Air Flow Modeling with AIR/W

An Engineering Methodology

February 2012 Edition

GEO-SLOPE International Ltd.

GEO-SLOPE International Ltd

1400, 633 – 6th Ave SW

Calgary, Alberta, Canada T2P 2Y5

E-mail: info@geo-slope.com

Web: http://www.geo-slope.com

Table of Contents

1 Introduction

AIR/W is a module that runs inside the SEEP/W program where it solves for air pressure and flow in response to pressure boundary conditions or changes in water pressure. If coupled with TEMP/W, then AIR/W also solves for thermally induced density dependent air movement and pressure changes.

Unsaturated soil mechanics has, as it primary stress state, a variable known as matric suction. Matric suction is composed of two parts: air pressure and water pressure; and is computed as the difference between these two (Ua – Uw). Traditional seepage models have assumed that the air component is atmospheric such that the matric suction reduces to the negative value of water pressure, -Uw. This is the case when using SEEP/W alone and this is a valid assumption in many applications. However, it is a limiting assumption for a wide range of engineering problems – which is the topic of this book.

1.1 Typical applications

Air pressure and flow in soils has significance in many engineering problems. Some of these include:

- Air pressures in tunneling for support and seepage control
- Air pressure build up in advance of wetting fronts (Lisse Effect)
- Air flow and convective heat transfer in arctic roads, railway embankments, and mine waste rock piles where permafrost is jeopardized
- Air flow for soil contaminant vapor extraction systems
- Air flow in landfills and mine waste dumps where air feeds biological breakdown of waste

1.2 About this book

Modeling the movement of air through soil with a numerical solution can be very complex. Natural soil deposits are generally highly heterogeneous and non-isotropic. In addition, boundary conditions often change with time and cannot always be defined with certainty at the beginning of an analysis. In fact, the correct boundary condition can sometimes be part of the solution as is the case for an air pressure exit review boundary, where air release may be impeded if water begins ponding on the ground surface.

The movement of air cannot be modeled without a valid model for groundwater flow in the system. That is why AIR/W is a module within SEEP/W and not a product on its own. The flow of water and air are inseparable. This book is NOT about seepage modeling and it is assumed from this point onward, that the reader is familiar with and has read the SEEP/W Engineering Methodology book. This book is not a stand-alone reference. It is about taking seepage and thermal modeling to the next level…. by including air pressure and air flow.

While part of this document and the SEEP/W book are about using AIR/W to do air flow analyses, they are also about general numerical modeling techniques. Numerical modeling, like most things in life, is a skill that needs to be acquired. It is nearly impossible to pick up a tool like AIR/W and immediately become an effective modeler. Effective numerical modeling requires some careful thought and planning and it requires a good understanding of the underlying physical fundamentals. Aspects such as discretization of a finite element mesh and applying boundary conditions to the problem are not entirely intuitive at first. Time and practice is required to become comfortable with these aspects of numerical modeling.

Chapter 2 of the SEEP/W book is devoted exclusively to discussions on the topic of How to Model. The general principles discussed in that book apply to all numerical modeling situations, even though the discussion there focuses on seepage analysis.

Broadly speaking, there are three main parts to a finite element analysis. The first is discretization – dividing the domain into small areas called elements. The second part is specifying and assigning material properties. The third is specifying and applying boundary conditions. Details of discretization are provided in the SEEP/ book, while material properties and boundary conditions as pertaining to air flow analysis are discussed in detail in their respective chapters here.

Air flow modeling is numerically challenging when coupled with thermal modeling because of the presence of a first order transport term in the main thermal differential equation. For this reason, it is important to have an understanding of how that term affects the solution of the equations and, in particular, how mesh size and time steps are critical to that solution. The importance of the Courant numbers will be introduced and discussed, along with other numerical considerations in a chapter titled Numerical Issues.

One chapter has been dedicated to presenting and discussing illustrative examples.

A full chapter is dedicated to theoretical issues associated with air flow and the solution the seepage, air and thermal finite element equations. Additional finite element numerical details regarding interpolating functions and infinite elements are given in Appendix A of the SEEP/W and TEMP/W books.

The chapter entitled “Modeling Tips and Tricks” should be consulted to see if there are simple techniques that can be used to improve your general modeling method or to help gain confidence and develop a deeper understanding of finite element methods, AIR/W conventions or data results.

In general, this book is not a HOW TO USE AIR/W manual. This is a book about how to model. It is a book about how to solve air flow problems using a powerful calculator; AIR/W. Details of how to use various program commands and features are given in the on line help inside the software.

2 Material Models and Properties

2.1 Soil behavior models

This chapter describes the various soil material models and related soil properties that are required in the solution of the AIR/W partial differential equations. It is important to have a clear understanding of what the soil properties mean and what influence they have on the type of results generated. This chapter is not meant to be an all inclusive discussion of these issues. It is meant to highlight the importance of various parameters and the implications associated with not defining them adequately.

Well defined soil properties can be critical to obtaining an efficient solution of the finite element equations. When is it acceptable to guess at a function and when must you very carefully define one? This chapter will address these issues.

There are five different material models to choose from when using AIR/W. Four of these are inside the SEEP/W program and one is inside the TEMP/W program if you are solving thermally coupled air flow. A summary of these models and the required soil properties are given below. A discussion of the individual parameters and function are provided in the next section.

AIR/W material models in SEEP/W

- None (used to removed part of a model in an analysis)
- Saturated / Unsaturated model
  - Water conductivity function, ratio and direction
  - Water content function
  - Air conductivity function
- Saturated only model
  - Water saturated conductivity (Ksat), ratio and direction
  - Saturated water content
  - Coefficient of compressibility (Mv)
  - Air conductivity set to zero
- Interface model
  - Water normal and tangent conductivity
  - Air conductivity

AIR/W material model in TEMP/W

- Coupled Convective Thermal model
  - Thermal conductivity vs water content function
  - Volumetric specific heat function
  - Unfrozen volumetric water content function

The Saturated Only soil model is very useful for quickly defining a soil region that will always remain below the phreatic surface, but it should not be used for soils that will at some point during the analysis become partially saturated. If this happens, the model will continue to solve but you will be saying, in effect, that the unsaturated zone can transmit the water at the same rate as for the saturated soil and you will over estimate flow quantity. A saturated zone does not have any volumetric air content.

If there is any doubt about which model to choose for a soil region (as opposed to an interface region), you should select the Saturated / Unsaturated model. Also, if you intend to do a coupled analysis with CTRAN/W you MUST use the fully specified soil property functions. At this time, CTRAN/W does not consider transport of contaminants in the air phase.

The Interface soil model is to be used in conjunction with “interface elements” that are added to the mesh to represent geo-membranes, wick drains, or cut off walls. In these cases, you can specify a different tangent and normal conductivity – which would be the case in a wick drain where the smearing effect caused during installation results in a lower conductivity in the normal flow direction than in the tangent direction.

2.1 Soil water storage – water content function

It is important to understand the relationship between pore-water pressure, pore-air pressure and water content in an AIR/W analysis. Soil consists of a collection of solid particles and interstitial voids. The pore spaces or voids can be filled either with water or air, or with a combination of both. In a saturated soil, all the voids are filled with water and the volumetric water content of the soil is equal to the porosity of the soil according to:

$$\Theta_w = nS$$

where:

Θ_w = the volumetric water content,

n = the porosity of the soil, and

S = the degree of saturation (in saturated soil equal to 1.0 or 100%).

The porosity, n, is related to the void ratio, e, by:

$$e = \frac{n}{1-n} = \frac{wG_s}{S}$$

where:

w = the gravimetric water content, and

G_s = the particle specific gravity.

In an unsaturated soil, the volume of water stored within the voids will vary depending on the matric suction within the pore-water, where matric suction is defined as the difference between the air and water pressure as follows: Ua-Uw.

There is no fixed water content in time and space and so a function is required to describe how the water contents change with different pressures in the soil.

The volumetric water content function describes the capability of the soil to store water under changes in matric pressures. A typical function for a drying soil is shown in Figure 2-1 where the function was measured for an air pressure of zero.

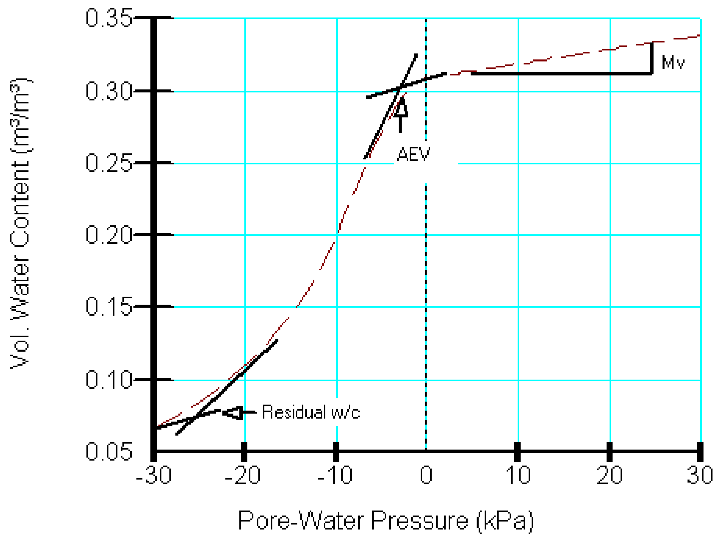


Figure 2-1 Volumetric water content (storage) function

The volumetric water content function describes what portion or volume of the voids remains water-filled as the soil drains. The three main features that characterize the volumetric water content function are the air-entry value, (AEV), the slope of the function for both the positive and negative pore-water pressure ranges (designated as mw), and the residual water content or saturation. The air-entry value (AEV) corresponds to the value of negative pore-water pressure when the largest voids or pores begin to drain freely. It is a function of the maximum pore size in a soil and is also influenced by the pore-size distribution within a soil. Soils with large, uniformly shaped pores have relatively low AEV's.

Adequately representing the slope of the water content function, in both the positive and negative pore-water pressure regions, can be very important in a seepage analysis, since water can be released from the soil in two ways. Water can be released by draining the water-filled voids and de-saturating the soil profile by gravitation forces, or by compressing the soil skeleton and reducing the size of the voids, effectively squeezing water out of a saturated system. In the positive pore-water pressure region, mw becomes equivalent to mv, the coefficient of compressibility for one-dimensional consolidation. The slope of the volumetric water content function in the negative pore-water pressure range represents the rate at which the volume of water stored within the soil changes as the pressure changes, over a range of values from the AEV to the pressure at the residual water content.

Another key feature of the volumetric water content function is the residual volumetric water content, which represents the volumetric water content of a soil where a further increase in negative pore-water pressure does not produce significant changes in water content. This point can also be expressed in terms of the degree of saturation by dividing the residual volumetric water content by the porosity of the soil. It is possible to remove water to a state less than the residual water content value, but this process is controlled by evaporation and / or osmotic forces. Evaporative drying is excluded from SEEP/W, but is included in VADOSE/W by considering simultaneous coupled heat, mass and vapor flow.

Water can be released by draining the water-filled voids and de-saturating the soil profile by gravitation force; by compressing the soil skeleton and reducing the size of the voids, effectively squeezing water out of a saturated system; or by applying evaporative demand.

Factors affecting the volumetric water content

The three main features of the volumetric water content function as described above are most strongly influenced by the size of the individual soil particles and the distribution of particle sizes in the soil. By way of example, consider the differences between the volumetric water content functions for sand, silt and clay shown in Figure 2-2

There are several factors that influence the shape of the volumetric water content function. As mentioned earlier, the air-entry value (AEV) reflects how much negative pore-water pressure can be applied to the pore-water before the largest pores or voids start to drain.

Consider the uniform sand function. Since the particles are relatively large and approximately the same size, water can easily be removed under relatively small negative pore-water pressures and the resulting air entry value is smaller than that for the silt or clay. The uniform nature of the pores means that all the pores drain over a small range of negative pore-water pressures, which makes the slope of the function steeper than the others.

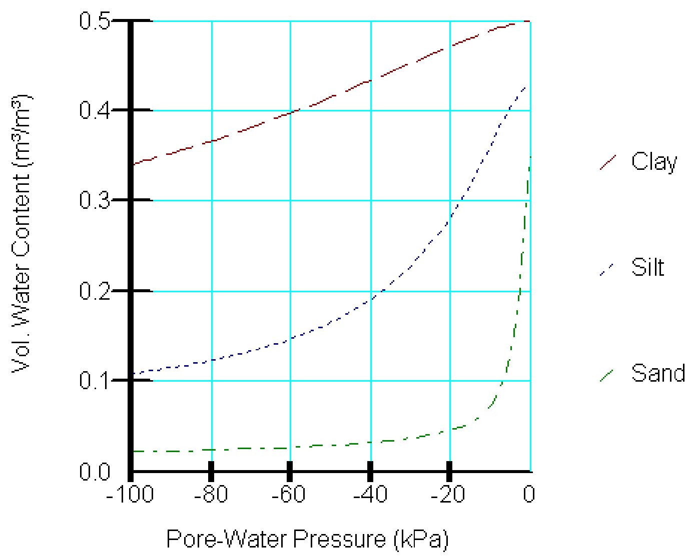


Figure 2-2 Typical storage functions for 3 soil types

The silt has a wider distribution of pore sizes. Some of the pores between the larger sand particles are filled with silt particles, making the largest pores sizes smaller than those of uniform sand. Consequently, a more negative pore-water pressure must be applied before drainage begins, thereby increasing the AEV. In addition, the pore sizes are not as uniform and the slope of the function is less steep.

The pores between the individual clay particles in the clay are small. It is often difficult to identify a specific air-entry value for clay, as consolidation and changes to the structure of the clay can release water from the system over a significant range before air actually enters the pores. Since clay tends to be compressible, the slope of the function in the positive pore-water pressure region tends to be steeper and becomes a significant parameter to consider in saturated seepage analyses.

2.2 Storage function types and estimation methods

It is not especially difficult to obtain a direct measurement of a volumetric water content function in a laboratory, but it does require time and it requires finding a geotechnical laboratory that performs the service. It is, however, standard practice to obtain a grain-size distribution curve and many companies have the capability and facilities to develop their own curves. The development of the grain-size distribution curve is inexpensive and can be quickly accomplished.

One of the required input parameters for a transient analysis is the volumetric water content function. Since it can sometimes be difficult or time consuming to obtain a volumetric water content function, it may be of benefit to be able to develop an estimation of the volumetric water content function using either a closed-form solution that requires user-specified curve-fitting parameters, or to use a predictive method that uses a measured grain-size distribution curve. SEEP/W has four methods available to develop a volumetric water content function. One is a predictive methods based on grain size, one is to base your function of a sample set of functions built into the software, and two are closed form equations based on known curve fit parameters.

Estimation method 1 (grain size - Modified Kovacs)

Aubertin et al (2003) presented a method to predict the volumetric water content function which is modified from the method proposed by Kovacs (1981). The modifications were made to Kovac's method to better represent materials such as tailings from hard-rock mines. A further modification extended the method for clay type soils. The Aubertin et al. method predicts the volumetric water content function using basic material properties which can be useful, particularly for preliminary analysis. It should be cautioned that, especially for clay type materials, it is critical to base final design on measured material properties.

The function is initially determined as a degree of saturation function and then is later converted to a volumetric water content function. The function is developed by defining the degree of saturation for two main components. The first component contributes to the amount of water that is stored in a soil by capillary forces that exist at relatively small negative pore-water pressures. The second component contributes to the volumetric water content function at large negative pore-water pressures where the amount of water that exists in the soil is primarily a function of adhesion. Both of these components can be evaluated from the negative pore-water pressure and material property information such as particle-size, the shape of the particles and the porosity.

The degree of saturation as determined based on the capillary and adhesive components is as follows:

$$S_r = \frac{\Theta_w}{n} = S_c + S_a^*(1 - S_c)$$

where:

S_r	=	the degree of saturation,
Θ_w	=	the volumetric water content,
n	=	the porosity,
S_c	=	the degree of saturation due to capillary forces, and
S_a^*	=	the bounded degree of saturation due to adhesion (S_a).

where:

$$S_a^* = \left\langle 1 - S_a \right\rangle + 1$$

The adhesive component is a bounded value since it is possible at low suctions for the value Sa to be greater than 1. The bounded value ensures that for a Sa greater or equal to 1, Sa* = 1 and if Sa is less than 1, then Sa* = Sa.

The adhesion component is associated with the thin film of water that covers the surface of the soil grain and depends on basic material properties such as the negative pore-water pressure in the soil and the particle-size, shape coefficient and porosity of the soil. It is determined by the following equation:

$$S_a = aC_\Psi \frac{\left(\frac{h_{co}}{\Psi_n}\right)^{2/3}}{e^{1/3}\left(\frac{\Psi}{\Psi_n}\right)^{1/6}}$$

where:

a = a curve fitting parameter,

Ψ = the suction,

Ψ_n = a suction term introduced to ensure dimensionless component,

e = the void ratio,

h_{co} = the mean capillary rise (cm) determined for capillary soils by:

$$h_{co} = \frac{b(cm^2)}{eD_{10}(cm)}$$

or

$$h_{co} = \frac{\xi w_L^{1.75}}{e}$$

for cohesion type soils where:

D_{10} = the particle diameter (cm) corresponding to 10% passing on a grain-size curve,

$b(cm^2)$ = is given by:

$$b(cm^2) = \frac{0.75}{1.17\log C_u + 1}$$

where:

C_u = the coefficient of uniformity,

w_L = the liquid limit (%),

ξ = a constant approximately equal to 402.2 cm^2 ,

C_Ψ = a correction coefficient that allows a progressive decrease in water content at high suctions, forcing the function through a water content of zero at one million kPa suction as initially proposed by Fredlund and Xing (1994) and described by:

$$C_{\Psi} = 1 - \frac{\ln\left(1 + \frac{\Psi}{\Psi_r}\right)}{\ln\left(1 + \frac{\Psi_o}{\Psi_r}\right)}$$

where:

Ψ_r = the suction corresponding to the residual water content at which point an increase in suction will not effectively remove more liquid water from the soil and given by:

$$\Psi_r = 0.86\left(\frac{\xi}{e}\right)^{1.2} w_L^{1.74}$$

The capillary saturation, which depends essentially on the pore diameter and the pore size distribution, is given by:

$$S_c = 1 - \left[\left(\frac{h_{co}}{\Psi}\right)^2 + 1\right]^m \exp\left[-m\left(\frac{h_{co}}{\Psi}\right)^2\right]$$

where:

m = a fitting parameter that takes into account the pore size distribution and controls the shape and position of the volumetric water content function in the capillary zone.

For plastic-cohesive soils considered here, both the value of parameters m and a can be taken as constants with m=3x10-5 and a=7x10-4 in the predictive applications. For the capillary based soils, m and a can be taken as 1 and 0.01 respectively.

Estimation method 2 (sample functions)

In previous versions of GeoStudio a list of over 20 fully defined water content functions were provided. Based on experience and user feedback, we no longer provide this list. Instead, we provide several “typical” water content functions for different types of soils. In using these sample functions, it is up to you to specify the saturated water content and the residual water content (if any) based on your understanding of field conditions. These functions are provided as a means to letting you set up some test models quickly, change functions easily, decide how sensitive your results are to function shape, and ultimately to have you decide if you need to spend more time and money obtaining more accurate data. In the past, our list of “real” functions ended up being used as final design material properties with little thought about their relevance. This is not good modeling practice.

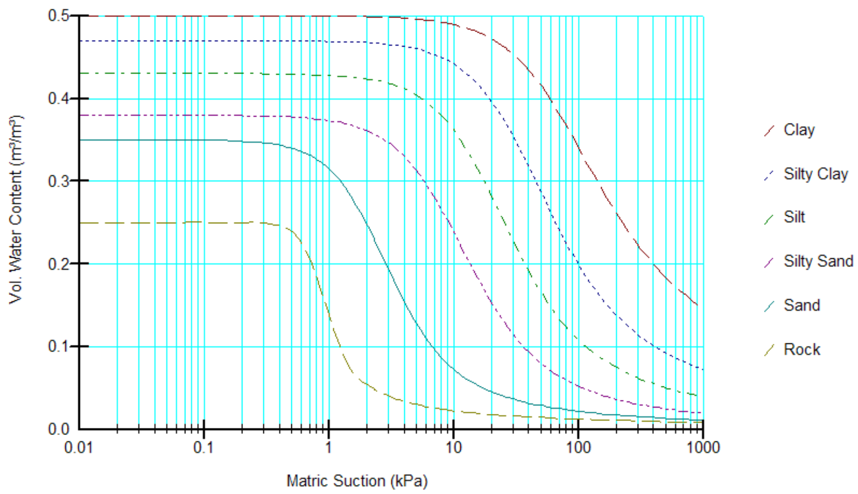


Figure 2-3 Sample Functions in GeoStudio

Closed form option 1 (Fredlund and Xing, 1994)

The Fredlund and Xing (1994) method is a closed-form solution that can be used to develop the volumetric water content function for all possible negative pressures between zero and minus one million kPa based on the user's knowledge of a group of three parameters. The governing equation is as follows:

$$\Theta_w = C_\psi \frac{\Theta_s}{\left\{ \ln\left[e + \left(\frac{\Psi}{a} \right)^n \right] \right\}^m}$$

where:

Θ_w	=	the volumetric water content,
C_ψ	=	the correction function described above,
Θ_s	=	the saturated volumetric water content,
e	=	the natural number (2.71828),
Ψ	=	the negative pore-water pressure, and
a, n, m	=	curve fitting parameters.

The 'a' parameter, which has units of kPa, is the inflection point of the volumetric water content function. It is generally slightly larger than the air-entry value. The parameter n controls the slope of the volumetric water content function and the m parameter controls the residual water content. The three parameters a, n, and m are determined as follows:

$$a = \Psi_i$$

$$m = 3.67 \ln\left(\frac{\Theta_s}{\Theta_i}\right)$$

$$n = \frac{1.31^{m+1}}{m\Theta_s} 3.72 s \Psi_i$$

where:

Ψ_i	=	the suction pressure corresponding to the water content occurring at the inflection point of the curve, and
s	=	the slope of the line tangent to the function that passes through the inflection point.

The Fredlund and Xing, 1994 method is only functional if you know values of a, n and m. In general, the a, n and m values can be determined using a fitting algorithm and applying it to measured data points. SEEP/W has this ability.

It is important to understand that this method is not intended to predict a volumetric water content function from grain-size curves, but was developed to obtain a smooth function over the complete range of negative pore-water pressure values (0 to one Million kPa).

Closed form option 2 (Van Genuchten, 1980)

In 1980, van Genuchten proposed a four-parameter equation as a closed form solution for predicting the volumetric water content function. The governing equation is as follows:

$$\Theta_w = \Theta_r + \frac{\Theta_s - \Theta_r}{\left[1 + \left(\frac{\Psi}{a}\right)^n\right]^m}$$

where:

Θ_w	=	the volumetric water content,
Θ_s	=	the saturated volumetric water content,
Ψ	=	the negative pore-water pressure, and
a, n, m	=	curve fitting parameters (note: “a” has units of pressure, not 1/pressure head as in some formulations of this equation)

Although the terminology of the a, n and m parameters are similar to those of Fredlund and Xing (1994), the definitions are slightly different. The a parameter in particular cannot be estimated by the air-entry value, but instead is a pivot point about which the n parameter changes the slope of the function. The parameter m affects the sharpness of the sloping portion of the curve as it enters the lower plateau. The van Genuchten closed form method can only be used then the curve fit parameters are known, but there are some references to these values in the literature that can be applied in the model.

CAUTION: the units of the “a” value should be checked to make sure they are consistent between your data source and SEEP/W, which requires the units be in terms of pressure and not 1/pressure.

2.3 Coefficient of volume compressibility

The coefficient of volume compressibility m_v is the slope of the volumetric water content function in the positive pore pressure range. The coefficient characterizes the volume of water stored or released from the soil when the pore-water pressure changes. More fundamentally, the coefficient of volume compressibility embodies the compressibility of the soil structure and water. The coefficient of volume compressibility can be calculated from specific storage, which is given by:

$$S_s = \rho g\left(\alpha + n\beta\right) = \rho g\left(m_v\right)$$

where:

ρ	=	the water density,
g	=	the gravitational constant,
α	=	the compressibility of the soil struture,
n	=	the soil porosity, and
β	=	the compressibility of the water.

Water can be assumed incompressible for most geotechnical applications. The coefficient of volume compressibility is often assumed equivalent to the coefficient obtained from a 1D compression test; however, this assumption is not strictly valid unless a constrained flow system is being simulated. In that case, or as as an approximation, the coefficient of volume change can be calculated as:

$$m_v = \frac{1}{M} = \frac{a_v}{1+e_o}$$

where:

M	=	the modulus of elasticity in confined compression. It is also given by:
a_v	=	the coefficient of compressibility, and
e_0	=	the initial void ratio.

The metric units for the coefficient of volume compressibility are 1/kPa and the value generally ranges from 1e-6 1/kPa to 1e-3 1/kPa for most soils.

