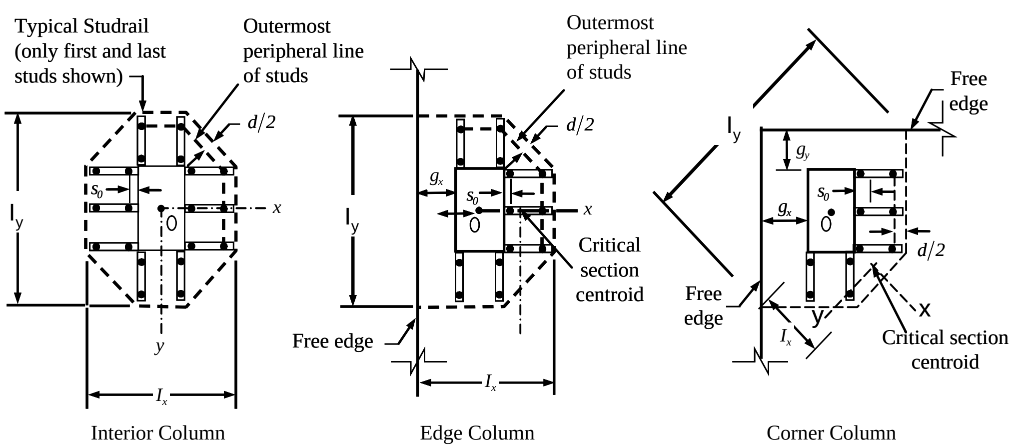


Figure 17-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed

$0.4d$ for shear studs (CSA 13.3.8.6)

$0.25d$ for shear stirrups (CSA 13.3.8.6)

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

17.6.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CSA 7.9 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than $0.45d$ to $0.4d$. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.4d \qquad \text{(CSA 13.3.8.6)}$$

$$s \leq \begin{cases} 0.75d & v_f \leq 0.56\lambda\phi_c\sqrt{f'_c} \\ 0.50d & v_f > 0.56\lambda\phi_c\sqrt{f'_c} \end{cases} \qquad \text{(CSA 13.3.8.6)}$$

For shear stirrups,

$$s_o \leq 0.25d \qquad \text{(CSA 13.3.9.5)}$$

$$s \leq 0.25d \qquad \text{(CSA 13.3.9.5)}$$

The minimum depth for reinforcement should be limited to 300 mm (CSA 13.3.9.1).

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