2.4 Hydraulic conductivity

The ability of a soil to transport or conduct water under both saturated and unsaturated conditions is reflected by the hydraulic conductivity function. In a saturated soil, all the pore spaces between the solid particles are filled with water. Once the air-entry value is exceeded, air enters the largest pores and the air-filled pores become non-conductive conduits to flow and increase the tortuosity of the flow path as shown schematically in Figure 2-4. As a result, the ability of the soil to transport water (the hydraulic conductivity) decreases. As pore-water pressures become increasingly more negative, more pores become air-filled and the hydraulic conductivity decreases further. By this description, it is clear that the ability of water to flow through a soil profile depends on how much water is present in the soil, which is represented by the volumetric water content function. Actually measuring the hydraulic conductivity function is a time-consuming and expensive procedure, but the function can be readily developed using one of several predictive methods that utilize either a grain-size distribution curve or a measured volumetric water content function and the saturated hydraulic conductivity. SEEP/W has built-in predictive methods that can be used to estimate the hydraulic conductivity function once the volumetric water content function and a Ksat value have been specified.

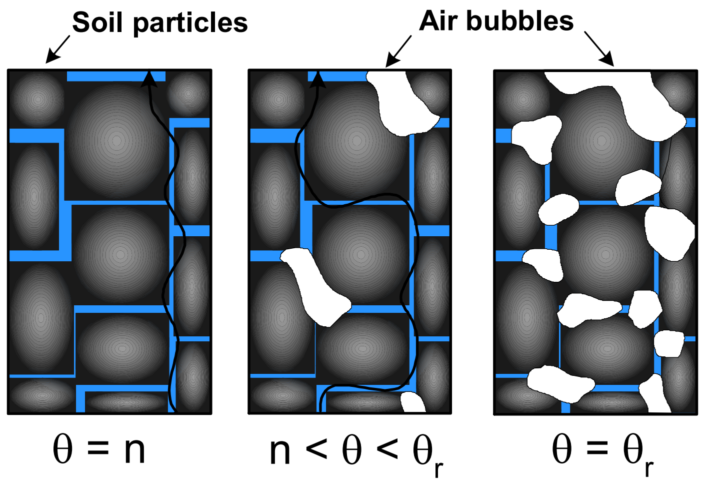


Figure 2-4 Availability of water filled flow paths from saturation to residual

A hydraulic conductivity function should be specified for all materials in a problem that will have an unsaturated zone. Even if the hydraulic conductivity function is an estimate, the results will be more realistic than if the function is omitted or entered as a flat horizontal line. In a unsaturated seepage analysis with negative surface fluxes (such as evaporation) where negative pressures can become extreme, the conductivity function should be defined over a pressure range that exceeds several hundred thousand kPa (or equivalent) of negative pressure. If the function is not defined over the full range, the lowest specified value will be used for any increasingly negative pressures.

Adopting a perfectly flat hydraulic conductivity function (i.e., a constant conductivity) for an unsaturated soil can lead to unrealistic results. The phreatic surface may end up at an unrealistic position, and the proportion of flow through the unsaturated zone may be too high. This occurs because with a horizontal conductivity function, water can flow through the unsaturated zone with the same ease as through the saturated zone. In other words, for a given constant head differential, the volume of flow is the same in the unsaturated zone as in the saturated zone when the hydraulic conductivities in the two zones are the same. In general, water cannot flow through unsaturated soil with the same ease as through saturated soil, because the unsaturated hydraulic conductivity is lower than that of a saturated soil.

To illustrate the effect of assuming that hydraulic conductivity is independent of negative pore-water pressure (i.e., a perfectly flat conductivity function), consider the example of seepage flow through a rectangular screened box, as shown in Figure 2-5. Initially, the box is filled with clay. The phreatic surface will have the form of a hyperbolic curve (top of figure). In the bottom of the figure, the box is enlarged, the upstream half is filled with clay, and the downstream half is filled with sand. The sand is assigned a perfectly flat hydraulic conductivity function. In this case, the phreatic surface in the clay will be at a lower position. The reason for this is that a significant portion of the flow passes through the unsaturated sand. Since the resistance to flow is the same in the unsaturated sand as in the saturated sand, there is no reason for the sand to be saturated in order to conduct the water. Intuition alone reveals that this is not the case. The phreatic surface in the clay should be approximately the same in both configurations, and the seepage that arrives at the clay-sand contact should flow vertically down the contact and then horizontally along the bottom of the box to the exit point at the lower right corner.

To model the clay-sand configuration, the sand needs to be assigned a very steep function, such that as soon as the sand de-saturates, the hydraulic conductivity drops dramatically. This ensures that there is no significant flow in the unsaturated sand. However, a nearly vertical function may cause convergence difficulties. A compromise would be to use a moderately steep hydraulic conductivity function, which would eliminate the majority of the flow in the unsaturated sand and yet produce a reasonable result. It would certainly be closer to the correct solution than for the first case where the sand has a perfectly flat hydraulic conductivity function.

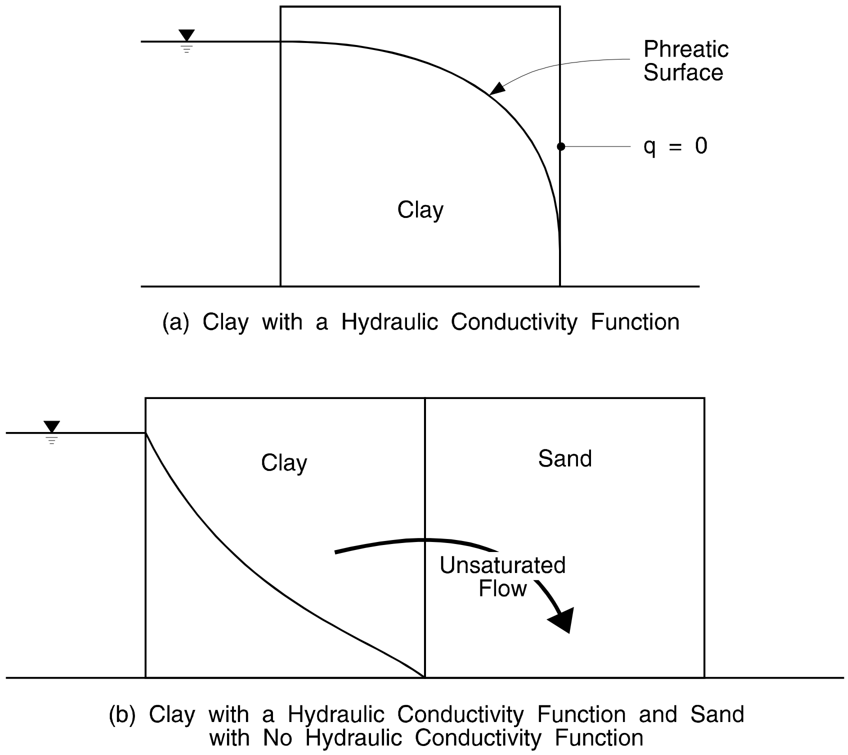


Figure 2-5 Comparison of flow with and without a K function

Coarse granular materials essentially have an infinitely steep (vertical) hydraulic conductivity function when unsaturated. The soil de-saturates completely when the pore-water pressure is zero or negative; consequently, no flow passes through such a soil when it is unsaturated. As a result, the hydraulic conductivity in the unsaturated zone should be infinitely low.

Whenever a problem contains a coarse granular soil that ideally has a near vertical hydraulic conductivity function when unsaturated, it is necessary to ask the question, "Does the material contribute to the dissipation of the head?" If it does not, then consideration should be given to excluding the material from the analysis. In the clay-sand box example, the sand may not contribute to dissipating the head. Consequently, a reasonable solution might be obtained by excluding the sand from the analysis and treating the vertical contact between the two materials as a boundary. The decision as to whether the sand should be included in the analysis must also be made in light of the question, "Does the negative pore-water pressure in the sand contribute to increasing the gradient in the clay?" If it does, the sand must be included in the analysis.

The accuracy with which the hydraulic conductivity needs to be specified depends to some extent on the objective of the analysis. If the primary objective is to compute the distribution of pore-water pressure, then an approximate function may be adequate. On the other hand, if the objective of the analysis is to make reliable time predictions, then it may be necessary to define the storage and hydraulic conductivity with the assistance of laboratory tests.

The level of effort required to define the material functions can be evaluated by performing several analyses with different assumed functions. Performing such a sensitivity analysis can greatly increase the confidence level of the computed results.

In summary, a hydraulic conductivity function must be specified for each material included in an analysis, even if the function is only an approximation.

An approximated curved conductivity relationship in the unsaturated zone results in a much better solution than using a straight, horizontal line.

2.5 Frozen ground hydraulic conductivity

AIR/W and SEEP/W can be used in conjunction with TEMP/W to model transient seepage behavior in frozen, partially frozen, or actively freezing ground. An example of seepage flow being diverted around an active freezing region is illustrated in Figure 2-6 and Figure 2-7. This type of analysis is controlled by the TEMP/W program, but information is passed back and forth between the two solvers as the solution progresses. TEMP/W requires knowledge of the water content in the soil as well as water and air velocities, so that it can compute the convective heat transfer associated with flowing water or moving air. SEEP/W on the other hand requires knowledge of the soil temperature so that it can estimate the reduction in hydraulic conductivity associated with pore-water becoming pore-ice. This estimate is based on knowledge of the soil's unfrozen water content function. The temperature is also used by AIR/W to adjust the air density.

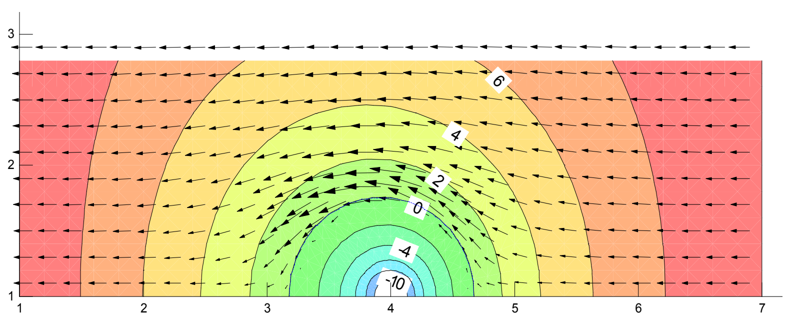


Figure 2-6 Seepage diversion around actively freezing soil region (Temperature contours from TEMP/W displayed)

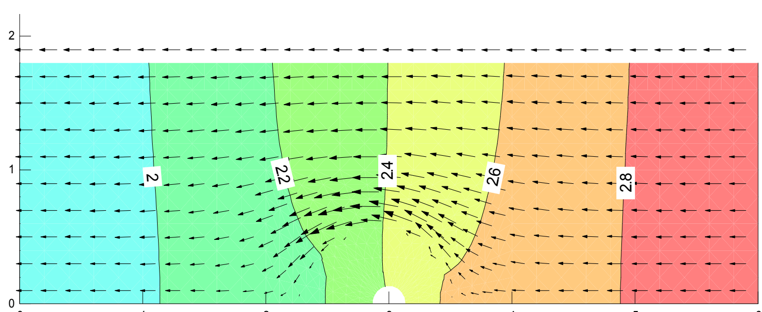


Figure 2-7 Seepage diversion around actively freezing soil region (Head contours from SEEP/W displayed)

The unfrozen water content function relates the amount of unfrozen water to a temperature below freezing and its curve is very similar in appearance to a soil water characteristic curve when plotted on a semi-log scale. The unfrozen water content curve can serve three purposes. It can be used to determine the freezing point depression for pore-water in soils at a given water content below saturation; it can be used to determine the amount of water that remains unfrozen at any given temperature below freezing; and the slope of the curve determines how much latent heat is added to the system by the phase change during the heat and mass transfer analysis.

Ideally, a soil freezing curve should be measured, but this is difficult to do. It is possible to estimate the curve using a measured soil water characteristic (storage) curve and the Clapeyron equation, which relates changes in suction to change in temperature based on equilibrium thermodynamics. Analysis of the Gibbs free energy for any two phases in equilibrium can be used to derive the Clapeyron equation, which relates how the equilibrium pressure changes with a change in temperature. The reduced form of the Clapeyron equation as applied to the soil freezing scenario is given by Black and Tice (1989) as follows:

Equation 2-1 $$\Delta\Psi = -1110\Delta T$$

where:

$\Delta\Psi$ = the soil matric suction (kPa), and

T = the soil temperature below zero Celsius.

The constant value equal to 1110 kPa/oC combines the latent heat of fusion value, specific volume, and the conversion between the freezing temperature of water in Kelvin and degrees Celsius.

If the soil temperature below zero Celsius is passed to SEEP/W from TEMP/W, then the seepage program can use Equation 2-1 to estimate what the approximate frozen condition suction would be such that this suction is used to determine the hydraulic conductivity at each gauss point with freezing temperatures (Newman, 1996).

Seepage analysis in freezing ground can be very complicated, especially in the direct vicinity of the phase change region. At this location it is possible for certain types of soil to experience “cryogenic suction” which results in very steep pressure gradients that can draw water towards the freezing front where it can accumulate and cause frost heave. In the SEEP/W model, this phenomenon is not accounted for. While the suctions are estimated based on temperature and used to determine frozen ground hydraulic conductivity, they are not directly coupled with the thermal equation and therefore only change due to the solution of the seepage partial differential equation.

2.6 Conductivity function estimation methods

The difficult task of measuring the unsaturated hydraulic conductivity function directly is often overcome by predicting the unsaturated hydraulic conductivity from either a measured or predicted volumetric water content function, such as the one illustrated in Figure 2-1. Consequently, this is the preferred approach if a suitable predictive model is available. These estimation methods generally predict the shape of the function relative to the saturated conductivity value which is easily obtained.

SEEP/W has three separate methods built into the model that can be used to predict unsaturated hydraulic conductivity functions using either a measured or estimated volumetric water content function or a saturated hydraulic conductivity function.

Method 1 (Fredlund et al, 1994)

This method consists of developing the unsaturated hydraulic conductivity function by integrating along the entire curve of the volumetric water content function. The governing equation of this method is:

$$k_w = k_s \frac{\sum_{i=j}^{N} \frac{\Theta(e^{y}) - \Theta(\Psi)}{e^{y_i}} \Theta'(e^{y_i})}{\sum_{i=1}^{N} \frac{\Theta(e^{y}) - \Theta_s}{e^{y_i}} \Theta'(e^{y_i})}$$

where:

k_w = the calculated conductivity for a specified water content or negative pore-water pressure (m/s),

k_s = the measured saturated conductivity (m/s),

Θ_s = the volumetric water content,

e = the natural number 2.71828,

y = a dummy variable of integration representing the logarithm of negative pore-water pressure,

i = the interval between the range of j to N,

j = the least negative pore-water pressure to be described by the final function,

N = the maximum negative pore-water pressure to be described by the final function,

Ψ = the suction corresponding to the j[th] interval, and

Θ' = the first derivative of the equation …

$$\Theta = C(\Psi) \frac{\Theta_s}{\left\{ \ln\left[e + \left(\frac{\Psi}{a} \right)^{n} \right] \right\}^{m}}$$

where:

a = approximately the air-entry value of the soil,

n = a parameter that controls the slope at the inflection point in the volumetric water content function,

m = a parameter that is related to the residual water content, and

$C(\Psi)$ = a correcting function defined as

Equation 2-2

$$C(\Psi) = 1 - \frac{\ln\left(1 + \frac{\Psi}{C_r}\right)}{\ln\left(1 + \frac{1,000,000}{C_r}\right)}$$

where:

C_r = a constant related to the matric suction corresponding to the residual water content.

A typical value is about 1500 kPa. The value 1,000,000 in Equation 2-2 corresponds to the matric suction (in kPa) at which there is zero moisture remaining in the soil in a liquid or vapor phase.

Method 2 (Green and Corey, 1971)

The Green and Corey equation is:

Equation 2-3 $$k(\Theta)_i = \frac{k_s}{k_{sc}} \cdot \frac{30\ T^2}{\mu g \eta} \cdot \frac{\xi^p}{n^2} \cdot \sum_{j=i}^{m} \left[\left(2j + 1 - 2i\right) h_i^{-2} \right]$$

where:

$K(\Theta)_i$	=	the calculated conductivity for a specified water content or negative pore-water pressure (cm/min),
$\frac{k_s}{k_{sc}}$	=	the matching factor (measured saturated conductivity / calculated saturated conductivity),
i	=	the last water content class on the wet end (e.g. i=1 identifies the pore class corresponding to the lowest water content, and i = m identifies the pore class corresponding to the saturated water content),
hi	=	the negative pore-water pressure head for a given class of water-filled pores (cm of water),
n	=	the total number of pore classes between i and m,
Θ	=	volumetric water content (cm3/cm3),
T	=	surface tension of water (Dyn/cm),
ξ	=	the water-saturated porosity,
η	=	the viscosity of water (g/cm •s-1),
g	=	the gravitational constant (cm/s-1),
μ	=	the density of water (g/cm3), and
p	=	a parameter that accounts for the interaction of pore classes.

The following are some suggested values of p given by various authors: Marshall (1958): 2.0; Millington and Quirk (1961): 1.3; and Kunze, Vehara and Graham (1968): 1.0.

The shape of the conductivity function is controlled by the term:

$$\sum_{j=i}^{m} \left[\left(2j + 1 - 2i\right) h_i^{-2} \right]$$

in Equation 2-3.

The term:

$$\frac{30\ T^2}{\mu g \eta} \cdot \frac{\xi^p}{n^2}$$

is a constant for a particular function and can be taken to be 1.0 when determining the shape of the hydraulic conductivity function. This is the assumption made in SEEP/W.

SEEP /W first computes the hydraulic conductivity at the zero pressure value using the equation,

$$k_{sc} = \sum_{j=i}^{m} \left[(2j + 1 - 2i)\ h_i^{-2} \right]$$

The saturated conductivity ks is a user-defined value in SEEP /W. When ks is specified, the entire conductivity function is moved up or down by a constant ratio of ks /ksc.

In summary, SEEP /W uses the Green and Corey equation to estimate the shape of the conductivity function and then moves the curve up or down so that the function passes through the user-specified value of ks.

Method 3 (Van Genuchten, 1980)

Van Genuchten (1980) proposed the following closed form equation to describe the hydraulic conductivity of a soil as a function of matric suction:

$$k_w = k_s \frac{\left[1 - \left(a\Psi^{(n-1)}\right)\left(1 + \left(a\Psi^n\right)^{-m}\right)\right]^2}{\left(\left(1 + a\Psi\right)^n\right)^{\frac{m}{2}}}$$

where:

k_s	=	saturated hydraulic conductivity,
a,n,m	=	curve fitting parameters,
n	=	1/(1-*m*), and
ψ	=	required suction range.

From the above equations, the hydraulic conductivity function of a soil can be estimated once the saturated conductivity and the two curve fitting parameters, a and m are known.

Van Genuchten (1980) showed that the curve fitting parameters can be estimated graphically based on the volumetric water content function of the soil. According to van Genuchten, the best point to evaluate the curve fitting parameters is the halfway point between the residual and saturated water content of the volumetric water content function.

The slope of the function can be calculated as:

$$S_p = \frac{1}{\left(\Theta_s - \Theta_r\right)} \left| \frac{d\Theta_p}{d\left(\log \Psi_p\right)} \right|$$

where:

Θ	=	the saturated and residual volumetric water contents respectively,
Θ_p	=	the volumetric water content at the halfway point of the volumetric water content function, and
Ψ_p	=	the matric suction at the same point.

Van Genuchten (1980) proposed the following formula to estimate the parameters m and a when Sp is calculated:

$$m = 1 - \exp\left(-0.8S_p\right)$$

for Sp between 0 and 1;

$$m = 1 - \frac{0.5755}{S_p} + \frac{0.1}{S_p^2} + \frac{0.025}{S_p^3}$$

for Sp > 1; and

$$a = \frac{1}{\Psi}\left(2^{\frac{1}{m}} - 1\right)^{(1-m)}$$

2.7 Air phase conductivity and storage

In order to model the flow of air in response to user-specified air pressure or temperature gradients, it is necessary to define the conductivity function for the air phase in the unsaturated soil.

When we estimate the water conductivity function we require the saturated water conductivity, Ksat. Likewise, when we estimate the air conductivity function we require the dry soil air conductivity, Kair-dry. This value can be estimated using the saturated water conductivity value along with the understanding that the relative difference in conductivity between air and water is related to the absolute viscosity difference between the two phases (Fredlund and Rahardjo, 1993). The absolute viscosity of air is 56 times less than that of air under fixed and similar conditions, so this can be used as an estimate. However, various factors affect this relationship so instead of making the assumption inherent in the finite element solution, the user is required to enter the dry state air phase permeability value.

With a water content function fully specified the air phase permeability can be estimated by a Brooks and Corey type equation as follows:

$$K_{air} = K_{dry\_air}(1-S)^{0.5}(1-S^{\frac{1}{q}})^{2q}$$

where:

q = a value of 2.9.

Examples of estimated functions for three soil types are shown in Figure 2-8.

If a transient analysis is carried out, then it is also necessary to define an air phase storage function. The user must input a water phase storage function, which relates volumetric water content to soil water pressure. Due to the inherent assumption that the soil is incompressible, any water that leaves the soil is replaced by air. Thus, the volume of air in the soil is equal to the porosity of the soil minus the volumetric water content for any given suction. This is a convenient fact because the change in volume of air in response to a change in soil water suction is the negative of the change in volume of water in response to the same change in suction. For example, if 10% of the water by volume leaves the soil, there is a corresponding increase of 10% air by volume. In other words, the user does not need to enter a storage function for the air phase because it is equal to the negative inverse of the storage function for water.

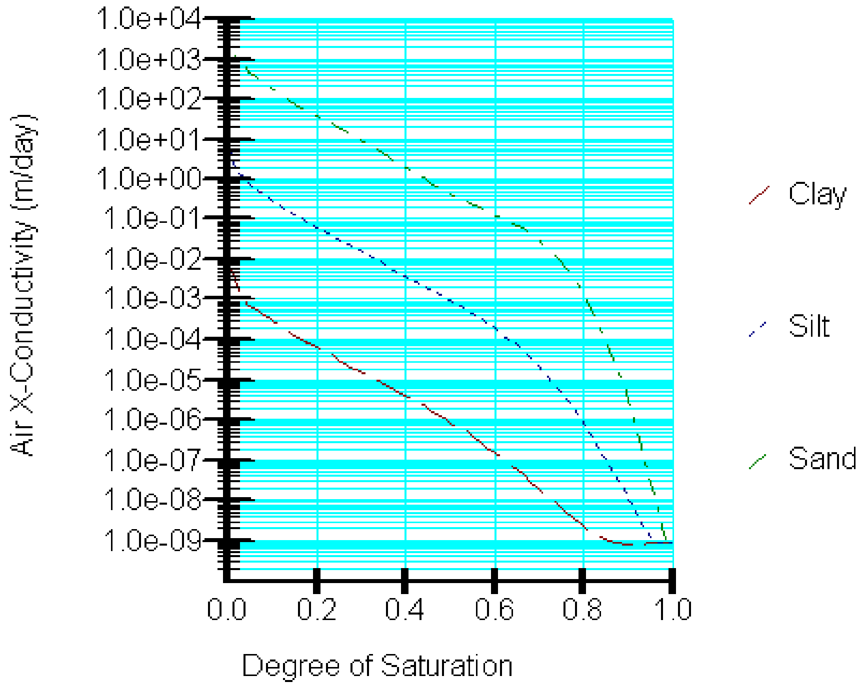


Figure 2-8 Air permeability relationships

2.8 Interface model parameters

The interface model allows you to assign a material model to a line and to give that line a thickness. In a seepage application, you may want to use an interface model to simulate a thin liner or a wick drain. When you assign an interface model to a line you must give it hydraulic conductivity values that are both normal and tangent to the direction of the line as shown in Figure 2-9. The air conductivity value you specify is independent of direction.

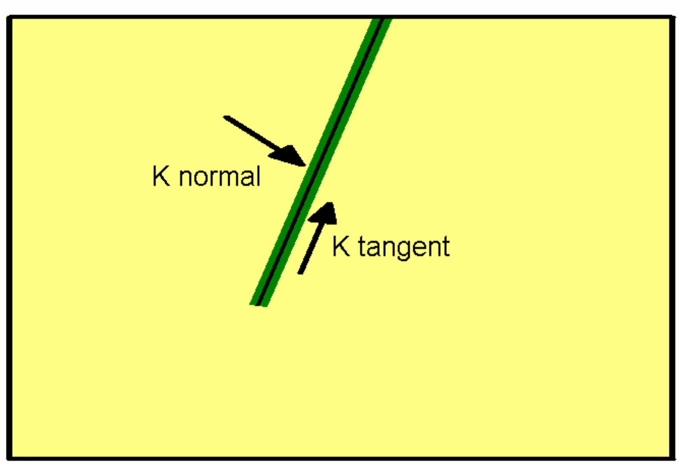


Figure 2-9 Illustration of interface model assigned to a line

2.9 Thermal functions (when coupled with TEMP/W)

Unfrozen water content function

Fundamental to the formulation of a general freeze-thaw thermal analysis of soil systems is an understanding of how energy stored in the soil varies as the soil temperature changes. An example of this relationship is shown in Figure 2-10. The function represents the relative energy required for the soil medium to sustain a certain temperature. The steep part of the function in the region of the freeze-thaw front represents the latent heat absorbed or released by a soil due to phase changes of the soil water. The slope of the function away from the freeze-thaw zone represents the volumetric heat capacity of the frozen and unfrozen zones.

Except in the case of a pure water medium, the water within the soil changes from liquid to ice or from ice to water over a temperature range. In other words, not all of the water within the soil experiences a phase change at a single temperature. The percentage of the soil water volume that remains unfrozen at a certain temperature is referred to as the unfrozen water content.

The unfrozen water content characteristic of a soil can be expressed as a function of temperature. An example of the unfrozen water content function for a coarse material is illustrated in Figure 2-10. In TEMP/W, the unfrozen water content is expressed as a value between 0 and 1.0. An unfrozen water content of 1.0 means that 100% of the water in the soil medium is unfrozen. Similarly, an unfrozen water content of 0.0 means that 0% of the water in the soil medium is unfrozen.

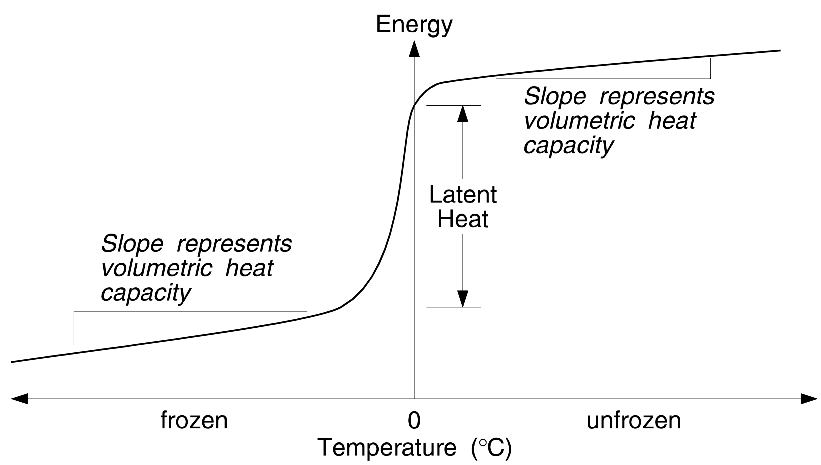


Figure 2-10 An example of an energy storage function

Thermal conductivity

Thermal conductivity k, which characterizes the ability of a soil medium to transmit heat by conduction, is defined as the quantity of heat that will flow through a unit area of a soil medium of unit thickness in unit time under a unit temperature gradient. Thermal conductivity units commonly are J/(sec.m.°C), kJ/(day.m.°C) or Btu/(hr.ft.°F).

At temperatures above zero (0°C), the thermal conductivity tends to increase with increasing water content (see Harlan and Nixon, 1978, pp. 103-163).

At temperatures below zero (0°C), the thermal conductivity is usually greater than that of an unfrozen soil, since the thermal conductivity of ice is much higher than that of water. The thermal conductivity of frozen soil is highly dependent on the unfrozen water content within the soil.

Furthermore, coarse-grained soils are commonly dominated by quartz, which has a relatively large thermal conductivity; fine-grained soils are dominated by clayey minerals, which generally have a substantially lower thermal conductivity.

TEMP/W users need only specify the function for the unfrozen zone and, based on this, the thermal conductivity in the frozen zone will be computed internally by the solver as a function of ice, water, air and soil mineral content.

If the ice content is greater than zero TEMP/W back calculates the soil particle conductivity. It then computes the frozen soil conductivity at the given water content assuming all water is frozen according to:

$$Kf = \left(K_{ice}\right)^{wc}\left(K_{soil}\right)^{1-wc}$$

This relationship is based on that proposed by Johansen (1975) and described in the following section.

Once the fully frozen conductivity is known, the partially frozen thermal conductivity of the ground is assumed to be linearly partitioned between the unfrozen and frozen states by the ratio of ice content to water content.

Estimating thermal conductivity for soils

Various empirical or semi-empirical methods have been developed for estimating the thermal conductivity of soils. These methods have been evaluated in detail by Farouki, (1981), who concluded that different types of soils may require different estimation methods. The method developed by Johansen, (1975), however, appears to be the most general method. Since it is beyond the scope of this chapter to present the various estimation methods, only Johansen's method is presented in this section.

For dry, natural soils, the thermal conductivity kdry can be estimated based on its dry density using the following equation:

$$K_{dry} = \frac{0.135\gamma_d + 64.7}{2700 - 0.947\gamma_d} \pm 20\%$$

where the dry density γd is in kg/m3 and the unit weight of soil particles is taken as 2700 kg/m3.

For dry crushed rock materials, the thermal conductivity kdry can be estimated based on its porosity n using the following equation:

$$K_{dry} = 0.039n^{-2.2} \pm 25\%$$

For a saturated unfrozen soil, the thermal conductivity ksat is estimated based on the thermal conductivities of its components and their respective volume fractions.

$$K_{sat} = \left(K_s\right)^{1-n}\left(K_w\right)^{n}$$

where, ks is the thermal conductivity of the soil particles, and kw is the thermal conductivity of the pore water.

For a saturated frozen soil containing some unfrozen water content, wu, the thermal conductivity ksat becomes:

$$K_{sat} = \left(K_s\right)^{1-n} \left(K_i\right)^{n-w_u} \left(K_w\right)^{w_u}$$

where ki is the thermal conductivity of ice.

For an unsaturated soil, the thermal conductivity kunsat is estimated based on its saturated conductivity, dry conductivity and degree of saturation S using the following equation:

$$K_{unsat} = (K_{sat} - K_{dry})K_e + K_{dry}$$

where:

K_e = 0.7 LogS + 1.0 for unfrozen coarse grained soil,
K_e = LogS + 1.0 for unfrozen fine grained soil, and
K_e = S for frozen soil.

The above equations may be used for a rough, general estimation of the thermal conductivity of a soil. It is your responsibility to ensure the applicability of the above equations to the respective soils.

Typical values of thermal conductivity

The data in Table 2-1 below provides typical values of thermal conductivity for various materials, (extracted from Johnston, Ladanyi, Morgenstern, and Penner, 1981). When using this data in the function estimation routine in GeoStudio, make sure the engineering units of the data you select match the units set you have chosen for the model and make sure you have set your time units prior to estimating these functions.

Table 2-1 Unit weight and thermal conductivity of various materials

Material	Unit Weight	Conductivity		
	lb/(ft^3)	Btu/(hr·ft·°F)	J/(sec·m·°C)	kJ/(day·m·°C)
Water	62.4	0.35	0.605	52.27
Ice	57	1.29	2.23	192.7
Air (dry, still)	0	0.014	0.024	2.07
Snow				
loose, new	–	0.05	0.086	7.43
on ground	–	0.07	0.121	10.45
dense, compacted	–	0.20	0.340	29.37
Soil and rock minerals				
shale	–	0.9	1.5	129.6
evaporites	–	3.1	5.4	466.6
limestone	168	0.75-2.9	1.3-5.0	112-432
dolomite	178	2.9	5.0	432
sandstone	–	1.1-2.4	1.8-4.2	155-86.4
schist	–	0.90	1.6	138.24
gneiss	–	1.4	2.5	216
greenstone	–	1.9	3.3	285.12
slate	–	2.2	3.8	328.3
argillite	–	1.9	3.3	285.12
quartzite	–	2.6-4.1	4.5-7.1	388.8-613
granite	–	1.0-2.3	1.7-4.0	146.9-345.6
diabase	–	1.2	2.1	181.4
gabbro	–	1.4	2.5	216
grandiorite	–	1.5	2.6	224.6

Volumetric heat capacity

The heat capacity of a material is defined as the quantity of heat required to raise the temperature of the material by a unit degree. When expressed on a per unit weight basis, this quantity of heat is referred to as the specific heat capacity; when expressed on a unit volume basis, the quantity is known as the volumetric heat capacity. The units for specific heat capacity are J/(kg.°C) or kJ/(kg.°C) and Btu/(lb.°F), and the units for volumetric heat capacity are J/(m3.°C) or kJ/(m3.°C) and Btu/(ft3.°F).

TEMP/W users need only specify the function for the unfrozen zone and, based on this, the volumetric specific heat in the frozen zone will be computed internally by the solver as a function of ice, water, air and soil mineral content using the relationships described in the following section.

Estimating volumetric heat capacity

TEMP/W uses the volumetric heat capacity in its formulation. The volumetric heat capacity of a soil can be approximated by the dry density of the soil and the sum of the specific heat capacities of its different constituents (namely, soil particles, water, ice, and air). The air component is very small and generally is neglected. A general equation for estimating unfrozen and frozen volumetric heat capacity is: (see Johnston, Ladanyi, Morgenstern and Penner, 1981, pp. 73-147).

$$C = c\gamma = \gamma_d \left[c_s + c_w w_u + c_i w_f \right]$$

where:

C	=	volumetric heat capacity of the soil,
c	=	specific heat capacity of the soil,
γ	=	bulk density of the soil,
γ_d	=	dry density of the soil,
c_s	=	specific heat capacity of soil particle,
c_w	=	specific heat capacity of water,
c_i	=	specific heat capacity of ice,
w_u	=	unfrozen water content expressed in % of dry weight of the soil, and
w_f	=	frozen water content expressed in % of dry weight of the soil.

Examples

The following examples illustrate how to estimate the volumetric heat capacity for various soils.

Example 1

For a dry mineral soil with dry density of 120 lb/ft3 or 1923 kg/m3, the volumetric heat capacity of the soil can be estimated as:

C = 120 lb/ft3 * 0.17 Btu/(lb . °F)
= 20.4 Btu/(ft3 . °F)

or in SI units as:

C = 1923 kg/m3 * 0.71 kJ/(kg . °C)
= 1365 kJ/(m3 . °C)

Example 2

For an unfrozen mineral soil with dry density of 120 lb/ft3 or 1923 kg/m3, and unfrozen water content of 0.25, the volumetric heat capacity of the soil can be estimated as:

C = 120 lb/ft3 * [0.17 Btu/(lb . °F) + 1.0 Btu/(lb . °F) * 0.25]
= 50.4 Btu/(ft3 . °F)

or in SI units as:

C = 1923 kg/m3 * [0.71 kJ/(kg . °C) + 4.187 kJ/(kg . °C) * 0.25]
= 3378 kJ/(m3 . °C)

Typical values of volumetric heat capacity

Table 2-2 below provides typical values the specific and volumetric heat capacity of various materials, (extracted from Johnston, Ladanyi, Morgenstern and Penner, 1981 and Harlan and Nixon, 1978):

Table 2-2 Specific and Volumetric Heat Capacities of Various Materials

Material	Mass Specific Heat Capacity		Volumetric Heat Capacity	
	Btu/(lb·°F)	kJ/(kg·°C)	Btu/(ft³·°F)	kJ/(m³·°C)
Water	1.00	4.187	62.4	4187
Ice	0.50	2.094	28.1	1880
Air	0.24	1.0	0.0187	1.25
Soil minerals	0.17	0.71	28.0	1875
Organic soil minerals	0.40	1.674	37.5	2520
Extruded polystyrene insulation	0.24	1.0	0.65	43.5
Concrete	0.21	0.895	30.0	2010
Asphalt	0.40	1.674	37.5	2520
Snow, fresh	–	–	3.11	209
Snow, drifted and compacted	–	–	7.80	523.5
Granite	–	–	37.1	2490
Limestone	0.29	1.2	48.9	3285
Dolomite	0.21	0.88	37.4	2510
Sandstone	–	–	37.4	2510
Shale	–	–	27.4	1840
Glass	–	–	26.2	1760
Steel	0.11	0.46	56.0	3890
Wood	0.19	0.8	7.79	523

2.10 Sensitivity of hydraulic results to material properties

How sensitive is a model to changes to the air-entry value, the slope of the function, the residual volumetric water content and the saturated hydraulic conductivity? The effect of altering each of the four material property functions is highlighted below, through a series of steady-state and transient analyses where only one parameter is changed at a time to clearly evaluate the influence of each parameter. Figure 2-11 shows a cross-section of a system where both saturated and unsaturated conditions exist. This cross-section represents a two-dimensional view of a flow system in which water from a canal passes through an unconfined, homogeneous, fine sand aquifer and is collected in a series of collection wells located along the right edge of the cross-section.

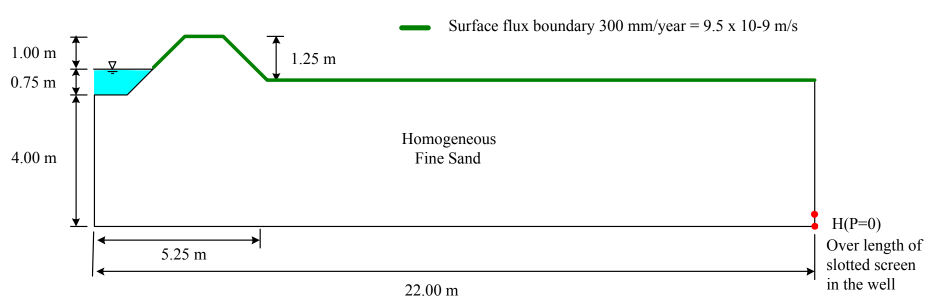


Figure 2-11 Diagram of system used in sensitivity analysis

Changes to the air-entry value (AEV)

To show how sensitive the model is to changes in air-entry value, a steady-state simulation was conducted where the AEV of a function was increased from 3 to 10 kPa. A volumetric water content function is not required for a steady-state analysis, however it is good practice to always make changes to the volumetric water content function when conducting a sensitivity analysis, and then let the changes be reflected in the hydraulic conductivity function by recalculating the function from the value of the saturated hydraulic conductivity and the newly specified volumetric water content function. Figure 2-12 shows the adjusted volumetric water content function and the predicted hydraulic conductivity functions used in the simulations. It is important to note that if the K-function was presented on a log-log plot (hydraulic conductivity functions are usually presented in the literature on a log-log scale), the effect of increasing the air-entry value and adjusting the rest of the curve accordingly would appear to also steepen the slope of the function. In SEEP/W however, the functions are always presented on a log-arithmetic scale and the AEV can be increased while the slope of both functions remain similar in shape. Figure 2-13 and Figure 2-14 show the SEEP/W modeling results for both the 3 kPa and 10 kPa AEV simulations respectively.

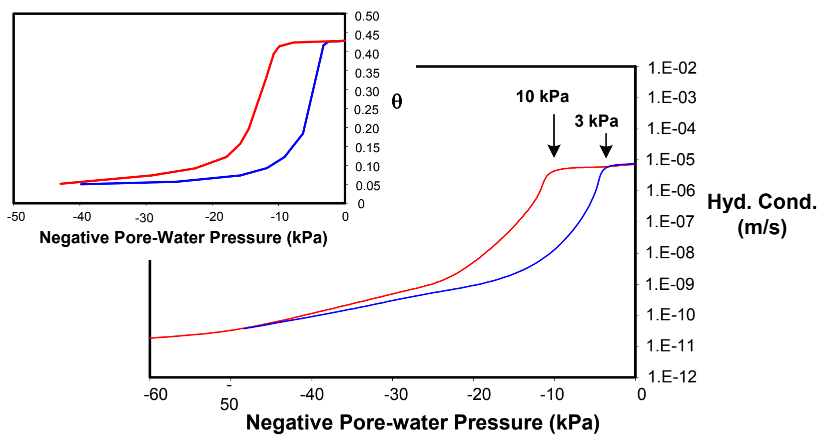


Figure 2-12 Material property functions used in the AEV sensitivity analyses

To compare the results in Figure 2-13 and Figure 2-14, it is probably easiest to consider the height of the capillary fringe that is emphasized in the magnified sections. A dimension arrow has been superimposed to show the extent of capillary rise that develops for both simulations. The capillary rise is the height above the water table where negative pore-water pressures exist, but the soil remains saturated due to capillary tension. The air-entry value, when converted from pressure (kPa) to a pressure head (m), is approximately equal to the height of the capillary fringe. In the capillary fringe, water is transported through the soil at a rate equal to the saturated hydraulic conductivity, so more water can be transported in a larger capillary fringe than in a smaller one. As an interesting aside, note how the structure of the model (i.e., the downstream side of the berm) controls the shape of the unsaturated flow system. Even though saturated flow occurs in the berm, the water table is still at depth and negative pore-water pressures exist on the downstream face so a seepage face never develops.

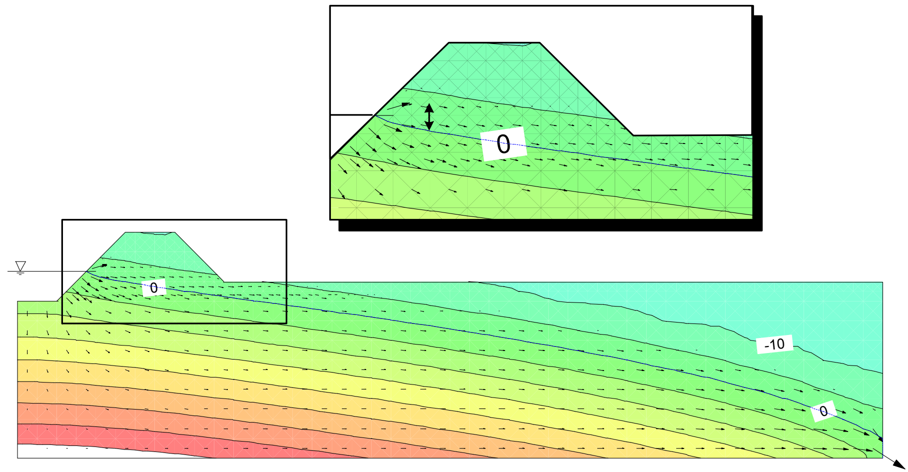


Figure 2-13 Pressure contours and flow vectors for a 3 kPa AEV material

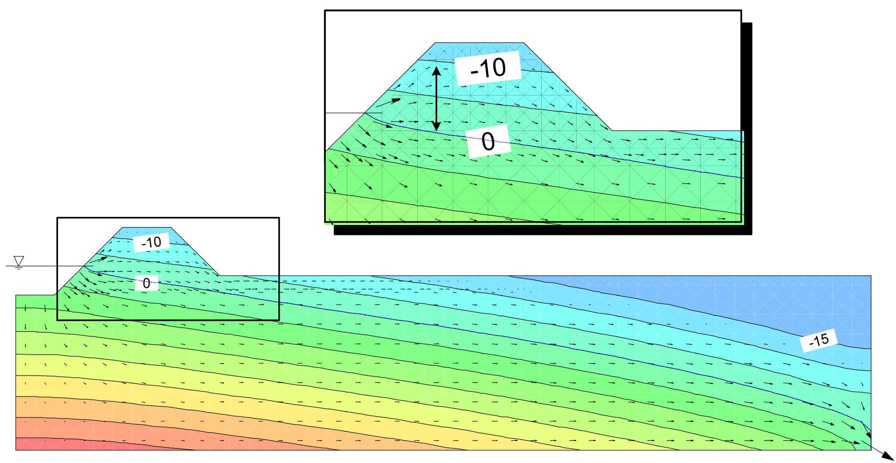


Figure 2-14 Pressure contours and flow vectors for a 10 kPa AEV material

Changes to the saturated hydraulic conductivity

Six steady state simulations were conducted to evaluate the effect of changing the saturated hydraulic conductivity (Table 2-3). Basically, the first three simulations compared the effect of both increasing and decreasing the saturated hydraulic conductivity and the last three showed the effect of conducting a saturated hydraulic conductivity sensitivity analysis with the added influence of an applied surface flux boundary condition.

Table 2-3 Summary of simulations for Ksat sensitivity analyses

Simulation	Ksat (m/s)	Surface Flux (m/s)
1	7.5×10^{-4}	none
2	7.5×10^{-6}	none
3	7.5×10^{-8}	none
4	7.5×10^{-4}	9.5×10^{-9}
5	7.5×10^{-6}	9.5×10^{-9}
6	7.5×10^{-8}	9.5×10^{-9}

The three hydraulic conductivity functions that were used in the simulations are presented in Figure 2-15. The general shape of the function remained unchanged while the saturated hydraulic conductivity was adjusted.

Figure 2-16 to Figure 2-18 show the results obtained from simulations 1 to 3 (see Table 2-3). A surface flux was not applied and the Ksat was varied by two orders of magnitude between each simulation. The resulting total head contours and total flux values that were determined near the well screen are included in the figures. One of the most significant comparisons to make is with respect to the total head contours. Altering the saturated hydraulic conductivity does not alter the shape of the flow net, so the total head contours should be and are the same. The only obvious difference between the results can be found in the value associated with the flux section near the well. The flux varies along the same order of magnitude that the Ksat was varied, so increasing the saturated hydraulic conductivity results in a greater flow rate to the well and decreasing it reduces the amount of flow to the well.

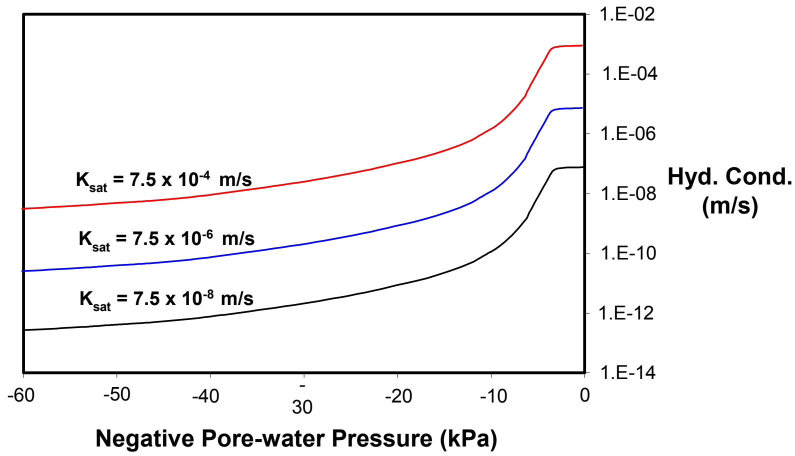


Figure 2-15 Hydraulic conductivity functions used for Ksat sensitivity analyses

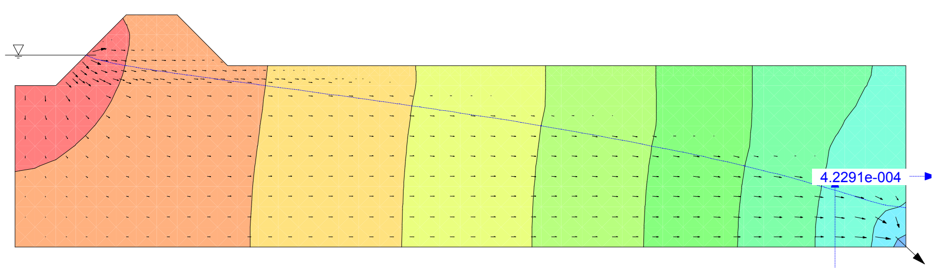


Figure 2-16 Simulation #1 total head contours (Ksat 7.5x10^{-4} m/s)

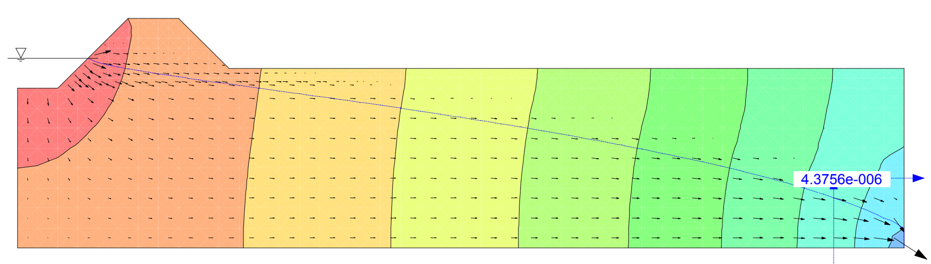


Figure 2-17 Simulation #2 total head contours (Ksat = 7.5x10^{-6} m/s)

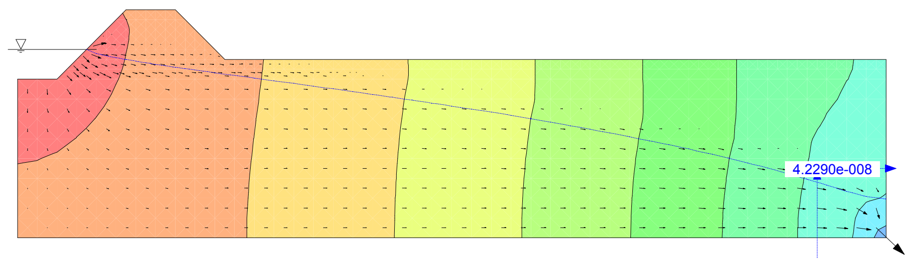


Figure 2-18 Simulation #3 total head contours (Ksat = 7.5x10^{-8} m/s)

The results presented in the above figures did not include a surface flux boundary condition. Results obtained from sensitivity analyses (regarding Ksat) where surface fluxes are applied, are not nearly as easy to compare. The next three figures show the results from simulations 4 to 6 from Table 2-3. In these cases, a surface flux boundary condition of 9.5 x 10-9 m/s (equivalent to 300 mm/year net infiltration) was applied over the surface and the Ksat was once again varied by two orders of magnitude between simulations.

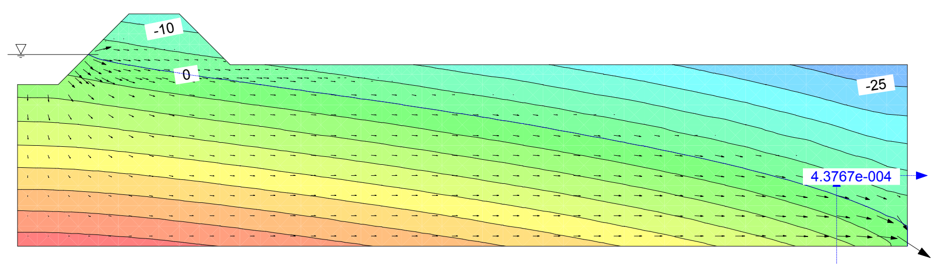


Figure 2-19 Simulation #4 pressure head contours (Ksat = 7.5x10^{-4} m/s)

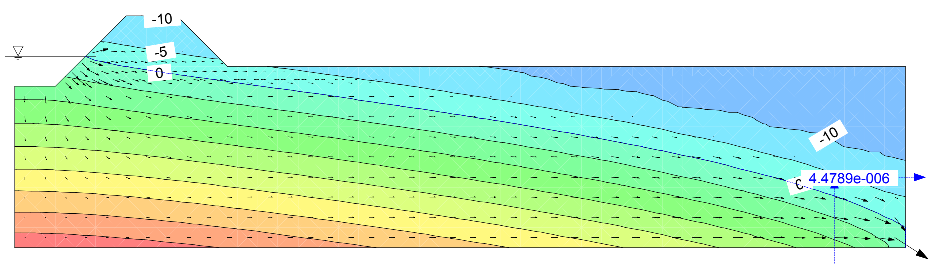


Figure 2-20 Simulation #5 pressure head contours (Ksat = 7.5x10^{-6} m/s)

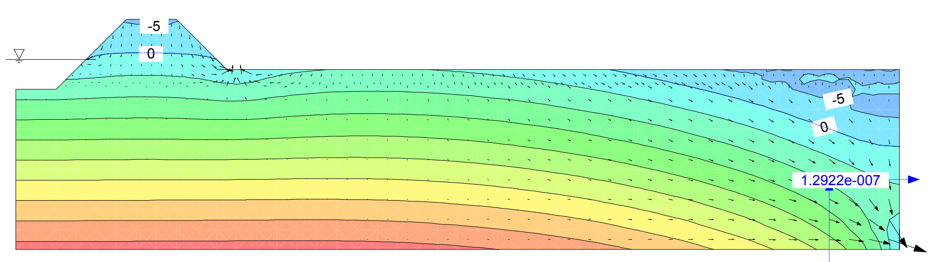


Figure 2-21 Simulation #6 pressure head contours (Ksat = 7.5x10^{-8} m/s)

The pressure head contours (and consequently the total head contours) are very different between the three simulations where a surface flux was applied. This time the system has two water sources to contribute to the total flow; the upstream ponded water and the applied surface flux. The effect on the flow system is minimal in Figure 2-19 because the Ksat is much greater than the applied surface flux (10-

4 compared to 10-9 m/s). As the difference between the Ksat and the applied flux gets smaller, the effect on the pressure profiles becomes more significant. In Figure 2-21, the system is close to becoming fully saturated and a seepage face develops on the downstream side of the berm. The rate at which the water can leave the system (i.e., the well), is still controlled by the saturated hydraulic conductivity, but in the case of analysis with a surface flux applied, the total flux value leaving the profile is greater.

Changes to the slope of the VWC function

In a steady-state analysis, the amount of flow into the system corresponds to the flow rate out of the system. However, in a transient seepage analysis the flow into the system may differ from the flow out of the system because the system stores or releases water. It therefore becomes necessary to account for the change of water stored within a soil profile with time. The amount of water stored or retained is a function of the pore-water pressures and the characteristics of the soil structure and is described by the volumetric water content function. A transient analysis can be used to evaluate the effect of altering the slope of the volumetric water content function (mw). In any transient analysis there are two main considerations; how fast the water is flowing (a function of the hydraulic conductivity) and how much water is flowing (a function of the change in storage and the amount of water in the system). As a result, both material property functions must be defined. Storage is the amount of water that remains in the pores of a soil under negative pore-water pressures. If the slope of the VWC function is flat, the change in volumetric water content for increasingly negative pore-water pressures would be less than for a soil with a steeper function. Figure 2-22 shows the volumetric water content and hydraulic conductivity functions used for the sensitivity analysis regarding the slope of the VWC function. Creating a function with a flatter slope represents a soil which is non-uniform and has a larger distribution of pore sizes. The air-entry value (a function of the largest pore size) has not been changed, nor has the residual water content (a function of the smallest pore size). The modifications were made to the VWC function and the changes were then reflected in the hydraulic conductivity function through the use of predictive methods.

In order to obtain initial head conditions, steady-state analyses were conducted using both the modified and unmodified material property functions. A pond depth of 0.75m was included for the steady-state analyses and was then removed for the start of the transient analyses, allowing the system to drain into the well for 40 days. The results from both transient analyses are presented in Figure 2-23.

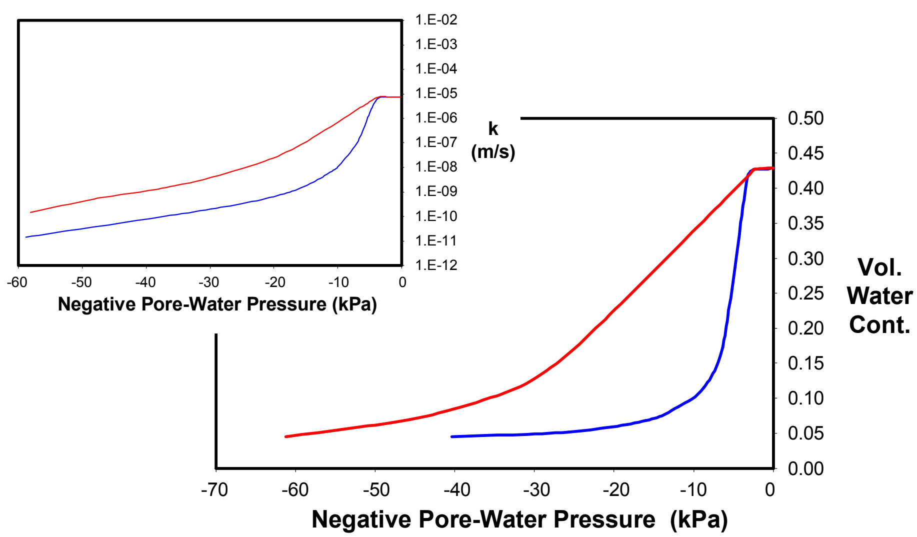


Figure 2-22 Material property functions used in the sensitivity analyses for changes to the slope of the storage function

One way to evaluate the effect of altering the slope of the VWC function is to compare how long it takes each simulation to lower the water table to the same elevation. As can be seen in Figure 2-23, it only took 22 days of drainage, using the modified function, to lower the P=0 contour (water table) to the same elevation as that of the unmodified function after 40 days of drainage. The time difference can be explained in part by comparing water content profiles taken at the same location. Figure 2-23 shows the initial water content for both simulations as a vertical, solid black line. The red line indicates the water content profile after 22 days of drainage using the modified function and the blue line indicates the water content profile after 40 days of drainage using the unmodified function (the one with the steeper slope). The amount of water removed from the system for each soil type can be estimated as the area between the black line and the red or blue line respectively. The water content profile of the modified soil (red) shows that the soil is wetter, having stored more water in the unsaturated zone than the unmodified soil (blue). As a result, the amount of water released from the system is less than that of the unmodified VWC function. The majority of the water removed for both soils was through the saturated flow system, and since the Ksat remained unaltered between the simulations, it took less time to drain less water.

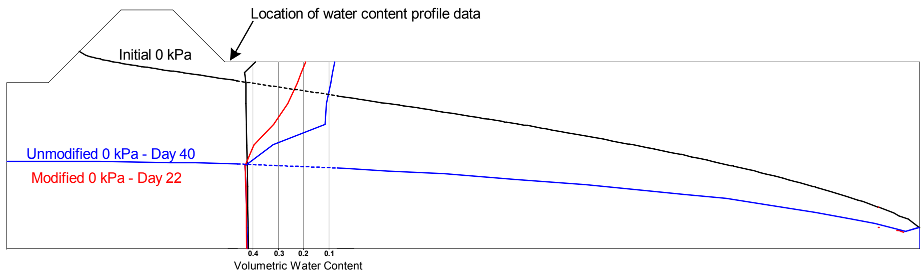


Figure 2-23 Location of P=0 kPa contour and water content profiles for the modified function (day 22) and unmodified function (day 40)

Changes to the residual volumetric water content

The last feature of the volumetric water content function to evaluate in the sensitivity analysis is the residual water content. The effect of changing the residual volumetric water content does not alter the hydraulic conductivity function, so only the VWC function was adjusted such that the residual water content for the modified function was much greater than the unmodified function as shown in Figure 2-24.

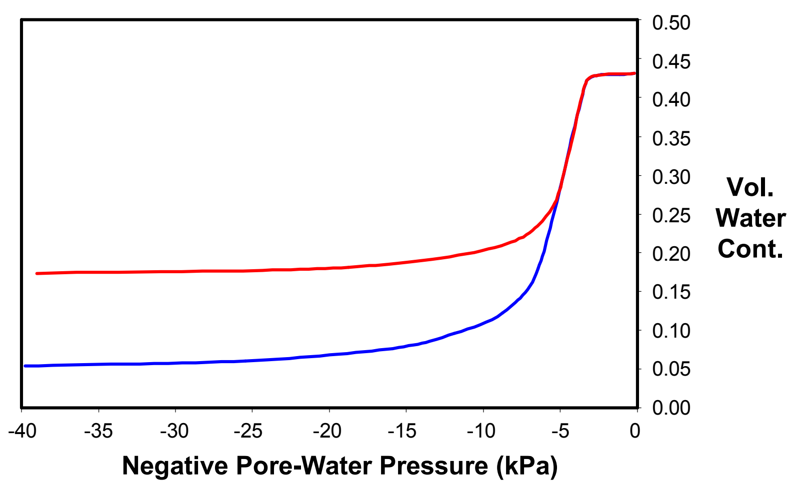


Figure 2-24 Modified and unmodified residual water contents used in analyses

Both steady state and transient analyses were conducted in a manner similar those described in the last section in terms of having the pond in place for the steady-state simulation and then letting the system drain over a 40 day period. Intuitively, altering the volumetric water content to have higher volumetric water content at residual should result in a wetter unsaturated profile. To confirm this thinking, the length of time that it took to lower the P=0 pressure contour (water table) to the same level as in Figure 2-23 was determined. The results are presented in Figure 2-25.

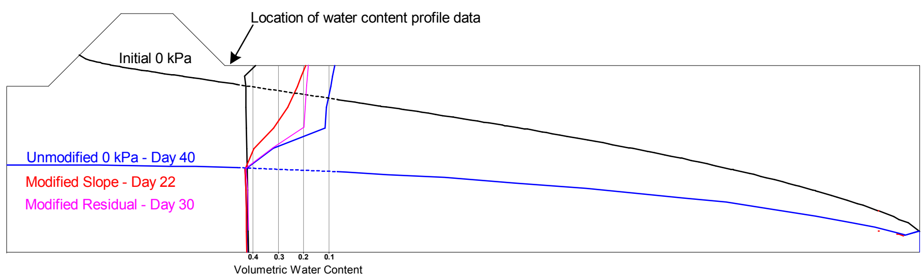


Figure 2-25 Location of P=0 kPa contours and water content profiles for the modified slope function (day 22), the modified residual (day 30) and the unmodified function (day 40)

With the completely unmodified function, it took 40 days to lower the P=0 contour (water table) to the location represented in Figure 2-25. It took the function with the shallower slope 22 days to have the P=0 contour lower to the same elevation. Increasing the residual water content of the unmodified function resulted in the P=0 contour reaching the same elevation after 30 days.

Therefore, increasing the residual water content of the volumetric water content function resulted in a wetter profile above the P=0 contour (as shown by the pink water content profile) than that of the unmodified function (as shown by the blue water content profile). Less water was released from the system and so it took less time to release the water. The greatest amount of water storage results in the least amount of water being released from the system. This occurred using the volumetric water content function with the shallow slope.

In summary, in modeling unsaturated systems, appropriate definition of the volumetric water content function is critical to achieving a representative solution. It is important to know how the results can be affected by varying the main features of the function and how to conduct a sensitivity analysis if you are unsure of your material property functions. Conducting a sensitivity analysis will help you gain confidence in the results and to increase your understanding of how the system being modeled will respond to changes in the material properties.

2.11 Sensitivity of thermal results to material properties and water content in soil

In seepage analyses where the hydraulic conductivity values can change many orders of magnitude a sensitivity study usually considers multiplying or dividing values by a factor of 10 in order to see a change in output data. With thermal properties, the change in values (be they thermal conductivity or heat storage) generally occur within a 1/3 order of magnitude. In addition, if freezing is taking place, the amount of unfrozen water that freezes and releases latent heat can be a significant factor.

In this discussion, the attempt to calibrate a thermal model to real site data is used for illustrative purposes. The geometry of the model was established and default material properties were used in the initial simulations based on previous modeling for the site. At iterative approach was then used to adjust material properties until a reasonable agreement between computed and actual observed temperatures was obtained. The calibrated model was needed for future site planning.

There are three primary material properties that can be adjusted. These are listed below with an explanation of the affect they have on the computed temperature trends. The comparison between actual measured and best fit calibrated data is provided in Figure 2-26.

Thermal Conductivity: Increasing this value increases the rate at which the temperature drops to the freezing point and then after the phase change has occurred. It is evident in the slope of the curves in the figure at temperatures above freezing and at temperatures below about –4 C when most of the pore-water is frozen.

Volumetric Water Content: Adjusting the water content affects the length of time the temperature profile "hovers" around the freezing point before it starts to drop off rapidly again. This length of time is directly related to the latent heat that is released so the more water there is, the more latent heat, and the longer it takes to be removed.

Slope of Unfrozen Water Content Function: This parameter controls the rate at which the water is frozen at temperatures just below freezing. If the function is steep, all the water freezes just below 0 C and the temperature decay curve drops off quickly at a rate almost equal to the rate before the freezing point was reached. If the function is shallow and unfrozen water is still present at temperatures around –2 to –4 C, then the temperature decay curve shows a slower drop in the temperature range just below 0 C. Steep functions are typical of very coarse-grained material and shallow functions are typical of fine-grained materials.

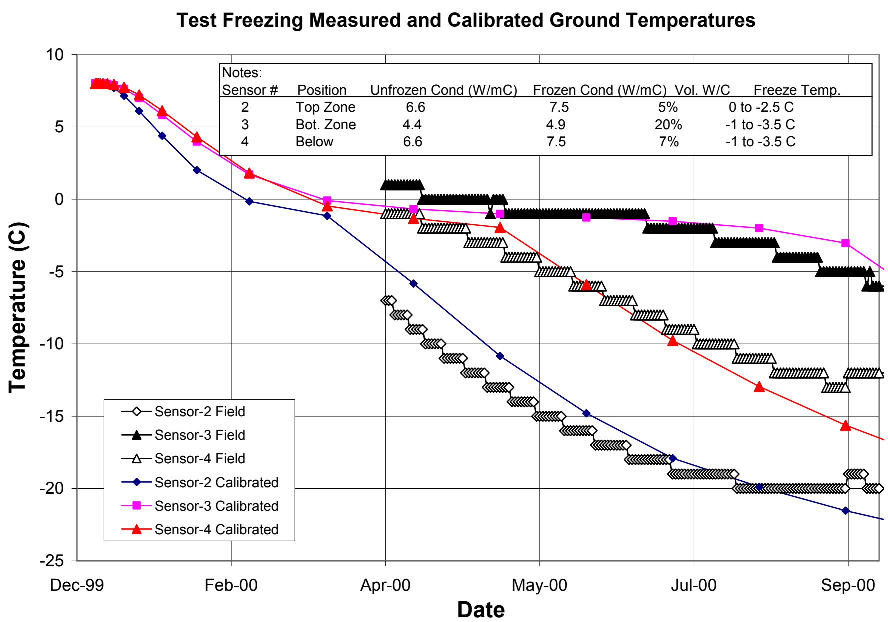


Figure 2-26 Comparison between actual and calibrated ground temperatures

When the model was initially run using assumed material properties the predicted freezing times were much longer than those observed. In fact, the temperatures were just starting to pass through the 0 C point by the end of the test freezing period. The slopes of the decay curves from the initial temperature of +8 C to the 0 C point indicated that the material thermal conductivity was too low. Sensors 2 and 3 were assumed to be in a clay/ore type ground with very low thermal conductivity and sensor 4 was assumed to be in a broken sandy type ground. The model was re-run a few times with slight increases to the thermal conductivity on each subsequent iteration. When the drop in temperature from +8 C to 0 C agreed somewhat with the measured data the thermal conductivity was held constant.

Once the thermal conductivity was adjusted it was necessary to adjust the volumetric water contents in order to match the time the temperature profile "hovered" near the freezing point. The final calibrated water contents are likely lower than one may expect, but even a slight increase in the calibrated values resulted in several more weeks of freezing delay near the freezing point. The final calibrated values are within the ranges originally reported by the project owner.

Finally, the shape of the unfrozen water content function was adjusted in order to approximate the rate of decay of the temperatures after they have passed through the freeze point. No extreme changes were made to the material properties (compared to earlier modeling), but they were adjusted somewhat to approximate a more fine-grained, than coarse-grained material.

In carrying out all of the above changes it was important to ensure that the final combination of material properties for any given soil made sense for that soil. In other words, care was taken to not have water contents typical of clay, combined with a thermal conductivity obtained from a solid rock core sample, applied to a soil with an unfrozen water content function found in beach sand

3 Boundary Conditions

3.1 Introduction

Specifying conditions on the boundaries of a problem is one of the key components of a numerical analysis. This is why these types of problems are often referred to as "boundary-valued" problems. Being able to control the conditions on the boundaries is also what makes numerical analyses so powerful.

Solutions to numerical problems are a direct response to the boundary conditions. Without boundary conditions it is not possible to obtain a solution. The boundary conditions are, in essence, the driving force. What causes air flow? It is the air total head difference between two points or some specified rate of flow into or out of the system. In most cases with air flow, the air pressure head is a much greater force than the air elevation head so it is easier to think in terms of air pressure. However, numerically, both elevation and pressure are considered as driving forces. The solution is the response inside the problem domain to the specified conditions on the boundary.

Sometimes specifying boundary conditions is fairly straightforward, such as defining the conditions that exist along the ground surface where it is often assumed that air pressure is atmospheric, or zero. Many times, however, specifying boundary conditions is complex and requires some careful thought and planning. Sometimes the correct boundary conditions may even have to be determined through an iterative process, since the boundary conditions themselves are part of the solution, as for instance along a seepage face. The size of the seepage face is not known and needs to be determined from the solution. Furthermore, the conditions on the boundaries may change with time during a transient analysis, which can also add to the complexity. If water starts to pond on a ground surface, then the air pressure boundary condition is no longer atmospheric and must change in response to what is happening in the soil.

Due to the extreme importance of boundary conditions, it is essential to have a thorough understanding of this aspect of numerical modeling in order to obtain meaningful results. Most importantly, it is essential to have a clear understanding of the physical significance of the various boundary condition types. Without a good understanding it can sometimes be difficult to interpret the analysis results. To assist the user with this aspect of an analysis, AIR/W has tools that make it possible verify that the results match the specified conditions. In other words, do the results reflect the specified or anticipated conditions on the boundary? Verifying that this is the case is fundamental to confidence in the solution.

This Chapter is completely devoted to discussions on boundary conditions. Included are explanations on some fundamentals, comments on techniques for applying boundary conditions and illustrations of boundary condition types applicable for various conditions.

3.2 Fundamentals

All finite element equations just prior to solving for the unknowns ultimately boil down to:

$$[K]\{X\}=\{A\}$$

where:

$[K]$ = a matrix of coefficients related to geometry and materials properties,

$\{X\}$ = a vector of unknowns which are often called the field variables, and

$\{A\}$ = a vector of actions at the nodes.

For an air flow analysis a simplified form of the finite element equation is:

$$[K]\{P_a\} = \{Q_a\}$$

Since the K matrix is a function of geometry and material properties it is the input into the solver and not part of the solution. Therefore, the equation either solves for the air pressure or the air flow. In a seepage analysis, it is possible to enter either the water pressure or water flow as a boundary condition. If you specify pressure, the equation solves for flow. If you specify flow as a boundary condition, then the only option is to solve for pressures.

In the case of an AIR/W analysis, it does not really make sense to specify an air flow boundary condition. In reality, when dealing with mechanical or natural air systems, the air pressure is the only known value and the flow depends on it. Therefore, in AIR/W, we do not allow for air flow boundary conditions to be specified. You can specify an air pressure or air pressure with review boundary condition and the model will compute the corresponding flows at those locations.

An important fundamental behavior that you need to fully understand is that when neither pressure nor flow is specified at a node, then the computed flow is zero. Physically, what it means is that the flow coming towards a node is the same as the flow leaving the node. Another way to look at this is that no flow is entering or leaving the system at these nodes. This is the case for most nodes in your model. Air leaves or enters the system only at nodes where a pressure boundary condition has been specified. At all nodes for no specified condition, flow is always zero.

3.3 Air pressure boundary conditions

An air pressure boundary condition can be a fixed value or a value that changes with time. It can be also be a positive value which would most likely force air into the soil, or a negative value which will most likely create a vacuum and suck air out of the soil. When specifying an air pressure, it is important to know that the value you specify is a gauge pressure, not an absolute pressure. So, atmospheric pressure would specified by the user as a value of zero.

An example of a time varying function intended to represent small changes to atmospheric pressure over a one month time interval is shown in Figure 3-3.

Figure 3-1

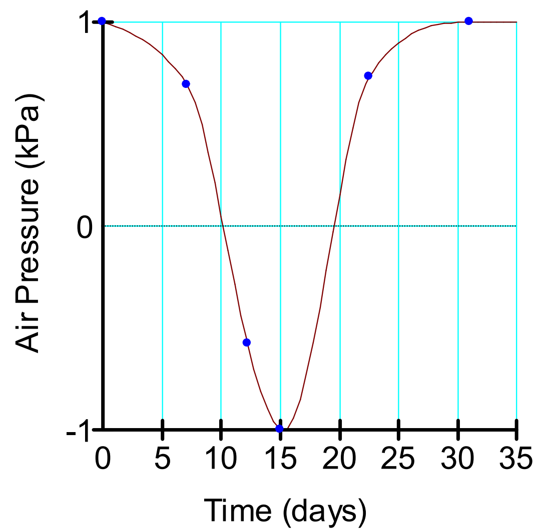


Figure 32

Figure 3-3 Atmospheric air pressure function

3.4 Air pressure on a vertical or sloping face

There are several cases where it is a bad assumption to assume that the air pressure is zero on a vertical or sloping face. While it is not always forefront in one's mind, air pressure does have an elevation component which means that the driving air "head" on a non-horizontal boundary must include an elevation component. Unfortunately in GeoStudio, there is no "Air Head" boundary condition option… only air pressure. Fortunately there is a solution. A GeoStudio Add-In function has been written that is shipped with the Sample Add-Ins called "Fix Pa as Pinitial". This Add-In can be applied to a Pa versus time boundary condition and what it does is read in the initial air pressure at the start of a model and apply it for all future time steps. The initial air pressure should be based on a previous steady state analysis in which the air pressure is set to zero at the top of the model and allowed to solve such that the air pressure increases with depth to account for elevation. Please refer to the Chimney.gsz example for an illustration of how this is done.

3.5 Air pressure review boundary conditions

There are many cases where the air pressure may change depending on the water pressure in the ground. Recall that the term matric suction is defined as the difference between pore-air and pore-water pressure: Ua – Uw. We know from the water content function that when the soil is fully saturated, the matric suction is zero. We also know that the soil can remain saturated under a small matric suction until the air entry value (AEV) is reached, at which point the larger pores of the soil will start to drain and the water in those pores will be replaced by air.

Now, with this in mind, think about a ground surface initially with atmospheric air pressure conditions which allow air to flow in and out of the soil. Then, it starts to rain and over time the ground surface saturates which no longer lets air enter or leave the soil. At this point, the air pressure is no longer atmospheric at the top of the soil and beneath the free water on the ground surface. As the rain continues, the air pressure in the soil will start to increase. It will, in fact, increase until it builds up enough pressure to push through the saturated soil and be released at the ground surface. The pressure it will rise to before escaping is approximately the AEV of the soil (which is sometimes referred to as the bubbling pressure).

AIR/W can take this mechanism into account. When you specify an air pressure review boundary condition, you can enter the review pressure. You can think of this review pressure as the Air Entry Pressure, or the Bubbling Pressure. The model will keep the specified boundary location at a pressure of zero atmospheric until the water pressure starts to build up on the ground surface. It will then review the pressures at each time step until it finds the pressure has exceeded the user entered value. At this point, it will set the ground pressure to be the user entered pressure and it will again let flow of air out of the soil. Prior to exceeding the user review pressure, the model will not let air pass the boundary if the ground is saturated. For more discussion on this boundary conditions, see the example titled Infiltration and Air Pressure Build Up in the examples chapter.

Surface regions and review BC's

In a normal seepage analysis a unit flux (rainfall) boundary condition will not result in pressures greater than zero at the ground surface. If this were always true, then it would not be possible to allow positive water pressures to build up such that the air trapped in the soil would also build up pressure. In order to capture this mechanism, you must draw a surface region across the top of your model so that the solver can track the amount of water that ponds in low points on the topography. Surface region meshing is discussed in the Geometry chapter of either the SEEP/W or TEMP/W engineering books.

3.6 Boundary condition locations

In GeoStudio all boundary conditions must be applied directly on geometry items such as region faces, region lines, free lines or free points. There is no way to apply a BC directly on an element edge or node. The advantage of connecting the BC with the geometry is that it becomes independent of the mesh and the mesh can be changed if necessary without losing the boundary condition specification. If you keep the concept of BC's on geometry in mind, you will find that you can specify any location for a BC quite easily. Consider the following examples which show the desired location of boundary conditions, the boundary condition applied to the geometry, and finally the underlying mesh with boundary conditions visible.

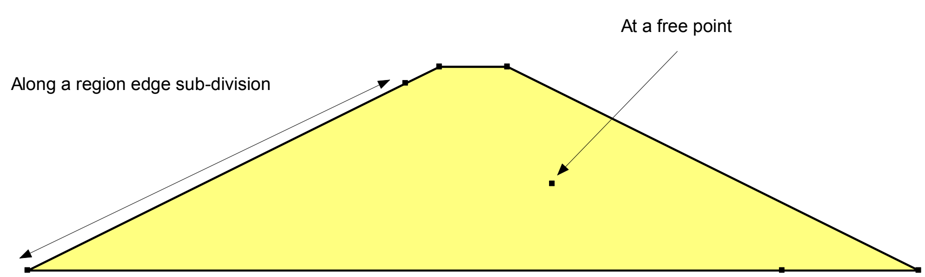


Figure 3-4 Desired BC locations

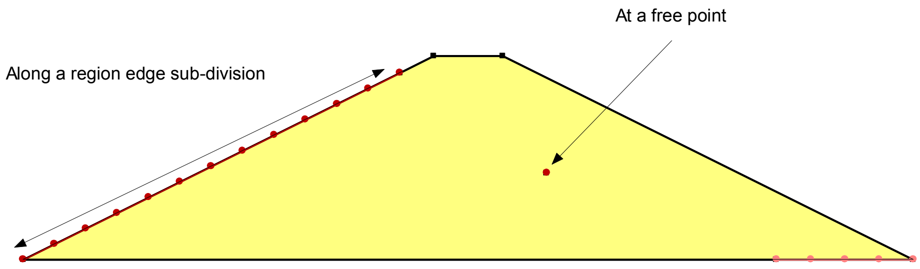


Figure 3-5 BC's attached to geometry

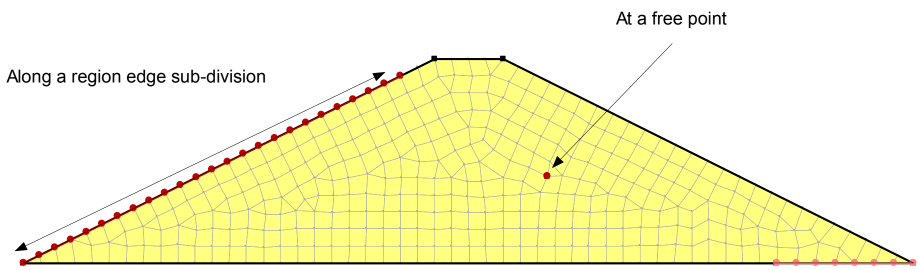


Figure 3-6 BC's with underlying mesh visible

If you look carefully at Figure 3-5 and Figure 3-6 you will see that the BC symbols along the slope edge are spaced differently. In the view with no mesh visible, the BC's are displayed at a spacing that depends on the scale and zoom factor of the page. In the image with the mesh visible, the BC's are drawn exactly where they will appear. They are always at a node for this type of BC. Notice also that the free point location forces a mesh node to be at the exact location. This way, you can always define a BC anywhere you want and when the mesh changes, the BC location will remain fixed.

3.7 Mixing air and hydraulic boundary conditions at the same location

Recall again that AIR/W and SEEP/W are really solving for matric suction, Ua-Uw. This concept can be a bit tricky to digest when you are thinking about specifying both hydraulic and air boundary conditions at the same location. You may be intending to specify a water table location as a boundary condition by fixing the water pressure but if the air pressure solves to a different (non zero) value, then Ua-Uw may no longer be zero and the water table will appear to move.

Always keep in mind the definition for matric suction, Ua-Uw, when specifying water and air pressure boundary conditions at the same location.

4 Analysis Types

AIR/W is a module within the SEEP/W program and also indirectly within TEMP/W. This means that AIR/W is not a stand alone program. The physical process of air flow through soil can be quite complex so it is important to know the options available to you when you set up your models. This chapter discusses these options, their assumptions, limitations and advantages.

4.1 Coupled and uncoupled air flow

Perhaps the first distinction that needs to be made is whether or not you intend to link AIR/W with both TEMP/W and SEEP/W. If you make the assumption that you have isothermal air flow then you are saying that you are not concerned about changes in temperature and how they affect the density of the air. Likewise, you are not concerned about the movement of air and its affect on heat transfer in the soil. If you are OK with these assumptions then you can use AIR/W with SEEP/W alone and you do not need to link it with TEMP/W.

If you are concerned about thermal affects, then read the additional information in section 4.4 of this chapter.

4.2 Steady state air flow

Think about the meaning of the words “steady state.” They describe a situation where the state of the model is steady and not changing. In an AIR/W analysis for example, the “state” means the air pressures and air flow rates. If they have reached a steady value, it means that they will be in that state forever.

In many cases where the geotechnical problem is exposed to nature with its cyclical conditions, it is possible that a steady state will never be reached. Daily heating and drying of a soil is not conducive to solving a steady state condition. Air flow through a vapor extraction well is possible to solve as a steady flow problem assuming the air pressure applied at the well is constant and assuming the ground surface atmospheric pressure is also constant.

It’s very important to understand that when you are doing a steady state analysis you are not making any estimation of how long it takes the final condition to develop, nor how long it will last. You are only predicting what the ground will look like for a given set of boundary values, and it is implied that you are pretending the boundary values have been in place forever and will be in place forever.

Because steady state analyses are taking out the “time” component of the problem it greatly simplifies the equations being solved. However, at the same time it can make convergence harder to achieve – depending on the degree of non-linearity of your soil property functions. The steady state air and water flow equations leave out the actual “time” variable and omits the entire volumetric water content function. They are not needed in the solution. The volumetric water content is used for telling how much water is gained or lost in the soil if there is a change in pressure. In a steady state analysis, there is no starting pressure, and so there is no change in pressure to worry about. Remember, it is only looking to find out what the pressures will be throughout the problem geometry, given the fact you know what they are at only a few known locations and for all points in time.

Unfortunately, simplifying the equations does not mean that there is a reduced set of numerical issues associated with solving them. Recall that the soil hydraulic and air conductivities can and usually do change several orders of magnitude between the wet and dry states of the soil. Because the solution is not starting from a known water content or pressure state at all points in the finite element mesh, it does not have the numerical luxury of letting the solution march forward in time from a start condition to its end condition. Since there is no start condition, the solver also has no idea what the soil air conductivity value is at all points in the soil. The objective, therefore, is to be able to get the pressures and conductivities to match throughout the mesh domain by only giving the solver a “known” condition at a very small part of the domain when the solve process begins. The way the solver gets the final answers is by iterating – that is, trying different things and moving slowly towards the singular answer. The answer is singular because for the fixed set of boundary conditions there is only ONE unique perfectly converged solution. The objective is to get as close to the unique solution as possible – without driving yourself crazy!

Boundary condition types in steady state AIR/W

In a steady state analysis there is only one type of air pressure boundary condition… the constant Pa. The review air pressure BC has no meaning because it requires a transient ground surface water flux process in order to trigger the review mechanism.

4.3 Transient air flow

A transient analysis by definition means one that is always changing. It is changing because it considers how long the soil takes to respond to the user boundary conditions. Examples of transient analyses include predicting the change in air volume in a soil due to changes in atmospheric pressures over time. Another example may be the change in water content in a pressure plate soil sample due to application of an air pressure to drive the water out.

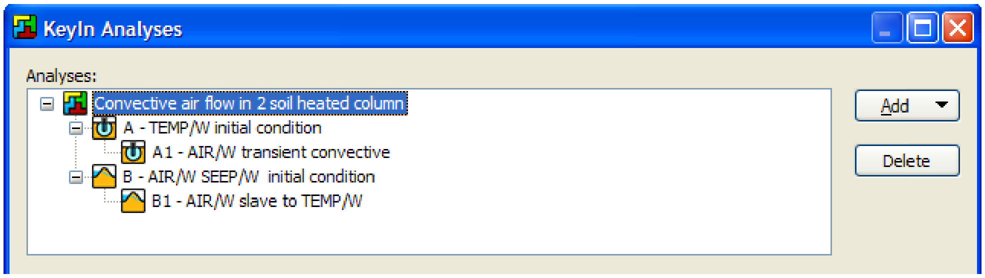
In order to move forward in time during a transient analysis, you must tell the solver what the soil pressure conditions are at the start of the time period in question. In other words, you must provide initial conditions as well as current or future boundary conditions. There are different ways to do this in an AIR/W analysis.

Using an initial conditions file

The most rigorous way to specify air pressures throughout the model is to first solve a steady state analysis and then to point to the results of this analysis as the starting conditions for the transient model. This process is very simple to do in GeoStudio with the introduction of multiple and staged analyses in a single project file. The hardest part about using this technique is that you have to have an estimate of steady state pressure conditions so that you can apply them as boundary conditions in the model. The advantage to using this approach is that you will have spatially varied air pressures and density throughout the model.

The initial conditions file does not need to be a steady state analysis in the same GeoStudio project file. It can be any other previously solved AIR/W analysis as long as the mesh in both files is identical. Keeping the various analyses within the same GeoStudio project will ensure the mesh is always compatible so this is the recommended approach.

An example of linking analyses for starting conditions is shown below. In this figure, both TEMP/W and AIR/W have their own steady stage initial condition files that are referenced by two transient processes. In this case, the transient process is a coupled convective analysis which is controlled by TEMP/W. This analysis type is discussed in the next section.



Keying in the initial conditions

The second option is to specify directly what you think the starting pressure conditions will be. You can assign an "activation air pressure" as a material property and it will be applied to the model in the absence of any other specified starting conditions the first time the soil becomes active in the analysis. This activation time may be the start of a transient model or, it may be at some point in a construction sequence when a new soil "lift" is placed and the soil is numerically included in the model for the first time.

Spatial function for the initial conditions

A new option is to specify directly what you think the starting values conditions will be by applying a spatial function. You can define a spatial function for pressure and have the solver point to this function result. An example of a spatial function is shown below.

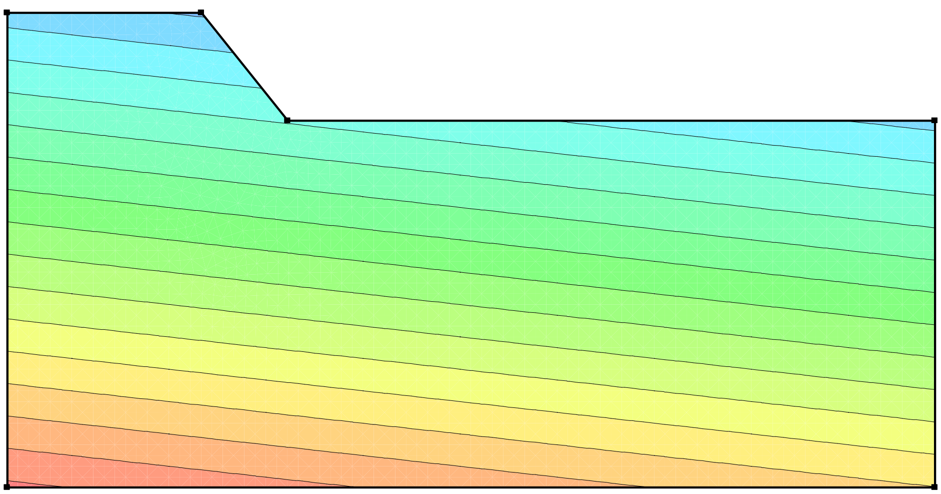
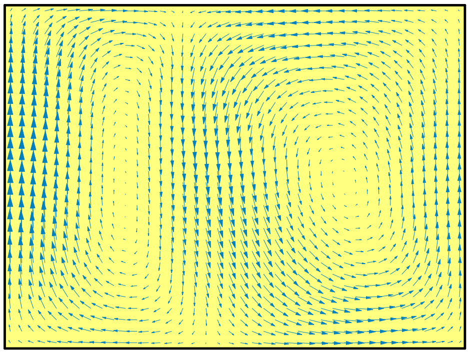
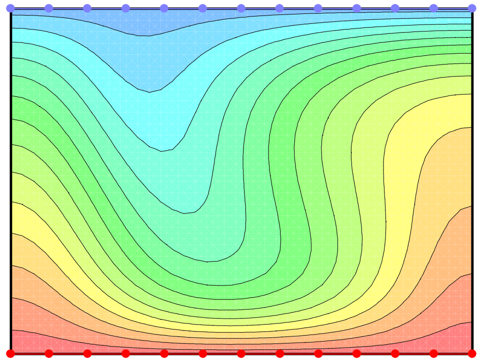


Figure 4-1 Spatial function assigned for initial pressures

4.4 Convective heat transfer in air phase

The most complex analysis you can do with AIR/W is to couple it with both SEEP/W and TEMP/W. The SEEP/W coupling links the pore-air and pore-water pressures and the TEMP/W coupling links the density of air to its temperature. With this full coupling, it is possible to model a wide range of phenomenon and to produce really cool pictures like this… where the left side is air movement in a closed box and the right side is temperatures. The only boundary conditions in the entire model are a warm bottom and cool surface.

When using the convective analysis it is important to know that the process is driven by TEMP/W. In other words once the models are set up the solving in launched from within TEMP/W and the time stepping is controlled by TEMP/W. TEMP/W will pass to AIR/W the ground temperatures which are used by AIR/W to know the air density. When combined with or without other air pressure boundary conditions the AIR/W program will solve the density dependent coupled air-pressure and water-pressure equations. At the end of each iteration, AIR/W passed back to TEMP/W the computed air content, water content, air velocity and water velocity so that TEMP/W can compute the convective heat transfer associated with the moving air. The process continues until TEMP/W sends a message to AIR/W that the model is complete.

4.5 Time stepping - temporal integration

An incremental time sequence is required for all transient analyses and the appropriate time sequence is problem dependent. In most cases it will likely be necessary to try a reasonable sequence and then adjust the sequence as necessary in response to the computed results.

When solving AIR/W as an isothermal analysis the equations are inherently quite stable. However, when solving AIR/W as a coupled thermal analysis with TEMP/W the equations become somewhat unstable due to the inclusion of the air and water velocity terms in the thermal equation. These terms are first order derivatives and in order to solve nicely require careful attention to time step size and mesh size.

In general, for either isothermal or coupled thermal AIR/W analyses, the accuracy of the computed results is dependent to some extent on the size of the time step. Over the period of one time increment, the process is considered to be linear. Each time step analysis is equivalent to a mini steady-state analysis. The incremental stepping forward in time is in reality an approximation of the nonlinear process. For the same rate of change, large time steps lead to more of an approximation than small time steps. It follows that when the rate of change is high, the time steps should be small, and when the rate of change is low, the time steps should be large.

Finite element temporal integration formulation

The following discussion is presented here and again in more detail in the Theory chapter where the full development of the equations is given. It is important now to just show the equation so that a key point can be highlighted that will enhance your understanding of the transient finite element method.

The finite element solution for a transient analysis has air pressure changes being a function of time as indicated by the $\{Pa_1\}$ term in the finite element equation below. In order to solve for the pressure at some point in the future, time integration can be performed by a finite difference approximation scheme.

Writing the finite element equation in terms of a "Backward Difference" finite difference form leads to the following equation (Segerlind, 1984):

Equation 4-1

$$\left(\Delta t\left[K\right]+\left[M\right]\right)\left\{Pa_1\right\}=\left[M\right]\left\{Pa_0\right\}$$

or

$$\left\{Pa_1\right\}=\frac{\left[M\right]\left\{Pa_0\right\}}{\Delta t\left[K\right]+\left[M\right]}$$

where:

dt	=	the time increment,
Pa_1	=	the pressure at end of time increment,
Pa_0	=	the pressure at start of time increment,
$[K]$	=	the element air conductivity matrix, and
$[M]$	=	the element air storage matrix.

Look at this equation carefully for a moment. You can even ignore the braces and brackets as they just indicate a grouping of node and element information with some geometry tied in. The thing to focus on is that in order to solve for the new pressures at the end of the time increment, it is necessary to know the pressures at the start of the increment along with the average material properties calculated at the average of the new and old pressures. If you do not have reasonable values for starting pressures, then you make the equation difficult to solve because you use these starting pressures directly in the equation AND also in the calculation of the average material properties.

4.6 Staged / multiple analyses

Multiple analyses can be included in a single GeoStudio project. Fundamentally, multiple analyses in a single Project allows different material properties and different boundary conditions to be specified across time and space. This facilitates the modeling of staged construction in which soil is added or removed over time and/or boundary conditions or material properties that change with time. Including multiple analyses in a single Project can be used for a variety of reasons such as:

1) Conducting sensitivity analyses for variations in material properties and boundary conditions;
2) Analyzing staged construction;
3) Establishing initial conditions for a transient analysis;
4) Integrating various GeoStudio products; and,
5) Linking together multiple transient analyses.

GeoStudio uses a parent-child terminology to describe the relative position of each analysis within a Project. Figure 4-2 displays an example of an Analysis Tree for a slope stability project. The SEEP/W steady-state analysis is the Parent and is used to define the initial pore-water pressure conditions for the two transient SEEP/W analyses. The indentation in the tree indicates that both analyses 2 and 3 have the same Parent. SLOPE/W analyses 2a and 3a are children of transient SEEP/W analyses. This naturally suggests that the pore-water pressure conditions for both stability analyses are defined using the transient seepage results.

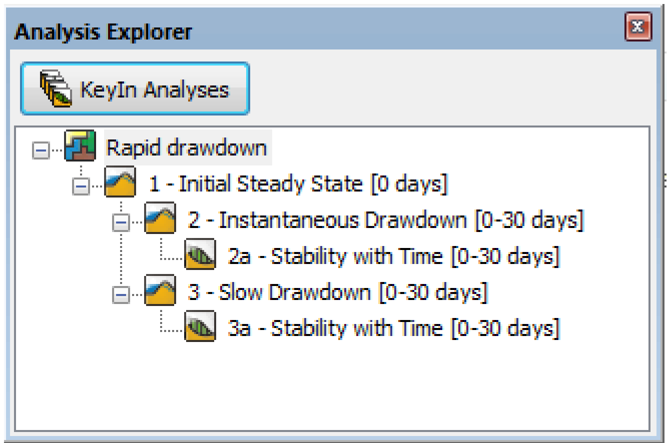


Figure 4-2 Example of an Analysis Tree in GeoStudio

One significant benefit of the Analysis tree is that all analyses related to a specific project are contained within a single file. It is no longer necessary to reference other files to establish initial conditions or integrate the various GeoStudio products.

4.7 Analysis View

In GeoStudio you can specify an analysis view as two dimensional or axisymmetric. In SEEP/W, for confined aquifer analysis, you can also specify a plan view analysis. Plan view in AIR/W has no real meaning.

The default view when you start to define a model is 2D. This means you are looking at vertical cross section of the geometry that extends one unit thickness into the image. So, if your length units are meters, then in 2D view the model is assumed to be 1m thick. If you change the default thickness to 10 m, then when you are considering output data, you must remember that any values which consider flow and cross sectional area are now based on a 10m depth into the image. The values will be 10x larger.

The axisymmetric view is a pseudo-3D analysis which considers that the viewed cross section is in reality a slice of a wedge that rotates around the x=0 axis as shown below. The default thickness in this view is 1radian but you can change it to be 2*Pi in which case the output data would consider a 360 degree rotation about the Y axis.

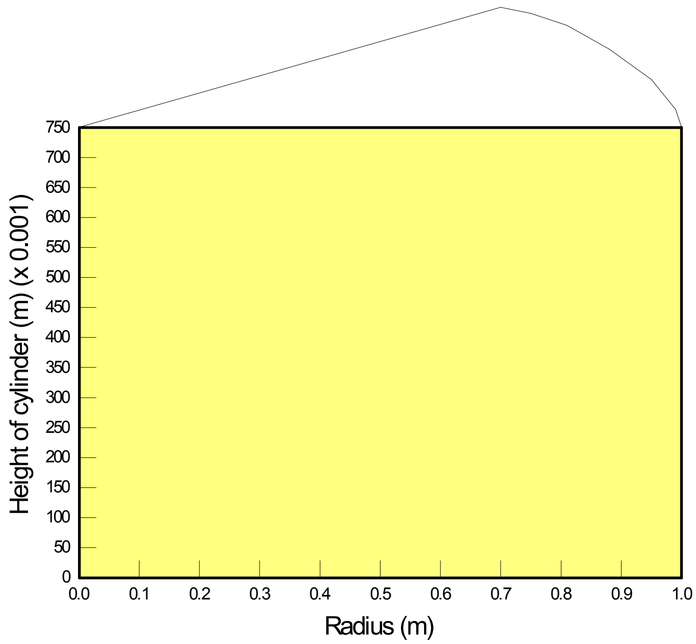


Figure 4-3 Unit radian thickness in axisymmetric view

5 Functions in GeoStudio

User specified functions are used throughout GeoStudio to specify soil material properties, to specify modifier parameters for constants or other functions, or to specify boundary conditions that change over time. It is important to have an understanding of how the functions are specified and used by the solver and also to know what your options are for inputting these functions. A functional relationship between "x" and "y" data can be defined using:

Natural and weighted splines between data points

Linear lines between data points

A step function between data points

A closed form equation that is based on parameters and does not require data points

A user written externally complied code (dll library) that connects with GeoStudio data or data from another process (eg, Excel)

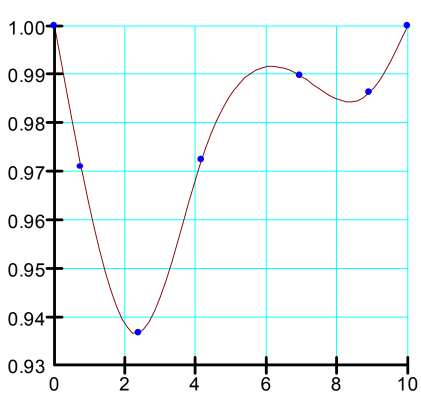
The type of function you choose to use will depend on what your needs are.

In many cases a function you need can be estimated from other data you have input. An example is the hydraulic conductivity function for soils that is based on a user input water content function. Several GeoStudio material models require functions that may be estimated if you do not already have a full set of data.

5.1 Spline functions

A spline function is a mathematical technique to fill in the gaps between adjacent data points with curved line segments. Unfortunately, all our data points do not always fit nicely along a path with a predictable curvature such as a logarithmic or exponential decay. Many of the functions in geo-technical engineering have double curvature with an inflection point between. Consider the water content function that is initially concave downwards, and then at higher suctions is concave upwards. Splining is an advantageous technique to fit lines through these data points because the spline settings can be altered to fit almost any set of data.

In GeoStudio you can control the look of a spline function by adjusting its degree of curvature and its level of fit with the input data points. Consider the two images below.



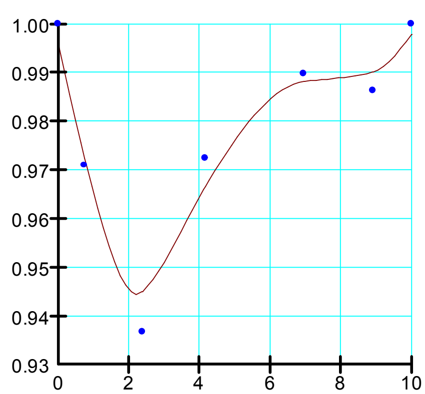


Figure 5-1 Spline functions with different settings

The left image has the spline fit almost exactly through the data points with fairly curved segments. The right image has more linear segments that only fit the data approximately. The range of fit and curvature is controlled by two "slider controls" and can range between values of zero and 100%. The important thing to note is that the solver will use the data represented by the splined fit. What you see in the function set up is EXACTLY what the solver will use when needed.

Slopes of spline functions

Sometimes, the solver does not require the "Y" value of a function at a given "X" value but the slope of the function at a given "X" value. This is the case for the water content function where the slope is used directly in the solution of the transient seepage and air flow equations. You must be careful when setting spline values because while a spline may look smooth, its slope may not be so.

The two images below are the slopes of the two functions shown above. You can see that the more natural curved function (left side images) with 100% curvature and exactness in the spline settings produces a much smoother slope function than the approximated function. While not usually critical, you should know if the function you are using is dependent on its slope being well behaved.

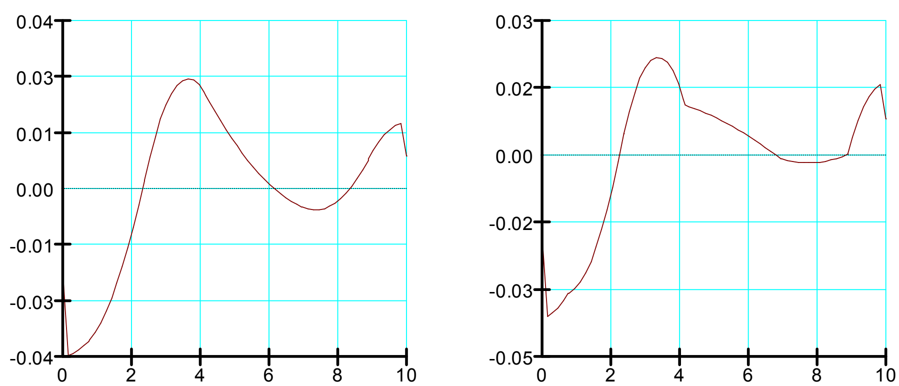


Figure 5-2 Slope of spline functions

5.2 Linear functions

A linear function is a spline function with the curvature setting to 0% and the fit set to 100% exact as shown below.

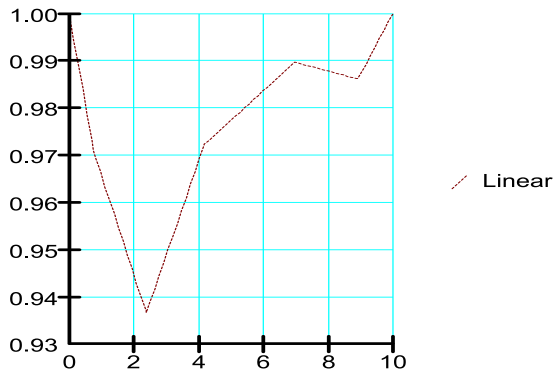


Figure 5-3 Linear fit spline

5.3 Step functions

GeoStudio has an option for functions that result in "steps" between data points. This can be useful if your data changes abruptly over time, for example, rainfall on different days. When you use a step function, you need to be careful of the location of the blue data point. You can see that the functions will assume the starting time of the step is at the data point and that its duration extends just up to but not reaching the next data point.

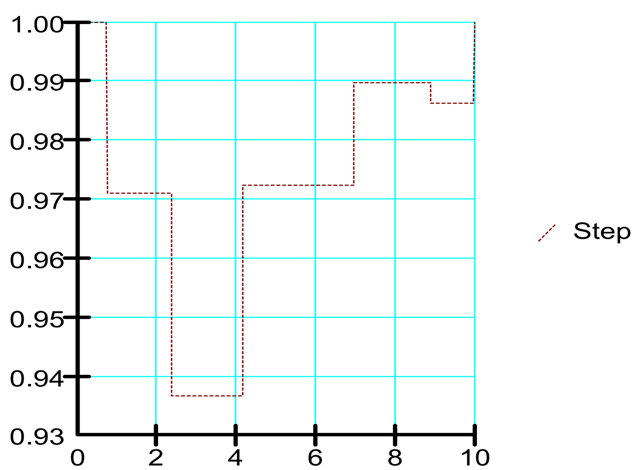


Figure 5-4 Step function

A comparison of all four data point functions is shown below on one image. When multiple functions are viewed simultaneously in GeoStudio, the data points are hidden and just the computed functions are displayed.

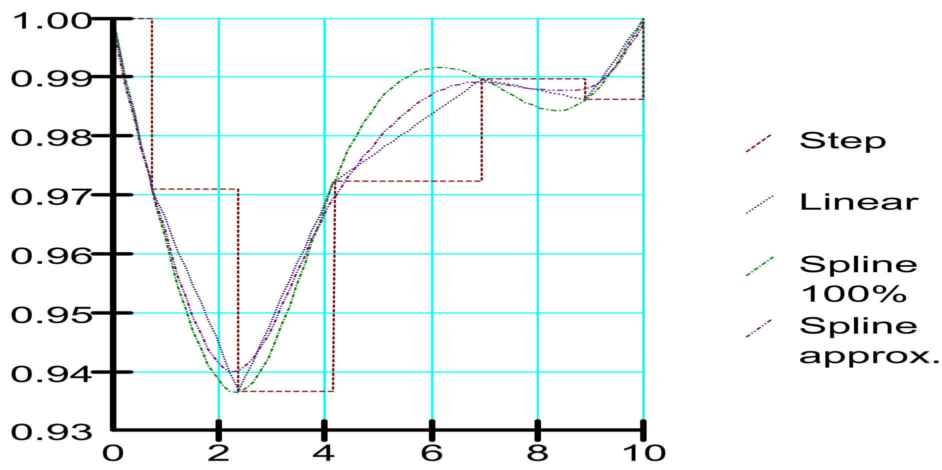


Figure 5-5 Comparison of all data point functions

5.4 Closed form curve fits for water content functions

The storage function is defaulted to be represented by a spline function; however, it is possible to have the function represented by a closed form equation that is fit to the data. Two common methods exist in the literature for water content functions: the Fredlund and Xing method, and the Van Genuchten method. Each of these curve fits require that you enter fitting parameters that are usually published or provided by soil laboratories. The only advantage to using these techniques in GeoStudio is that you do not have to enter a series of data points. If you know the fit parameters, you may enter them directly to obtain the function. More information about these two fit equations are provided in the chapter on Material Models and Soil Properties in this book.

5.5 Add-in functions

GeoStudio Add-Ins are supplemental programs run by the solver as part of a GeoStudio analysis. A Function Add-In is an object that takes the place of a function defined in GeoStudio, and offers the flexibility of computing function results that vary dynamically based on the current analysis state. For example, Add-Ins can be assigned to Slip Surface Slices (via strength functions), Mesh nodes (via boundary condition functions), and Mesh gauss points (via material property functions). Please consult the Add-In Developers Kit (SDK) available on the website (www.geo-slope.com/downloads) for full details.

5.6 Spatial functions

A spatial function in AIR/W can be used to establish starting pressure profiles over a two-dimensional domain. When you first create a spatial function you will not see its contoured colors appear on the geometry. However, once you assign the function as the initial condition in Key In Analysis Settings, you can return to the Key In Spatial Function command, make changes and edits to the function data, and see instantly what the new function will look like when applied to your model. An example of this is shown below for initial pore-water pressures which would be applied in the seepage part of the analysis.

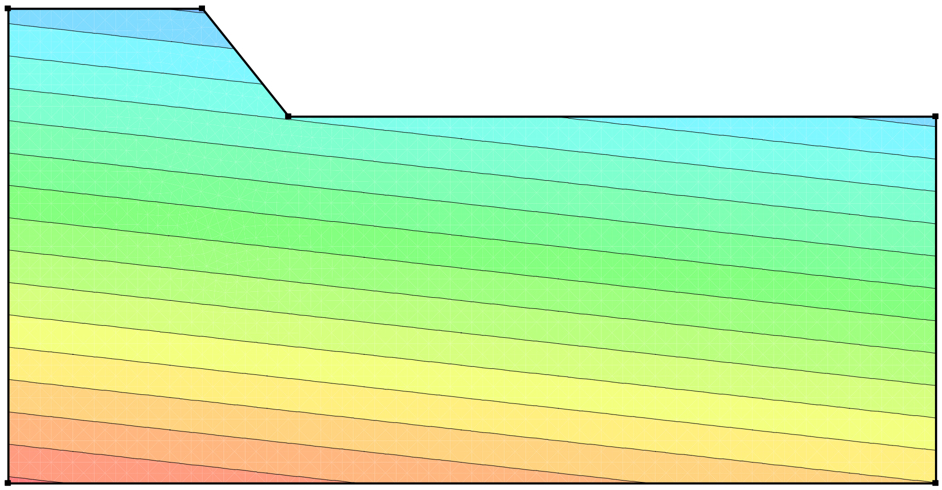


Figure 5-7 Example of spatial function assigned to model

6 Numerical Issues

Entire textbooks can be written on numerical issues related to finite element analysis. While modern computers and powerful graphics can make defining an analysis quite fast and easy, they can not necessarily deal with some of the intricate issues related to the concept of taking a natural process and breaking it down into finite time and special domains (i.e. individual elements within a soil geometry).

There are various ways to deal with many of the numerical issues, but the unfortunate part is that there is no single method, approach, or technique that can deal with all problems. Some numerical issues relate to restrictions in computer hardware such as rounding off of non-integer variables during math operations; some issues relate to non-linearity of soil properties; some issues relate to the fact the physical equations being solved do not apply to all cases; some issues relate to our inability to discretize a domain to small enough element sizes; and other issues relate to the fact we have made the elements too small!

There are numerical solvers that make use of adaptive meshing or adaptive time stepping or both in attempts to be more suited to a wider range of problems. All of these, however, have their limitations from a sound mathematical perspective – regardless of what the software developer will tell you. If you don't know what the limitations are, it becomes somewhat risky to rely on a solver that claims to "handle it all."

Some finite element solutions attempt to march forward in time by considering soil property values taken from the last, the current or the mid-time step average. Some solvers simply make assumptions that limit their applicability to real life in order to get a solution, such as moving the mesh to find the water table in a seepage program instead of solving for the physics of flow above and below the water table. Finally, some solvers may only work if you put in an initial guess of the solution for the dependent variable being solved that is close to the desired solution, in other words, start off the solution by pointing it in the right direction.

While it may appear all hope is lost, this is far from the case. Sound judgment and common sense can usually overcome most of these challenges and result in meaningful interpretations of how the soil will respond to changes in various parameters.

It is not always possible to get an exact solution in many challenging cases, so you should not necessarily be seeking an exact solution. If the problem is so difficult that it is not solving reasonably, then it is very likely that either mistakes have been made in the input, or, that you are pushing the envelope of the physical theory applied in the model. This chapter looks at some of these issues as they pertain to AIR/W.

6.1 Convergence

A discussion on convergence and under-relaxation can be found in the SEEP/W Engineering Book.

6.2 Mesh size and time steps

The Meshing chapter of the SEEP/W or TEMP/W books provides general modeling guidelines for the design of a finite element mesh for seepage and thermal analyses. This section provides additional guidelines which should be followed in order to extend the AIR/W analysis results for thermally coupled modeling.

In order to obtain stable solutions in a convective heat transfer analysis, the Courant number criteria must be considered in the design of the finite element mesh. To satisfy the Courant criteria, the spatial discretization requirements are:

$$\Delta x \leq v_x \, \Delta t$$

$$\Delta y \leq v_y \, \Delta t$$

The above equations indicate that the mesh element must be small enough such that an imaginary particle of water does not pass over the element within any given time increment. However, you will not know the velocity when you set up the mesh so it's hard to use this equation at the design stage. You will have to try something and then look at the computed velocity of air or water and return to the analysis settings where you can either re-define the mesh or alter the time step size.

General rules for setting time steps

So, now that we have made the issue of time steps somewhat unclear, what do we recommend are some methods to deal with it? Some of the following points have a strong theoretical basis and some are just common sense based on years of experience.

The finite element shape is important. Triangular elements should not have any interior angles greater than 90^0. Square elements can have double the time step size as triangular elements.

Since the element size is directly proportional to time step size, doubling the element size means you can double the time step. The corollary of this is that decreasing the element size and not decreasing the time steps accordingly will not improve the calculated results.

Numerical dispersion and oscillation are directly affected by the time step increments. To minimize numerical dispersion and oscillation, the Courant Number constraint should be satisfied. The Courant Number constraint requires that the distance traveled by the advective component of the transport process during one time step ideally should not be larger than one element; that is, the advective component should not jump across elements in one time step.

In order to satisfy the Courant Number constraint, the time increment should be:

$$\Delta t \leq \frac{\Delta x}{v_x}$$

and,

$$\Delta t \leq \frac{\Delta y}{v_y}$$

6.3 Gauss integration order

The details of numerical integration are provided in the appendices, along with a discussion of how different integration orders can affect results for various types of elements. Part of this discussion is repeated here as it pertains to improving solution convergence.

The appropriate integration order is a function of the presence of secondary nodes. When secondary nodes are present, the interpolating functions are nonlinear and consequently a higher integration order is required. Table 6-1 gives the acceptable integration orders.

Table 6-1 Acceptable element integration orders

Element Type	Secondary Nodes	Integration Order
Quadrilateral	no	4
Quadrilateral	yes	9
Triangular	no	1
Triangular	yes	3

It is also acceptable to use four-point integration for quadrilateral elements which have secondary nodes. This is called a reduced integration order (see Bathe, 1982). Acceptable results can be obtained with reduced integration.

It is also possible to use three-point and nine-point integration with elements that have no secondary nodes. However, the benefits of this are marginal, particularly for quadrilateral elements. Nine-point integration for a quadrilateral element involves substantially more computing than four-point integration, and there is little to be gained from the additional computations. As a general rule, quadrilateral elements should have secondary nodes to achieve significant benefits from the nine-point integration.

The situation is slightly different for triangular elements. One-point integration means the material properties and flow gradients are constant within the element. This can lead to poor performance of the element, particularly if the element is a zone of steep pressure gradient and there is active change in air or water content. Using three-point integration, even without using secondary nodes, can improve the performance, since material properties and gradients within the elements are distributed in a more realistic manner. The use of three-point integration in triangular elements with no secondary nodes is considered acceptable for triangular elements in a mesh that has predominantly quadrilateral elements. This approach is not recommended if the mesh consists primarily of triangular elements with no secondary nodes.

In general, it is sufficient to use three-point integration for triangular elements and four-point integration for quadrilateral elements. In situations where there is adsorption and steep gradients within an element, it is best to use quadrilateral elements with secondary nodes together with nine-point integration.

6.4 Equation solvers (direct or parallel direct)

AIR/W has two types of equation solvers built into it; a direct equation solver and a parallel direct equation solver. Both offer certain advantages.

Select the direct equation solver option if you want the system equations to be solved using a Gauss elimination skyline direct solver. The processing speed of the direct solver is bandwidth (the maximum node number difference of all the elements in a domain) dependent. In other words, the direct solver is very fast when solving simple problems with small bandwidth, but it can be quite slow when solving more complex problems with a large bandwidth. AIR/W automatically sorts the nodes so that the bandwidth is the smallest possible value, which helps the solution solve faster using the direct solver. By default, the direct equation solver is selected.

Select the parallel direct equation solver option if you have a larger mesh. The parallel solver will save the matrices in a compressed format to eliminate zero's and it has many advanced schemes to solve large systems of equations more efficiently. It also offers the ability to make use of multiple processors on a computer if they are available. The disadvantage of this solver is that it is a bit slower when the models are smaller in size.

If in doubt, try each solver and choose the one that offers the best performance.

7 Visualization of Results

When you get to the visualization of results stage of a finite element analysis you can congratulate yourself for having completed the hardest parts – setting up the geometry, defining meaningful soil property functions, and applying appropriate boundary conditions to the mesh. If, at this point, you do not have the tools or the understanding of how to interpret the massive amount of data that may have been generated by the solver, then you have wasted your time.

This chapter describes the various types of output data that are computed by the solver and it attempts to get you thinking about what the data is trying to tell you. For example, did the solution solve properly? Did the boundary conditions you applied get reflected in the actual solution? Did the soil respond how you thought it would respond? If not, how to you methodically determine what to check next?

The chapter is structured to explain what type of data is available for visualization. In the various sections, comments are provided that relate they type of result data in question to how is should be used in the overall thought process. It's a good idea to read this entire chapter.

7.1 *Transient versus steady state results*

The type of data you can view is somewhat dependent on the type of analysis you have completed. For example, if you have done a steady state analysis, you cannot interpret any data related to changes in time. You can view instantaneous flux values that have units of volume per time; however, you cannot see how these values may change with time because steady state, by nature, means things do not change with time.

In a steady state analysis you are not required to define a volumetric water content function, because it is the slope of this function that is needed in the solution of the transient, not steady state, finite element equation. If you do not define a water content function, then it stands to reason that you cannot view air or water contents. Air and water contents can only be viewed in a steady state solution if you have defined a water content function that the solver can access to report water contents based on solved pressures. No water content function… no reported air or water contents!

In a transient analysis, you can look at how all of the various output data values change with respect to time and/or position, whereas in steady state analysis, you can only graph how the data changes with position.

7.2 *Node and element information*

In order to understand what type of information can be viewed as results output, it helps a bit to know how the data is obtained. So, to recap, you set up the problem geometry, define material properties, and apply boundary conditions of either known head (or pressure) or flux. The solver assembles the soil property and geometry information for every Gauss point in every element and applies it to the flow equation that is written for every node. Therefore, at each node we have applied boundary data, interpolated soil property data and geometry data. The solver then computes the unknown value in the equation for each node – the unknown value being either head or flux. It is the Gauss point data that is used to set up the nodal equations, so the Gauss point data written to the output file is the actual data used in the solver.

In GeoStudio, all output data for nodes and gauss points anywhere in the model is accessible using the View Results Information command. With the command selected, you can click the mouse on any single node to view the output at the node. You can also hold down the shift key to muli-select many points. If you click beside a node and within the element itself, you will get the Gauss point data at that location. You can multi-select Gauss point to see a table of data.

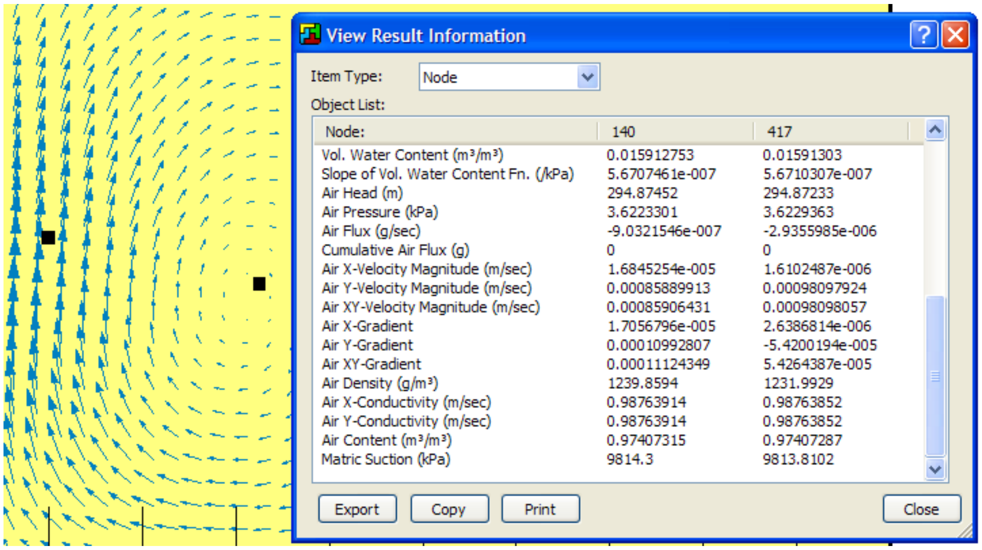


Figure 7-1 is an illustration of the type of information that can be viewed for each node in the finite element mesh. You can view the types of data in the list to see that there is a combination of heads, pressures, fluxes, velocities, gradients, conductivities and water contents. There is also a summary of the position of the node within the problem domain. In effect, the node information is a summary of the problem geometry, the soil material properties, and the boundary conditions – the three main parts of any finite element analysis. When you include an air flow analysis, the list contains all the related air flow parameters too as shown.

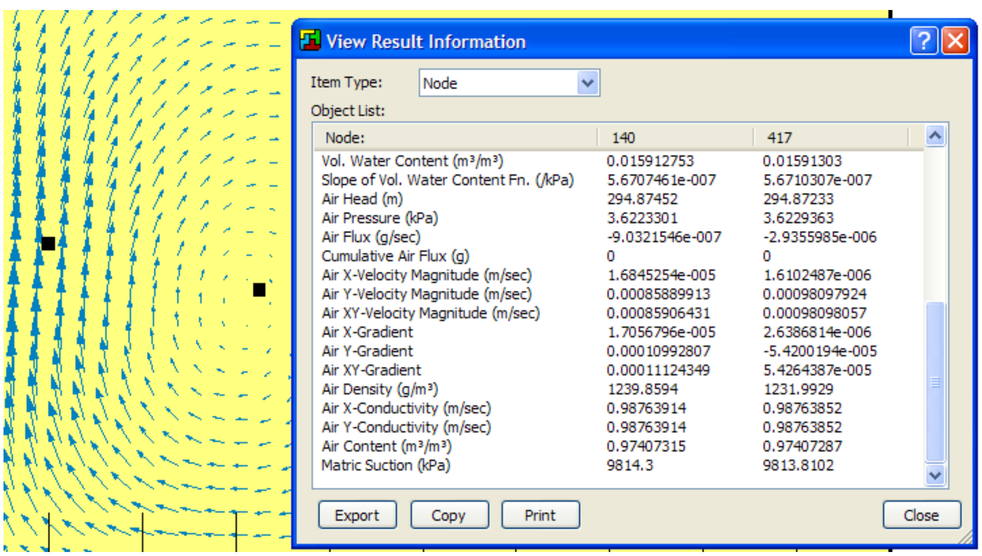


Figure 7-1 Visualization of node information

Figure 7-2 is the corresponding Gauss point information for the Gauss point located just below and to the right of the node illustrated in the previous figure. The shaded region in the figure shows the contributing area of that Gauss point and, in this case, because the element is rectangular, this Gauss area is equal to one quarter of the total area of the element. The inset in the figure below shows the type of data that can be viewed at each Gauss point. If you consider the air content value of 0.974%, for example, you should realize that this air content is assumed to exist throughout the Gauss point area displayed; and you should next realize that if the element size is increased, the estimate of the air content becomes less accurate, as we are averaging it over a larger area. The real trick to getting good finite element analysis results is to create a finite element mesh with just the right sized elements that are not too big or too small, that can represent the highly non-linear soil properties within them, and that can handle the potentially extreme boundary conditions you apply. It is not always easy and there is no sure quick or automatic method to make that happen.

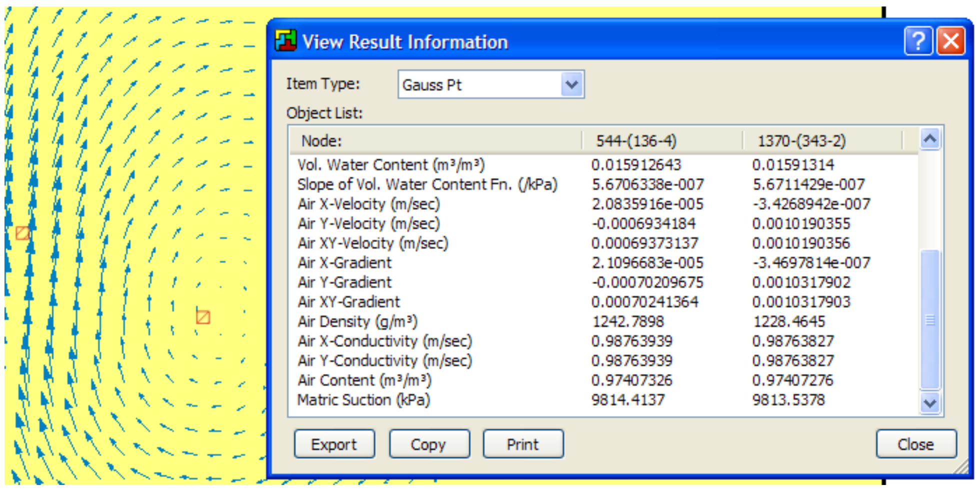


Figure 7-2 Visualization of element Gauss point information

7.3 Graphing node and gauss data

The Draw Graph command allows you to plot a graph of any computed value as a function of time, position or both time and position. In past versions of GeoStudio, all graphing was based on user selected nodes. Moving forward, GeoStudio now requires the user to select graph data locations based on one or more points, a cut line, or a region of points. It is possible to select all three types of data locations within a single graph. Figure 7-3 shows a combination of all three graph data objects in a single dam cross section.

The advantage of using this type of data selection is that the location and type of data used in any graph can be named and saved. Each time you return to the graphing command, you can choose from your saved list of graphs and you do not have to re-define them. Even if you change the mesh, the model will know the new nodes nearest to your graph selections and it will draw the graph using the most recent solution.

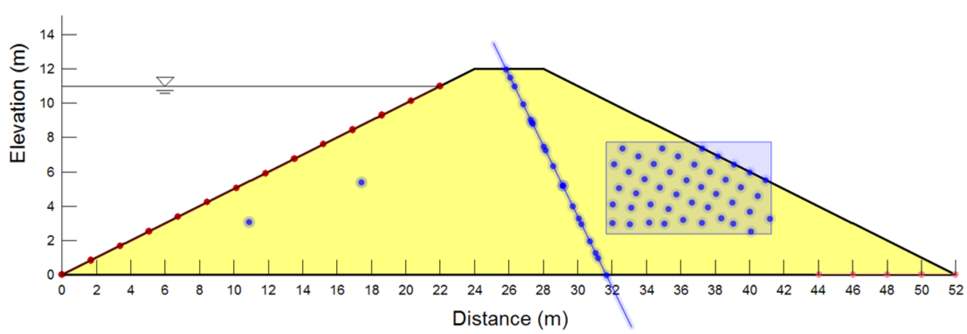


Figure 7-3 Graph data selection options (points, lines, planes)

In the previous image, the graph points were selected at any point in the domain. Sometimes it is easier to select all points along a given geometry object such as a region line or point. Consider Figure 7-4 where the entire up stream region edge line has been selected for graphing. In this case, it was easier to just select one point along the entire edge and have the model capture all nodes along that edge. The option of selecting custom points or geometry points is totally a user preference.

Once the graph is visible there are many options to change the font, apply a legend, rotate the image, copy the image to paste it into a report, copy the data to paste to Excel or another program, or export it as a comma separated text file.

You can even hover the mouse directly on a graphed point to see the actual data as shown in Figure 7-5 below.

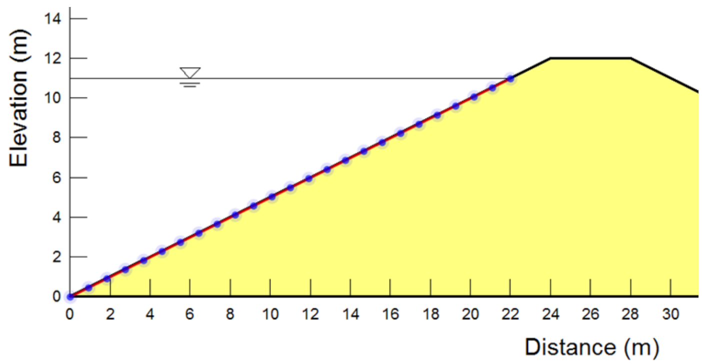


Figure 7-4 Graph selections based on geometry item (upstream region edge line)

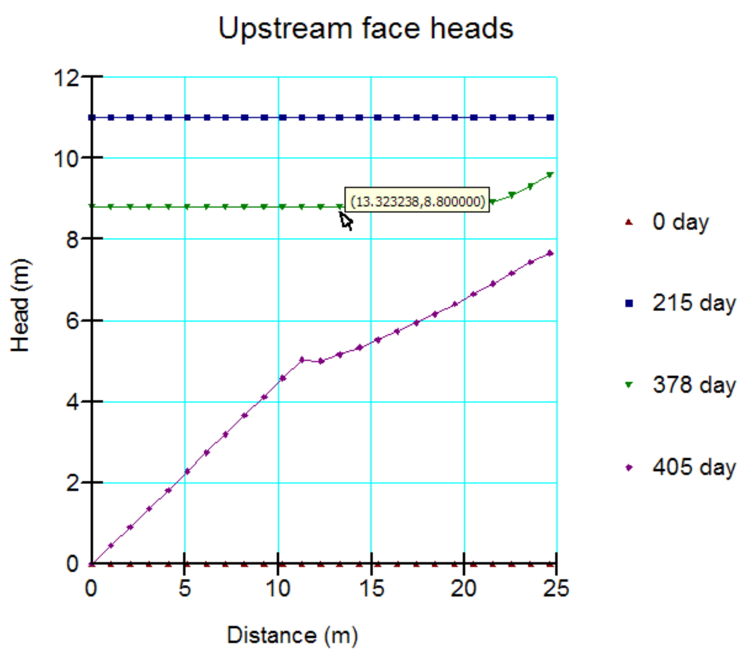


Figure 7-5 Upstream total head as a function of position for each time

7.4 *"None" values*

In GeoStudio, an attempt is made to distinguish between data values that have a true value of zero, and those that are missing. A missing value is labeled as "none" in a data list or is not printed to file when you save the data for export or pasting into another program such as Excel. A missing value is simply a data type that is not relevant to the current set of analysis parameters. For example, in Figure 7-6 below, the node boundary flux values are set to “none”. This is because there are no nodal flows at internal, non boundary condition nodes.

“None” or missing values, are simply a way for GeoStudio to not erroneously report data values as zero (which has meaning) when they really just do not exist. Consider the following graph generated by GeoStudio of pore-water pressures in a soil as it is placed during a construction sequence. At the 0 second time, the soil surface is at 10m. At 10 seconds, 2 meters of more soil is added. At 8010 seconds, another 2 meters is added. Notice that for the two added lifts of soil, the pressure values are not graphed as zero prior to their placement time. The data is “missing” in the program so is not reported or graphed.

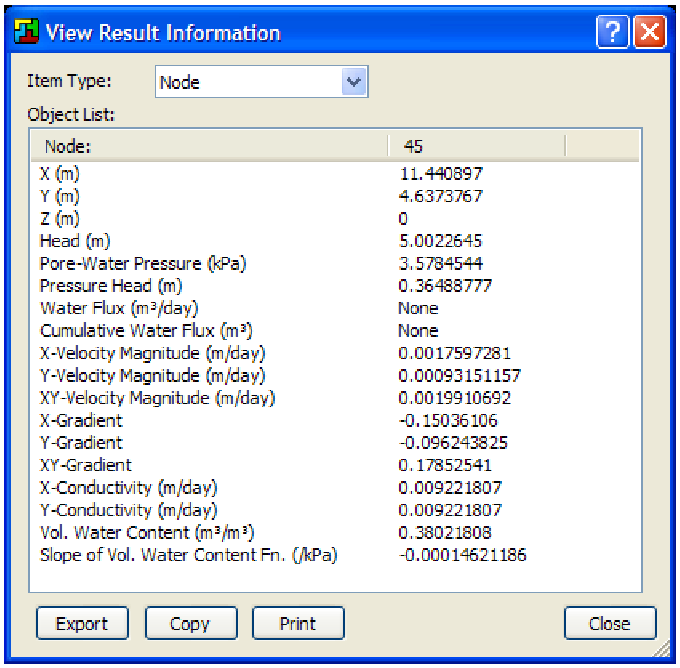


Figure 7-6 Illustration of "none" or "missing" data

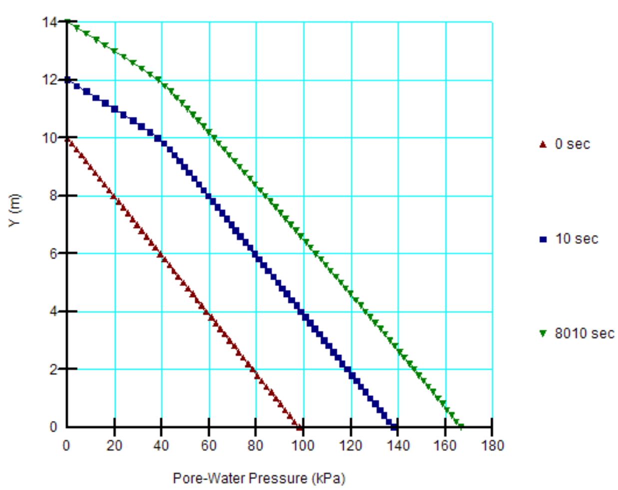


Figure 7-7 Graph showing how missing data is excluded and not printed as zero

7.5 Water table in AIR/W

In SEEP/W, the location of the water table is drawn in Contour along an isoline where the water pressure is zero. This is technically correct for the case where the air pressure is assumed to be zero but it is not correct when air pressures are considered. In SEEP/W with the AIR/W plug in, the location of the water table is actually where the matric pressure (Ua – Uw) is zero.

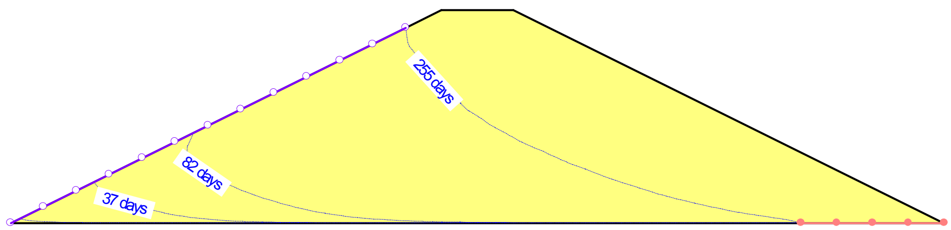


Figure 7-8 Water table (isolines) over time

7.6 Isolines

You can use the Draw Time Contours command to choose which parameter you want the water table calculation based on. It can be water pressure or matric suction. You can also choose to draw an isoline contour of any other parameter at an instance in time or over multiple times. If you draw an isoline at multiple time steps then you can not also view contour shading as it only exists for any instance in time. The isolines are a way to track a single value of a parameter as it changes over time… such as a water table.

7.7 Projecting Gauss point values to nodes

AIR/W performs contouring calculations based on parameter values at the nodes. Since the primary parameters, (air pressure, total head, water pressure, and pressure head), are computed at the nodes, these parameters can be contoured directly. However, secondary parameters, (velocity, gradient, conductivity, and volumetric air or water content), are computed at the element Gauss points and must therefore be projected to the nodes for contouring purposes.

In triangular elements, the Gauss point values are projected on the basis of a plane that passes through the three Gauss points. For one-point integration, the value at the Gauss point is also taken to be the value at the nodes (i.e., the Gauss point value is constant within the element).

In quadrilateral elements, the Gauss point values are projected using the interpolating functions. (For more information about interpolating functions, see the appendix). In equation form,

$$x = \langle N \rangle \{X\}$$

where:

x	=	the projected value outside the Gauss points at a local coordinate greater than 1.0,
<N>	=	a matrix of interpolating functions, and
{X}	=	the value of Gauss point variable.

The local coordinates at the element nodes are the reciprocal of the Gauss point local coordinates when forming the element characteristic matrix. Figure 7-9 is an example of the local coordinates at the element corner nodes when projecting outwards from the four Gauss points in the element. The value of 1.7320 is the reciprocal of the Gauss point coordinate 0.57735.

This projection technique can result in some over-shoot at the corner nodes when variation in the parameter values at the Gauss points is large. For example, consider that we wish to contour volumetric water content and that in some elements the water content at the Gauss points varies over the complete range of the volumetric water content function. Projecting such a large variation to the nodes can result in projected nodal water contents beyond the range of the volumetric water content function.

Extreme changes in the parameter values at the Gauss points within an element often indicate numerical difficulties (the over-shoot at the nodes being just a symptom of the problem). This over-shoot can potentially be reduced by a finer mesh discretization. Smaller elements within the same region will result in a smaller variation of parameter values within each element, therefore lowering the potential for encountering unrealistic projections.

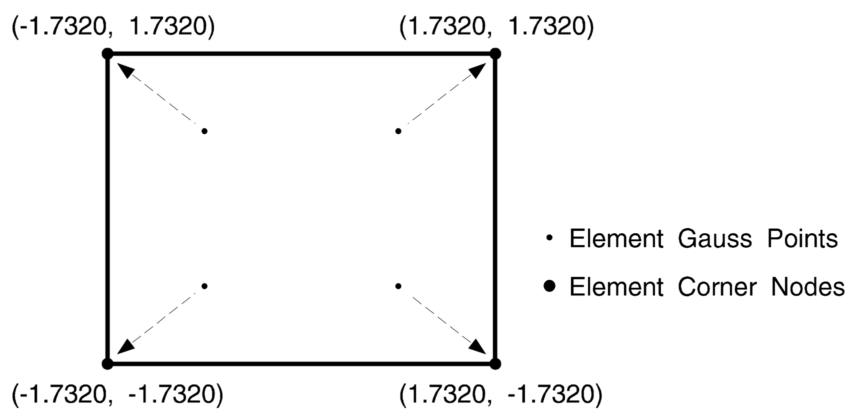


Figure 7-9 Local coordinates at the corner nodes of an element with four integration points

7.8 Contours

The power of using advanced graphical interfaces with finite element analysis is that the computer can quickly convert thousands of pieces of data into meaningful pictures. In a section above we introduced isolines and showed how their relative positions give an indication of the change in a parameter over time and space. In this section, we show that it is quite simple, and much more meaningful, to interpret parameters over space if we can view all values for a model at one time over the entire domain. GeoStudio has the power to let you contour any of the output data over your problem domain.

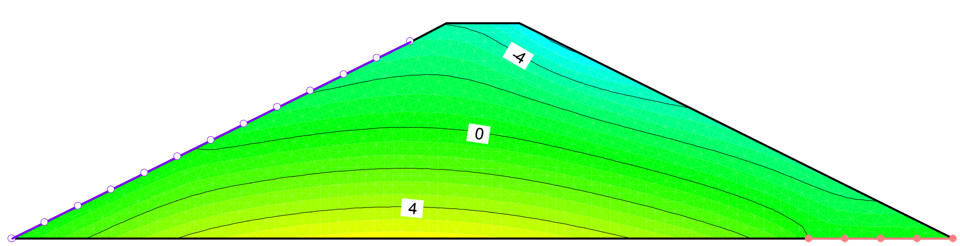


Figure 7-10 Contour of pressures at a fixed time

7.9 Animation in GeoStudio

Movie files (*.avi) can be created in GeoStudio to illustrate a physical process in a transient analysis. The first step in creating a movie is to define the contours and specify any View Preferences that need to be visible (e.g. flux vectors or the displaced mesh). The View Animation command is selected and the time steps and viewing area are defined. After saving the movie file to the appropriate location, GeoStudio joins together all of the individual images for each time step, creating a seamless animated movie.

7.10 Velocity vectors and flow paths

Calculating gradients and velocities

The specific discharges have slightly different meaning in the different analysis types. In seepage analysis, the volumetric flux within the finite element is often called a Darcy velocity (volume flow per unit area normal to direction of flow) with units of volume of water / cross sectional area of soil / time which reduces to length/time and is computed as

$$v_w = -\left[K_w\right]\left[B\right]\left\{H_w\right\}$$

In air flow, the specific discharge includes a density component as follows:

$$q_a = -\left[K_a\right]\left[B\right]\left\{\frac{P_a}{\gamma_{oa}}\right\} + \left[\frac{\rho_a}{\rho_{oa}}K_{ay}\right]i_y$$

The AIR/W velocity is actually the specific discharge, q, which is the total flux Q divided by the full cross-sectional area (voids and solids alike); it is not the actual speed with which the air moves between the soil particles. The actual microscopic velocity is:

$$v = \frac{q}{n}$$

where:

v	=	the average linear velocity,
q	=	the specific discharge, and
n	=	the soil porosity.

Velocity vectors

Velocity vectors are a useful way of seeing not only where the flow is occurring, but how much flow there is relative to other regions of the domain. An examples showing both water and air vectors is given in Figure 7-11. AIR/W uses the magnitude of the actual velocity in its calculation of how large to display the vector so that you have a visual representation of where the velocities are high or low. For each element, the average x-velocity and average y-velocity from the Gauss point velocity values are computed and then vectorally summed to obtain an average velocity vector for the element. This average velocity vector is plotted with the tail of the vector at the center of the element.

When displaying vectors, AIR/W finds the maximum velocity vector and draws it at the length specified in the Draw Vectors dialog box. All other vectors are drawn in proportion to the element velocity relative to the maximum velocity. For example, if the element velocity is one quarter of the maximum velocity, then the length of the velocity vector is one-quarter of the length specified in the Draw Vectors dialog box.

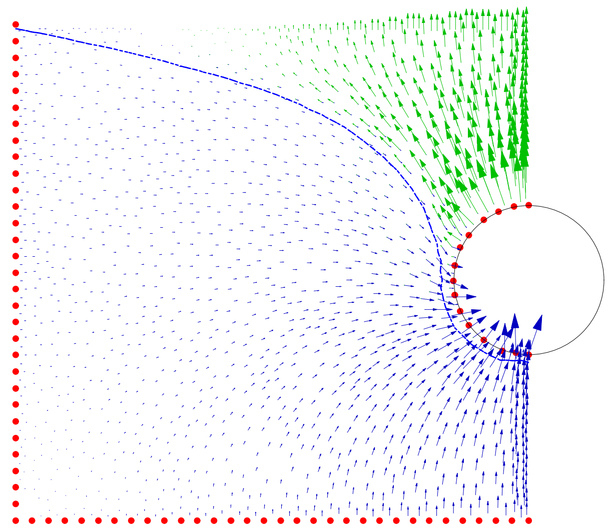


Figure 7-11 Simultaneous water and air vectors

You have the option of controlling the size of the vectors as they are displayed by controlling the magnification scale when you issue the command to create the vectors. Specifying a magnification value allows you to control the scale at which all vectors are drawn. When you type a value in the magnification edit box, the maximum length edit box is updated to display the length at which the maximum vector will be drawn. You can control the vector length either by specifying a magnification value or by specifying a maximum display length value. If you specify a length, the magnification value is computed by dividing the maximum length by the maximum velocity and adjusting the value for the scale of the page and engineering units.

7.11 Flux sections

SEEP/W with AIR/W has the ability to compute the instantaneous seepage or air volume rate that flows across a user-defined section for either a steady state or transient analysis. This is a very useful tool for isolating flow volumes to specific regions of interest and it can save you manually adding up individual nodal flows in the case of a drain or seepage face that is comprised of many nodes.

Flux section theory

This value can be computed from the nodal heads and the coefficients of the finite element equation. For example, consider a mesh with only one element, as illustrated in Figure 7-12. The objective is to compute the total flow across a vertical section of the element.

In the Theory chapter, the finite element form of the air mass flow equation is presented. It can be re-written in a simplified form with the flux value isolated on one side as follows:

$$[K]\{H\}+[M]\frac{\Delta H}{\Delta t} = \{Q\}$$

This form excludes the density and water pressure component for illustrative purposes. The full equation is used in the solver for computed flux data.

In a steady-state analysis, the storage term $[M]\dfrac{\Delta H}{\Delta t}$ becomes zero, and the equation can be reduced to:

$$[K]\{H\} = \{Q\}$$

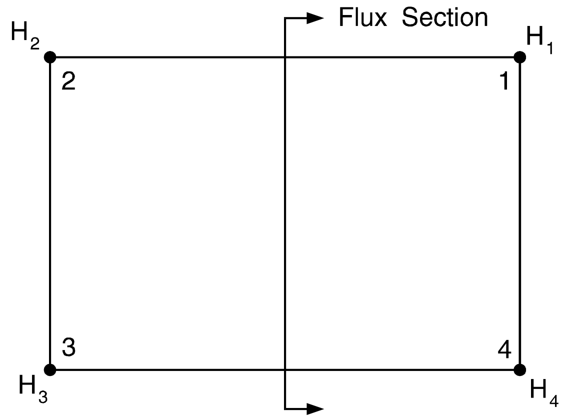


Figure 7-12 Illustration of a flux section across a single element

The global set of finite equations for one element is as follows:

Equation 7-1
$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}$$

From Darcy's Law , the total flow between two points is:

Equation 7-2
$$Q = k\,A\frac{\Delta H}{l}$$

The coefficients, c, in Equation 7-1 are a representation of $\dfrac{KA}{l}$ in Equation 7-2 Therefore, the flow from Node i to Node j is:

$$Q_{ij} = c_{ij}\left(H_i - H_j\right)$$

In a transient analysis, because of material storage, the calculation of the total flow quantity must include the storage effect. The change in flow quantity due to the storage term can be expressed as:

$$\frac{1}{\Delta t}\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{Bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta H_3 \\ \Delta H_4 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}$$

where $\Delta H_{1,2,3,4}$ etc. are the changes of total head at the various nodes between the start and the end of a time step. In general, the average change of total head from Node i to Node j can be expressed as:

$$\Delta H_{ij} = \frac{\Delta H_i + \Delta H_j}{2}$$

Therefore, the change in flow quantity from Node i to Node j due to a change in storage is:

$$Q_{ij} = m_{ij} \frac{\Delta H_{ij}}{\Delta t}$$

The total flow quantity from Node i to Node j for a transient analysis then becomes:

$$Q_{ij} = c_{ij} \left(H_i - H_j \right) + m_{ij} \frac{\Delta H_{ij}}{\Delta t}$$

The total flow quantity through the flux section shown in Figure 7-12 is:

$$Q = Q_{21+} + Q_{24} + Q_{31} + Q_{34}$$

The imaginary flow lines from one side of the section to the other side are known as subsections. AIR/W identifies all subsections across a user-defined flux section, computes the flow for each subsection, and then sums the subsection flows to obtain the total flow across the flux section.

Flux section application

Flux sections can be used in many ways, because they can be drawn any place across which you want to know the flux. You many want to check that an influx is equal to an out flux such as illustrated in Figure 7-13; or you may want to know the total drain flux as illustrated in Figure 7-14.

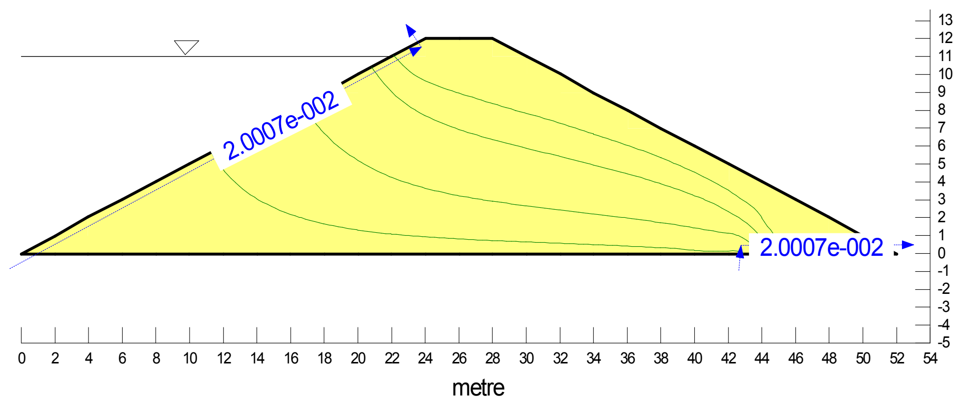


Figure 7-13 Flux section used to check balance of inflow and outflow

Flux sections do not have to be drawn as single straight lines. They can be made of continuous attached segments as illustrated in both figures above. When a multiple segmented flux section is drawn, the value of flux reported for the section applies to the entire section, not any individual segment.

The key point to note when defining a flux section is to make the flux section cross the sides of the elements and not the nodes of the elements. Also, if you want to check the flux around a closed loop as illustrated for the drain nodes in Figure 7-14, make sure the end of the flux section crosses over the tail of the first segment of the flux section.

Two words of caution: flux sections MUST be defined before you solve the problem, because the program needs to calculate the values during the solution sequence, not afterward. In addition, all flux values are reported as positive, which means direction is not taken into account. This is required because the sign of the flux value will depend on which way you draw the section. To avoid any misinterpretation, all flux section values are reported as positive, and then you can plot flux vectors in order to determine the direction of flow, if it is not obvious based on your problem definition.

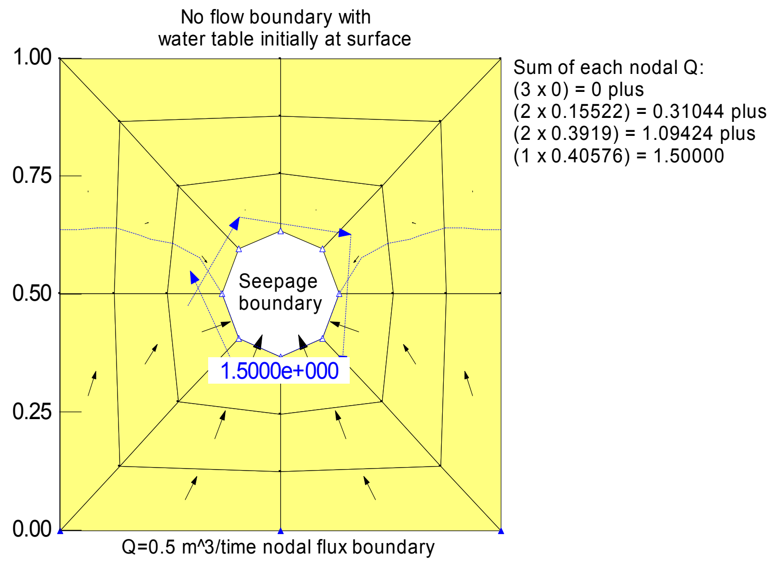


Figure 7-14 Flux section used around series of drain nodes to check flow

8 Modeling Tips and Tricks

8.1 Introduction

This chapter contains many useful hints about using the software and understanding what it does. READ THIS CHAPTER!

AIR/W is a powerful analytical tool, but it will only provide valid solutions if the boundary conditions, material properties, and time sequence are appropriately defined. It is your responsibility to properly define the problem parameters and ensure that the results produced are valid and reasonable.

This chapter presents some general modeling guidelines. The information presented is not an exhaustive statement on the "how-to" of modeling a contaminant transport problem. Instead, it is intended to provide suggestions on how you might model various conditions, as well as to outline the implications of certain modeling specifications.

There have been many occasions where GEO-SLOPE has been contacted by clients with questions about how the model behaves in response to changes in various parameters. If we do not know the answer, we conduct a numerical experiment to test what will happen. The first few sections of this chapter illustrate a few common examples of numerical experiments. You are strongly encouraged to learn why these types of simple tests are so powerful in testing how the program computes results BUT ALSO in enhancing your understanding of how the physical mechanisms of flow through porous medium occurs.

A numerical experiment is carried out by making a very simple finite element problem. It is useful to use a mesh that is one distance unit wide and one distance unit high. This makes hand calculating flux values very simple and they can easily be checked against the computed flux values. The following discussion illustrates how some simple numerical experiments have been carried out to test some simple, yet valid, questions.

When setting up these experiments, it is a good idea to input simple soil property functions. In most cases, two data points are sufficient to define the conductivity and storage function. Just as a reminder, give both functions some slope – don’t make them horizontal!

8.2 Problem engineering units

New in GeoStudio is the addition of units to all data input and output. The displayed units will be based on the set of units specified in the Set menu and any changes to units do not change the actual data. The units are present as an aide when setting up the model and considering output results.

Any system of units can be used for a seepage analysis; the only requirement is that you must be consistent. Fundamentally, you must select the units for length (geometry), time, and force. Once you have selected units for these parameters, all other units must be consistent. Table 8-1 presents some typical sets of consistent units.

Table 8-1 Consistent SI and Imperial units

Parameter	Symbol	SI Units	Imperial Units
Length	L	metres	feet
Time	t	seconds	hours
Force	F	kN	lbs
Pressure	F/L^2	kN/m^2	psf
Unit Weight of Water	F/L^3	kN/m^3	pcf
Conductivity	L/t	m/sec	ft/hr
Total / Pressure head	L	m	ft
Nodal Flux (Q)	L^3/t	m^3/sec	ft^3/hr
Boundary Flux (q)	L/t	m/sec	ft/hr
Flux Section	L^3/t	m^3/sec	ft^3/hr
Volume	L^3	m^3	ft^3
Density of air	M/ L^3	Kg/ m^3	Lb/ ft^3
Mass flux	M/t	Kg/sec	Lb/hr

The units of time are established once you select the units for hydraulic conductivity. The units of pressure are established once you select the unit weight of water. Generally, all units are defined by selecting the units of length for the problem geometry, units for hydraulic conductivity, and the units for the unit weight of water.

In summary, the key requirement is that the system of units be consistent.

8.3 Flux section location

Question: Does the location of a flux section within an element have any influence on the computed flux value?

Answer: No. The flux section value will be the same regardless of whether the section is drawn near the element edge or element middle. Figure 8-1 shows this to be the case and it is true for a transient and steady state solution.

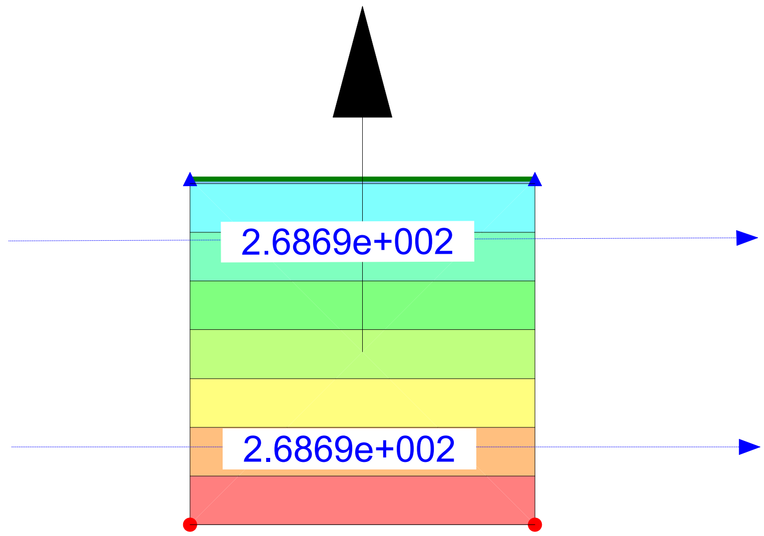


Figure 8-1 Test to check flux section locations

8.4 *Unit flux versus total flux?*

There are many people who are unsure of the difference between a unit flux and a total nodal flux. Do a simple test if you are unsure.

Question: How is a unit flux related to a total nodal flux in a 2D analysis?

Answer: The total nodal flux should be exactly equal to the unit flux multiplied by the total length of the element edges that contribute to that node.

In the figure below, a unit flux of -500 g / time / meter edge length has been applied to the top of the element. The top is a mass sink face which will let the contaminant out. The flux sections drawn in the element confirm that the total edge flux of -500 g / time has been converted by the solver into two equal total nodal fluxes of -250 g / time each. For such a simple mesh, it is also possible to use the View Node information option and click on each node to see the computed total flux at each node. The sum of the individual total nodal fluxes is the total flux across the element edge.

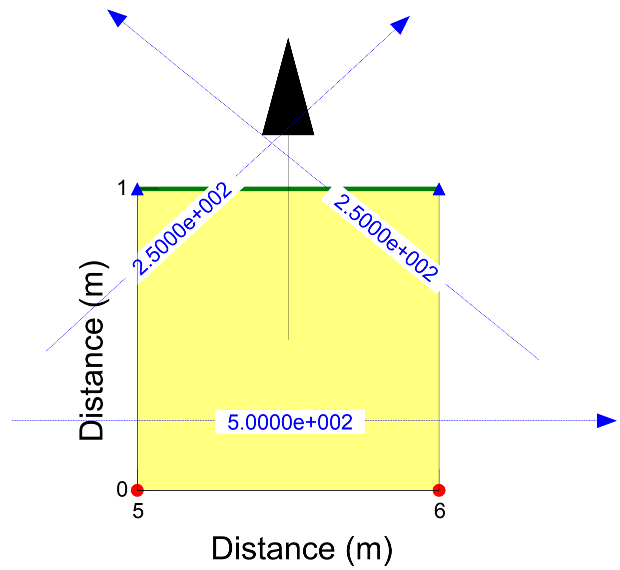


Figure 8-2 Test to compare unit flux and total flux

8.5 Staged construction

GeoStudio has the ability to do staged construction where new soil regions are added or removed. The key point to doing this is to know what the initial conditions are for soil regions that become newly active. Consider a new "lift" of soil in an embankment construction project that is carried out over time. When a new "lift" is placed, the model knows what the air and water pressures and temperatures are in the soil that existed previously because they were solved for in an earlier analysis. Before the new soil can be included in the next analysis, the model must know what its "state" is when it becomes active.

When you specify a soil material model, you have the option to specify an activation pressure or temperature. This value will be used for the soil when it first becomes an active part of the analysis. In this way, you can add soils and carry on an existing construction sequence.

9 Illustrative Examples

A variety of verification and illustrative examples has been developed and are available with the software. These examples can be useful for learning how to model various problems, particularly in the selection and application of boundary conditions. Each example comes with a PDF document that provides explanations on the problem setup, comments on modeling techniques and a commentary on interpreting the results. Verification examples are discussed in terms of closed-form solutions, published information and/or laboratory measurements.

All of the examples can be downloaded and installed from GEO-SLOPE's web site (www.geo-slope.com). Once installed, it is possible to search for a particular type of analysis on the GeoStudio desktop. Conversely, the search feature is available directly on the website. It should be noted that a product-specific search is possible (e.g. search for TEMP/W or SIGMA/W).

The GeoStudio example files can be reviewed using the **free** GeoStudio Viewer license.

10 Theory

This chapter presents the equations, procedures, and techniques used in the AIR/W formulation. AIR/W is not a stand-alone program and must be solved as part of SEEP/W and optionally TEMP/W. It is of value to be familiar with this information in order to use the software. An understanding of these concepts will be of great benefit in applying the software, resolving difficulties, and judging the acceptability of the results.

10.1 Simultaneous thermally coupled air and water flow

Unsaturated soil behavior is dependent on the stress state in the soil. In order to know the correct stress state, it is necessary to know the total stress, the pore-water and pore-air pressures. Traditionally it is assumed that the pore-air pressure is atmospheric. In many cases, this is not a good assumption and it is necessary to solve the coupled pore-air and pore-water conservation of mass equations. SEEP/W has been modified to include the AIR/W option to solve for compressible air flow in response to hydraulic, pneumatic or thermal boundary conditions.

This new formulation has many applications including: soil vapor extraction analysis, tunneling, infiltration into dry soils, density dependent air cooling of rock piles, dissipation of excess heat in mine waste rock dumps, generation and dissipation of excess pore-air and pore-water pressures in low permeability dam cores during reservoir filling etc. The AIR/W module can be solved for the isothermal case, or, if launched from within TEMP/W, it can consider changes in air density due to temperature and, in response, TEMP/W can compute the convective heat transfer associated with both moving water and moving air.

10.2 Conservation of mass (general form):

For one dimensional compressible flow the general form of the conservation of mass equation is:

$$\frac{\partial\left(\rho_f \theta_f\right)}{\partial t} = \frac{\partial}{\partial y}\left[-\left(\rho_f K_f\right)\frac{\partial H_f}{\partial y}\right] + Q_f$$

Where the subscript 'f' is for any fluid and H is the total energy potential comprised of both pressure and elevation potentials, where the elevation potential is scaled by the relative density as follows:

$$H_f = \left(\frac{P_f}{\gamma_{of}} + \frac{\rho_f}{\rho_{of}} y\right)$$

Water conservation of mass:

The left side of the general equation can be expanded using the chain rule as follows:

$$\frac{\partial\left(\rho_w \theta_w\right)}{\partial t} = \theta_w \frac{\partial \rho_w}{\partial t} + \rho_w \frac{\partial \theta_w}{\partial t}$$

The right hand side in the general equation can also be expanded using the chain rule as follows:

$$\frac{\partial}{\partial y}\left[(\rho_w K_w)\frac{\partial H_w}{\partial y}\right]=K_w\frac{\partial H_w}{\partial y}\frac{\partial \rho_w}{\partial y}+\rho_w\frac{\partial}{\partial y}\left[K_w\frac{\partial H_w}{\partial y}\right]$$

Since water is assumed incompressible, the time derivative AND spatial derivative of density terms in the above two equations are zero. The remaining density variable can be cancelled from all term so that the mass balance equation for water becomes a volume balance equation as follows:

$$\frac{\partial(\theta_w)}{\partial t}=\frac{\partial}{\partial y}\left[K_w\frac{\partial(H_w)}{\partial y}\right]+Q_w$$

We can introduce matric suction as the difference in capillary pressure between air and water as follows:

$$\frac{\partial \psi}{\partial t}=-\frac{\partial(P_a-P_w)}{\partial t}=\frac{\partial\left[\gamma_w H_w-P_a\right]}{\partial t}$$

The left side of the water flow equation can be changed to be the time derivative of total head by using the slope of the water content vs matric suction relationship as follows:

$$\frac{\partial \theta_w}{\partial t}=\frac{\partial \theta_w}{\partial \psi}\frac{\partial \psi}{\partial t}=m_w\frac{\partial \psi}{\partial t}=m_w\frac{\partial\left[\gamma_w H_w-P_a\right]}{\partial t}$$

The time derivative of matric suction excludes the elevation term above as elevation is constant with time. The air pressure term can be moved to the right side.

So the final water flow equation becomes…

$$m_w\gamma_w\frac{\partial H_w}{\partial t}=\frac{\partial}{\partial y}\left[K_w\frac{\partial H_w}{\partial y}\right]+m_w\frac{\partial P_a}{\partial t}+Q_w$$

where Qw has units of length per time.

Air conservation of mass:

If we take the general mass balance equation for air and apply the chain rule to expand the time derivate of the left term we arrive at:

$$\frac{\partial(\rho_a\theta_a)}{\partial t}=\theta_a\frac{\partial \rho_a}{\partial t}+\rho_a\frac{\partial \theta_a}{\partial t}=\frac{\partial}{\partial y}\left[(\rho_a K_a)\frac{\partial H_a}{\partial y}\right]+Q_a$$

The time derivative of density can also be expanded using the chain rule and the ideal gas law as follows:

$$\theta_a\frac{\partial \rho_a}{\partial t}=\frac{\theta_a}{RT}\frac{\partial P_a}{\partial t}+\frac{\theta_a P_a}{R}\frac{\partial\left(\frac{1}{T}\right)}{\partial t}$$

Where R = 287 J/(kg K) for dry air.

The change in air volume over time is the negative of the change in water volume over time and can be related to the change in matric suction as follows:

$$\rho_a \frac{\partial \theta_a}{\partial t} = -\rho_a \frac{\partial \theta_w}{\partial t} = \rho_a m_w \frac{\partial \psi}{\partial t}$$

Combining the previous two equations into the general air flow equation results in

$$\frac{\theta_a}{RT}\frac{\partial P_a}{\partial t} + \frac{\theta_a P_a}{R}\frac{\partial\left(\frac{1}{T}\right)}{\partial t} + \rho_a m_w \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial y}\left[\left(\rho_a K_a\right)\frac{\partial\left(\frac{P_a}{\gamma_{oa}} + \frac{\rho_a}{\rho_{oa}} y\right)}{\partial y}\right]$$

The air source/sink term on the right, Qa, has been removed from the model as it is practically difficult to inject a known mass of air into soil.

This equation can be rearranged as follows:

$$\frac{\theta_a}{RT}\frac{\partial P_a}{\partial t} = \frac{\partial}{\partial y}\left[\frac{\rho_a K_a}{\gamma_{oa}}\frac{\partial P_a}{\partial y} + \frac{\rho_a{}^2 K_a}{\rho_{oa}}\right] - \frac{\theta_a P_a}{R}\frac{\partial\left(\frac{1}{T}\right)}{\partial t} - \rho_a m_w \frac{\partial \psi}{\partial t}$$

and the change in matric suction term on the right can be expanded into air and water pressures as follows:

$$\frac{\theta_a}{RT}\frac{\partial P_a}{\partial t} = \frac{\partial}{\partial y}\left[\frac{\rho_a K_a}{\gamma_{oa}}\frac{\partial P_a}{\partial y} + \frac{\rho_a{}^2 K_a}{\rho_{oa}}\right] - \frac{\theta_a P_a}{R}\frac{\partial\left(\frac{1}{T}\right)}{\partial t} - \rho_a m_w \frac{\partial\left(P_a - P_w\right)}{\partial t}$$

The expanded Pa term on the right can be combined with the Pa term on the left as follows and the Pw can be put in terms of hydraulic head as follows:

$$\left(\frac{\theta_a}{RT} + \rho_a m_w\right)\frac{\partial P_a}{\partial t} = \frac{\partial}{\partial y}\left[\frac{\rho_a K_a}{\gamma_{oa}}\frac{\partial P_a}{\partial y} + \frac{\rho_a{}^2 K_a}{\rho_{oa}}\right] - \frac{\theta_a P_a}{R}\frac{\partial\left(\frac{1}{T}\right)}{\partial t} + \rho_a \gamma_w m_w \frac{\partial H_w}{\partial t}$$

10.3 Thermal energy balance

If air density is a function of temperature then it is necessary to solve the energy balance equation. The energy balance equation, with phase change in the water phase, is

$$(\rho_s c_{ps} + L\theta_w \frac{\partial \theta_u}{\partial T})\frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left[K_t \frac{\partial T}{\partial y}\right] + c_{pa}\frac{\partial(\dot{m}_a T)}{\partial y} + \rho_w c_{pw}\frac{\partial(q_w T)}{\partial y} + Q$$

Where

$\rho_s c_{ps}$ = volumetric heat capacity of soil,

$c_{pa/w}$ = mass specific heat of air or water,

$\dot{m}_a$ = mass flow rate of air,

$\frac{\partial \theta_u}{\partial T}$ = the slope of the unfrozen water content function,

q_w = the specific discharge (Darcy velocity) of water, and

L = latent heat of water.

10.4 Solution scheme

We now have two equations and two unknowns, namely total water head, Hw, and air pressure Pa. If we rearrange the two equations to isolate time derivative of the dependent variables on the left side of each equation, we arrive at:

For water…

$$m_w \gamma_w \frac{\partial H_w}{\partial t} = \frac{\partial}{\partial y}\left[K_w \frac{\partial H_w}{\partial y} \right] + m_w \frac{\partial P_a}{\partial t} + Q_w$$

On the first iteration, the air pressure term is not known. For all other iterations, it can be obtained from the solution at the previous iteration.

And for air…

$$\left(\frac{\theta_a}{RT} + \rho_a m_w \right) \frac{\partial P_a}{\partial t} = \frac{\partial}{\partial y}\left[\frac{\rho_a K_a}{\gamma_{oa}} \frac{\partial P_a}{\partial y} + \frac{\rho_a{}^2 K_a}{\rho_{oa}} \right] - \left[\frac{\theta_a P_a}{R} \frac{\partial \left(\frac{1}{T} \right)}{\partial t} \right] + \left[\rho_a \gamma_w m_w \frac{\partial H_w}{\partial t} \right]$$

The second and third and last terms on the right side can be explicitly applied as sources or sinks as follows: If SEEP/W is coupled with TEMP/W, we can obtain the second term on the right in the above equation which represents flow in response to thermally induced density changes. We can also use the actual temperature value in all terms where T appears. If SEEP/W is solved independently of TEMP/W, the second term will be zero and we will assume room temperature for all T terms. The third term in the above equation on the right side is known from the previously solved seepage equation at each iteration.

10.5 Air and water and energy equations in finite element form

For water:

$$(\Delta t[K] + [M])\{H_1\} = \Delta t\{Q\} + [M]\{H_0\} + [M_{a1}]\{\frac{\Delta P_a}{\Delta t}\}$$

Where

$$[M] = \tau \int_A \gamma_w m_w \left[N\right]^T \left[N\right] dA$$

$$[M_{a1}] = \tau \int_A m_w \left[N\right]^T \left[N\right] dA$$

$$[K] = \tau \int_A \left[B\right]^T \left[K_w\right]\left[B\right] dA$$

For air:

$$(\Delta t[K_a] + [M_{a2}])\{P_1\} = [M_{a2}]\{P_0\} + \Delta t[K2] - [M_{a3}]\{\frac{\Delta \frac{1}{T}}{\Delta t}\} + [M_{a4}]\{\frac{\Delta H_w}{\Delta t}\}$$

Where

$$[M_{a2}] = \tau \int_A \left(\frac{\theta_a}{RT} + \rho_a m_w \right) \left[N\right]^T \left[N\right] dA$$

$$[K_a] = \tau \int_A \frac{\rho_a}{\gamma_{oa}} \left[B\right]^T \left[K_a\right]\left[B\right] dA$$

$$[K_2] = \tau \int_A \frac{\rho_a^{\,2}}{\rho_{oa}} \left[B\right]^T \left[K_a\right] dA$$

$$[M_{a3}] = \tau \int_A \frac{\theta_a P_a}{R} \left[N\right]^T \left[N\right] dA$$

$$[M_{a4}] = \tau \int_A \rho_a \gamma_w m_w \left[N\right]^T \left[N\right] dA$$

For energy:

$$(\Delta t[K_t] + [M_t])\{T_1\} = \Delta t\{Q\} + [M_t]\{T_0\}$$

Where

$$[M_t] = \tau \int_A \left(\rho_s c_{ps} + L\theta_w \frac{\partial \theta_u}{\partial T} \right) \left[N\right]^T \left[N\right] dA$$

$$[K_t] = \tau \int_A \left[B\right]^T \left[K_a\right]\left[B\right] dA + \tau \int_A \rho_a c_{pa} \left[v_a\right]\left[B\right] dA + \tau \int_A \rho_w c_{pw} \left[v_w\right]\left[B\right] dA$$

10.6 Elemental specific discharges, mass flow rates, and velocities

The specific discharges have slightly different meaning in the different analysis types. In seepage analysis, the volumetric flux within the finite element is often called a Darcy velocity (volume flow per unit area normal to direction of flow) with units of volume of water / cross sectional area of soil / time which reduces to length/time and is computed as

$$q_w = -\left[K_w\right]\left[B\right]\left\{H_w\right\}$$

In air flow, the specific discharge includes a density component as follows:

$$q_a = -[K_a][B]\left\{\frac{P_a}{\gamma_{oa}}\right\} + \left[\frac{\rho_a}{\rho_{oa}} K_{ay}\right] i_y$$

Where as a mass flow it has units of mass/time.

10.7 Element equation assembly

Details regarding assembly of the element matrices and equations are included in the theory chapter of the SEEP/W and TEMP/W engineering books.

References

Aubertin, M., Mbonimpa, M., Bussière, B. and Chapuis, R.P. 2003. A model to predict the water retention curve from basic geotechnical properties. Canadian Geotechnical Journal, 40(6): 1104-1122 (2003)

Bathe, K-J., 1982. Finite Element Procedures in Engineering Analysis. Prentice-Hall.

Black, P., and Tice, A. 1989. Comparison of Soil Freezing Curve and Soil Water Curve Data for Windsor Sandy Loam. Water Resources Research 25(10):2205-2210.

Brooks, R.H., and Corey, J.C. 1966. Properties of Porous Media Affecting Fluid Flow. ASCE Journal, Irrigating and Drainage Division.

Farouki, O.T. 1981. Thermal Properties of Soils. CRREL Monograph 81-1. U.S. Army Corps of Engineers, Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire.

Fredlund, D.G. and Rahardjo, H., 1993. Soil Mechanics for Unsaturated Soils. John Wiley & Sons, Inc.

Fredlund, D. G., and Anqing Xing. 1994. Equations for the soil-water characteristic curve. Canadian Geotechnical Journal. Vol. 31, pp: 521-532.

Freeze, R.A., and Cherry, J.A., 1979. Groundwater. Prentice-Hall.

Green, R.E. and Corey, J.C., 1971. Calculation of Hydraulic Conductivity: A Further Evaluation of Some Predictive Methods. Soil Science Society of America Proceedings, Vol. 35, pp. 3-8.

Harlan, R.L., and Nixon, J.F., 1978. Ground Thermal Regime. Geotechnical Engineering For Cold Regions, Edited by O.B. Andersland and D.M. Anderson. McGraw-Hill Inc.

Kunze, R.J., Vehara, G., and Graham, K., 1968. Factors Important in the Calculation of Hydraulic Conductivity. Soil Science of America Proceedings, Soil Physics, Vol. 32.

Marshall, T.J., 1958. A Relation Between Permeability and Size Distribution of Pores. Soil Science of America Journal, Vol. 9.

Millington, R.J., and Quirk, J.P., 1961. Permeability of Porous Solids. Transaction of the Faraday Society, Vol. 57, pp. 1200 1207.

Newman, G. P. 1995. Heat and Mass Transfer of Unsaturated Soils During Freezing. M.Sc., Thesis, Univ. of Sask.

Segerlind, L.J., 1984. Applied Finite Element Analysis. John Wiley and Sons.

Johansen, O., 1975. Thermal Conductivity of Soils. Ph.D. thesis. Trondheim, Norway. (CRREL Draft Translation 637, 1977), ADA044002.

Johnston, G.H., Ladanyi, B., Morgenstern, N.R., and Penner, E., 1981. Engineering Characteristics of Frozen and Thawing Soils. Permafrost Engineering Design and Construction. Edited by Johnston, G.H. John Wiley & Sons.

van Genuchten, M. Th. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of America Journal 44:892-898.

Index